HISTORICAL PERSPECTIVES

AL-KARKHI: MATHEMATICIAN

Abu Bekr Mohammed Ibn Al-Hosain died in Bagdad in 1029 AD. His date and place of birth are less certain; he is variously known in near-contemporary sources as Al-Karkhi (possibly from Kharkh, near Bagdad) or as Al-Kardji (possibly after Kadj in modern Iran). It is certain, however, that most of his working life was spent in Bagdad, though he apparently spent some time near the end of his life in the 'mountain countries' near the Caspian Sea.

Al-Karkhi produced seminal work in geometry, algebra and arithmetic; in particular he contributed to the establishment of algebra as a discipline in its own right. free from geometrical interpretations but instead applying the manipulations of arithmetic to unknown, generalised, quantities.

It is clear from references in the works of his immediate successors that Al-Karkhi wrote a number of books that were subsequently lost. Among those that survive, two of the most famous are *Al-Kafi fi Al - Hisab* (Essentials of Arithmetic) and *Al-Fakhi*, a treatise on algebra, with special reference to Diophantine equations. *Al-Kafi fi Al - Hisab* is mainly a handbook on computation. The mathenatical formulae, as in all Al-Karkhi's surviving work, are expressed in words. Numerical notation was known to mathematicians of that time — Al-Karkhi himself wrote a treatise, since lost, on 'Indian Computation', as such notation was then known. But it was not yet in general use, and Al-Karkhi almost certainly wished his work to be accessible to merchants and others outside the ranks or professional mathematicians.

The treatise *Al-Fakhri* is named after the grand vizier of Bagdad, a personal friend of Al-Karkhi. The Arithmetic of Diophantus had been translated into Arabic by then, and Al-Karkhi reviews many of the problems contained in this earlier work. These Diophantine equations contain more unknowns than there are equations and are usually solved in terms of sets of positive whole numbers that satisfy them. Al-Karkhi extended and generalized these expressions. introducing a systematic study of algebraic exponents at the same time. He first stated such rules as $(x^n)^m = x^{nxm}$ and $x^n x\, x^m = {}^{n+m}$. He derived whole number solutions to the equation $x^3 + y^3 = z^2$, the simplest being ($x=1$, $y=2$, $z=3$), using a substitution method that can be generalized for equations of the form $ax^n + by^n = cz^{n-1}$. He stated, and proved, that $x^3+y^3 = z^3$ can have no positive integer solution for x, y and z. His proof has since been lost. The generalization, that $x^n + y^n = z^n$ has no integer solutions for $n<2$, has become known as 'Fermat's Last Theorem" because the French mathematician Pierre Fermat claimed in a famous marginal note to have a proof for all values of n. That proof has also been lost, and although proofs for many individual values of n have since been found no mathematician has suceeded in proving the general case since. Al-Karkhi dealt also with equations of the type $ax^{2p}+bx^p+c=0$, showing how they may be reduced to quadratics. In the field of algebraic surds Al-Karkhi showed, for example, that $\sqrt{8}+\sqrt{18}=\sqrt{50}$ and $\sqrt[3]{54}-\sqrt[3]{2} = \sqrt[3]{16}$. Binominal expressions are also considered in Al-Karkhi's work, and it is clear that he know and used the triangular array of binominal coefficients:

$$\begin{array}{ccccccccc} & & & & 1 & & & & \\ & & & 1 & & 1 & & & \\ & & 1 & & 2 & & 1 & & \\ & 1 & & 3 & & 3 & & 1 & \\ 1 & & 4 & & 6 & & 4 & & 1 \end{array}$$

often known as "Pascal's Triangle" though it predates Pascal by nearly a millenium:

Al-Karkhi also know the formula for the sum of the first n natural numbers:

$$\sum_{i=1}^{n} i = \tfrac{1}{2}n(n+1)$$

and derived and proved the relationships for the square and cubes of the same sequence:

$$\sum_{i=1}^{n} i^2 = \tfrac{1}{3}\,(2n+1)\sum_{i=1}^{n} i$$

$$\sum_{i=1}^{n} i^3 = \left(\sum_{i=1}^{n} i\right)^2$$

The last relation is so pleasingly simple that it would be interesting to know if it applies to any other powers of the first in natural numbers, i.e. if $(\Sigma i^k)^l = (\Sigma i^r)^s$ for any other integral values of k, l, r, and s, $k \neq 1$ and $r \neq 3$, and if Al-Karkhi or his contempories and successors ever considered this question.

Al-Karkhi was not merely a theoretician. He wrote on a number of subjects relating to engineering and surveying, including a treatise on surveying instruments and methods. His later works, while he was away from Bagdad, relate to such subjects as wells, qanats (or underground water channels) and aquaducts. He was concerned with the legal and sociological aspects of such projects, and in the question of how they could best serve the interests of the people in the regions where they were constructed. He know and used the work of the Greeks and others before him but extended it immensely, as others were to build upon his work in turn.

Brief historical perspectives of significant Arab scientists constitute a regular feature of the ARABIAN JOURNAL FOR SCIENCE AND ENGINEERING. The AJSE staff wishes to acknowledge gratefully the assistance of Dr. Ali A. Daffa', Chairman of the Mathematics Department at the University of Petroleum and Minerals, in providing this commentary on Al-Karkhi.