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Multiphasic Lactation Curve for Holstein Cows in the Kingdom of Saudi Arabia

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Abstract. Multiphasic function was used to describe the lactation curve of monthly test- day average of 12020 Holstein Friesian milk records in the Kingdom of Saudi Arabia. The data represented four lactations with 3793, 3500, 2470 and 2257 records for first, second, third and fourth or later lactation, respectively.

Records with lactation period less than 100 or greater than 450 days were excluded from the analysis. Monophasic function exhibited inadequate fitting for the data. The curve was flat with no peak and no steepest ascent at the early stage of lactation (15 day through 105 day). Diphasic function showed close fitting with actual data, but some residual (actual - predicted) existed, particularly at the first and second month of lactation. Triphasic function with nine parameters gave the best fit through the whole lactation. The contribution of each phase of triphasic function was different for the four lactations.

Introduction

The multiphasic function proposed by Koops [1] and Koops *et al.* [2] for describing the growth curve for mice has been applied by other researchers to describe the lactation curve of dairy cattle [3-5]. Multiphasic function ranked best among other functions of the lactation curve [6-7]. Moreover, multiphasic function has some advantages over other functions used to describe the lactation curve , including: (1) Fitting monthly test yield results in smaller and random errors as measured by small autocorrelation among residuals [2]. (2) Exact total of 305 day milk yield can be predicted [3]. (3) The multiphasic function has more than one phase and power transformation can be used with phase one to ensure appropriate modeling for the early portion of the lactation curve. (4) Reparameterization of multiphasic model reduces the collinearity between parameters. (5) The multiphasic function is appropriate to model multiple peaks, a situation occurring after injection of bovine somatotropic hormone [5].

The objectives of this study were two folds: Fitting monthly test- day milking data using multiphasic function as a mathematical model to describe the lactation curve and using goodness of fit measures to determine the appropriateness of each function in describing the lactation curve of Holstein cows.

Material and Methods

The data used in this study was based on 12020 monthly test-day milk records collected between 1980 to 1992 from six dairy farms located in the central region of the Kingdom of Saudi Arabia and having similar management conditions. The number of records for first, second, third and fourth or later lactation were 3793, 3500, 2470 and 2257, respectively.

Records with lactation periods less than 100 or greater than 450 days were excluded from the analysis. Holstein dairy cows in Saudi Arabia are characterized by a lactation period greater than 305 days due to their relatively low rate conception (45%) as reported by Salah and Mogawer[8].

Frequency distribution has shown that 73% of the records have lactation period greater than 305 days . Fifteen monthly test-day records (kg milk/day) were taken at 30 day intervals starting at day 15 after calving. Means of 15 monthly test-day yields for the overall data and for each parity were computed and used as input data to fit monophasic, diphasic and triphasic functions .

Model: The multiphasic function based on the sum of logistic functions as described by Grossman and Koops [3] is as follows :

$$Y_t = \sum_{i=1}^{n} \left[a_i b_i \left(1 - \tanh^2\left(b_i\left(t - c_i\right)\right)\right)\right] \quad (1)$$

where

Y_t = milk yield at time t (days).

n = number of phases.

tanh = the hyerbolic tangent.

a_i is half asymptotic total yield (kg),

b_i is the rate of yield relative to a_i (day^{-1}).

c_i is time of peak yield (day) for each phase.

The cumulative yield from 0 to 305 -d milk yield can be computed as:

$$MY_{305} = \sum_{i=1}^{n} \left\{ a_i \left(\tanh\left(b_i \left(305 - c_i \right) \right) - \tanh\left(b_i \left(0 - c_i \right) \right) \right) \right\} \quad (2)$$

Functions of parameters for three phases were:

initial yield computed from equation (1) at t = 0 ,

peak yield was represented by a_i b_i,

Duration, defined as the period in days required to attain about 75% of asymptotic total yield during that phase is computed as $2b^{-1}$.

Marquardt's method of nonlinear regression [PROC NLIN using Marquardt, SAS [9] was used to estimate the parameters because Marquardt is equivalent to performing a series of ridge regression which correct for colinearity or near singularity problems that arise from the correlation between the parameters of the lactation curve as given by Batts and Watts [10, p.81].

Residual mean square (MSE) and Durbin Watson statistics (D), a measure of first-order autocorrelation of residuals [11, pp.199-201]), were used to measure goodness of fit of the multiphasic model.

Results and Disucssion

Fitting multiphasic function by non-linear regression model resulted in estimating three parameters for monophasic, six for diphasic and nine for triphasic functions (Table 1). These parameters were inserted in function (1) to estimate the predicted value for each point of the lactation curve and in function (2) to compute 305-milk yield. Initial milk yield, peak yield, time of peak and the duration of each phase were also computed as functions of the estimated parameters.

Actual and predicted values of the fifteen loci of the lactation curve are given in Table 2. The magnitude of the difference between actual and predicted values (residual) of monophasic functions reflects the inadequacy of fitting the data. The curve of monophasic function (Fig.1) is flat with no steep ascent at early stage of lactation (15 days through 105 days) and consequently no distinct peak exists. Moreover, the monophasic function overpredicted the actual test-day milk at the first month of lactation,followed by underprediction from 45 day through 135 day and overprediction from 165 day through 315 day and from 357 day through the end of lactation.

Table 1. Lactation parameters for mono, di and triphasic functions for different lactations and overall data

Monophasic

	a_1	b_1	c_1
Overall	10724.9	.0026	38.4

Diphasic

Lactation	a_1	b_1	c_1	a_2	b_2	c_2
Overall	1615.5	.0101	68.0	5850.8	.0031	227.1
1 st	284.1	.0271	69.0	7612.8	.0028	160.5
2 nd	1354.6	.0115	67.7	6197.1	.0032	204.4
3 rd	4943.1	.0028	315.2	3048.3	.0081	72.6
4 th	2850.9	.0084	71.2	4767.3	.0029	271.0

Triphasic

Lactation	a_1	b_1	c_1	a_2	b_2	c_2	a_3	b_3	c_3
Overall	344.8	.0267	48.5	4610.8	.0029	337.0	2434.9	.0073	102.5
1 st	-296.7	.0295	64.0	7702.9	.0026	172.3	188.6	.0153	143.9
2 nd	339.1	.0295	50.0	47.0	.0029	335.8	2571.4	.0072	103.3
3 rd	359.3	.0283	47.8	4542.8	.0028	432.8	3252.1	.0073	102.2
4 th	495.9	.0227	47.9	4065.1	.0027	435.4	3246.2	.0071	107.3

a_i = a half asymptotic total yield, b_i = rate of yield relative to a_i, c_i = time of peak

Table 2. Actual (A) and predicted value(p)[1] of multiphasic function for overall data and for different lactations

	Overall				1st		2nd		3rd		4th	
Time	**A**	**Mon**	**Di**	**Tri**	**A**	**Di(p)**	**A**	**Di(p)**	**A**	**Di(p)**	**A**	**Di(p)**
15	22.7	27.9	24.5	22.8	18.9	19.7	22.9	25.0	25.0	27.3	25.7	27.7
45	31.2	28.0	28.7	31.1	24.9	24.5	32.7	29.9	34.9	31.6	34.8	32.1
75	30.5	27.7	30.7	30.7	27.7	27.8	31.6	32.1	33.8	33.7	34.2	34.1
105	28.7	27.1	29.8	28.2	24.0	24.3	29.5	30.8	31.7	33.0	32.1	33.2
135	26.6	26.3	27.2	26.9	23.1	22.2	27.3	27.7	29.0	30.2	29.1	30.0
165	25.0	25.1	24.4	25.2	22.5	21.2	25.8	24.8	26.8	26.4	26.1	26.1
195	23.1	23.8	22.1	23.1	21.7	21.3	23.8	22.6	24.0	22.9	23.4	22.5
225	21.1	22.3	20.4	20.9	20.1	20.7	21.8	21.1	21.0	20.2	20.8	19.7
255	19.2	20.6	19.2	19.1	19.5	20.0	19.7	20.0	18.4	18.2	18.1	17.6
285	17.5	19.0	18.1	17.6	18.5	19.0	18.0	18.8	16.1	16.8	15.4	16.1
315	16.3	17.3	17.1	16.4	17.3	17.9	16.6	17.6	15.2	15.8	14.3	15.0
345	15.6	15.6	16.0	15.5	16.8	16.6	15.6	16.3	14.8	15.0	13.5	14.0
375	14.6	14.1	14.8	14.7	15.4	15.2	14.8	14.9	14.0	14.2	12.7	13.1
405	13.7	12.5	13.6	13.9	13.4	13.8	13.6	13.5	13.2	13.5	12.3	12.2
435	13.2	11.1	12.3	13.0	13.4	12.4	13.4	12.0	13.5	12.7	12.1	11.3

[1] Predicted and actual values are almost the same for triphasic function

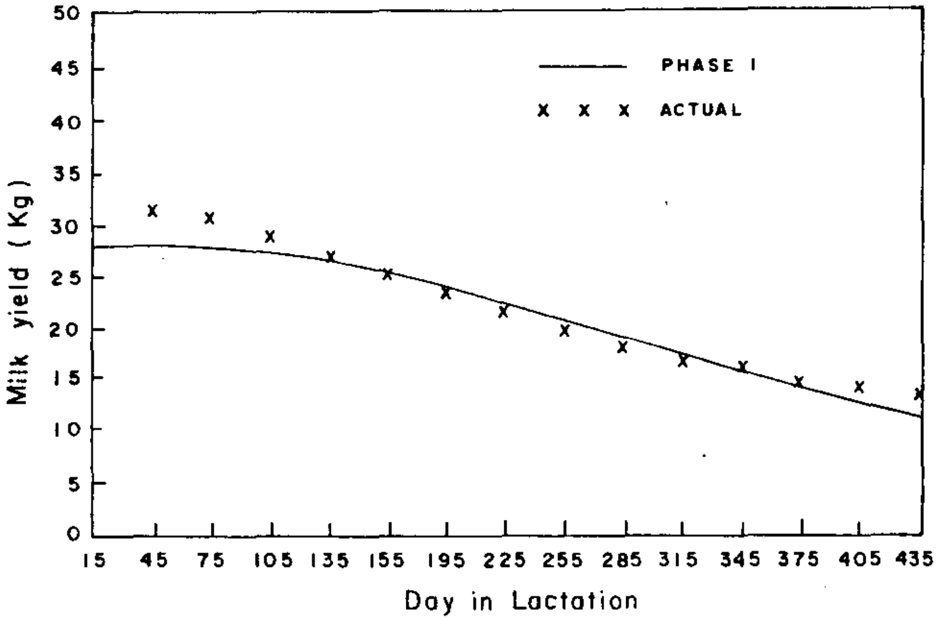

Fig. 1. Monophaic lactation curve and overall actual data.

Diphasic function showed some improvement in fitting the data since the predicted values were more adjacent to the actual fifteen points of the curve.Yet, diphasic function is not the most accurate function to represent the actual data, since some residual (actual-predicted) existed, particularly at the early stage of lactation i.e during first and second months (Fig. 2).

Triphasic function with nine parameters gave the best fit, minimum residual, because the actual values imposed exactly on the triphasic curve through the whole lactation (Figs. 3-7).

Grossman and Koops [3] concluded that two phases were sufficient to describe the lactation curve for their data. On the other hand , De Boer *et al.* [4] found that curves for monthly test day milk records were fitted accurately with a triphasic function.

Functions of parameters are given in Tables 3 and 4. These functions are important in describing the components of the lactation curve (initial yield, peak time, peak yield and duration), representing the differences among different phases of the lactation curve and exhibiting discrepancies between different curves of different classes like parities.

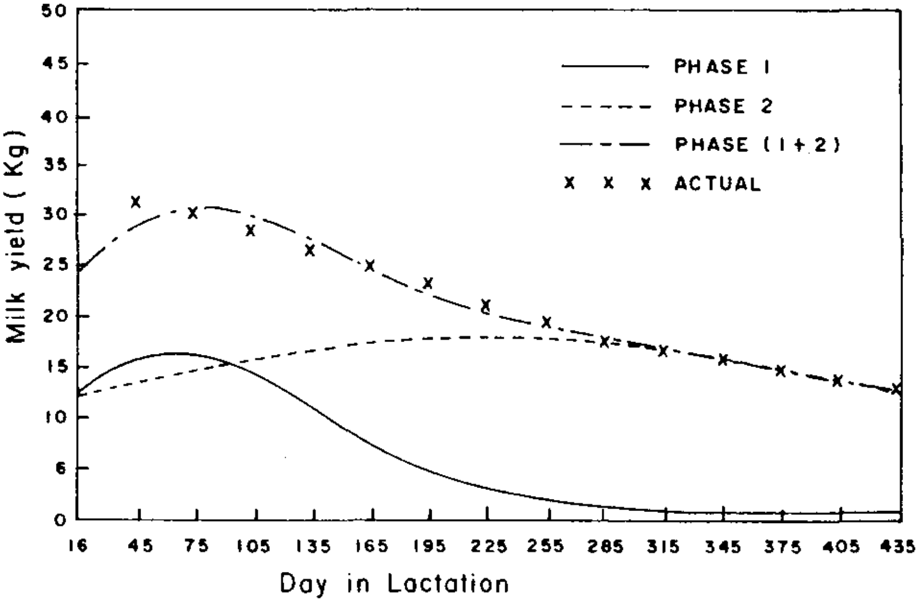

Fig. 2. Diphasic lactation curve and overall actual data.

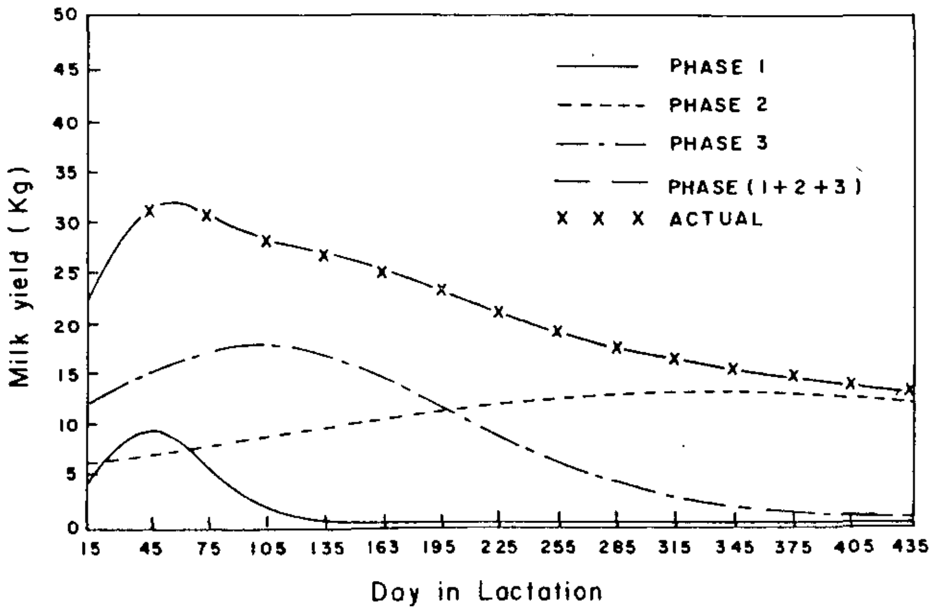

Fig. 3. Triphasic lactation curve for overall and actual data.

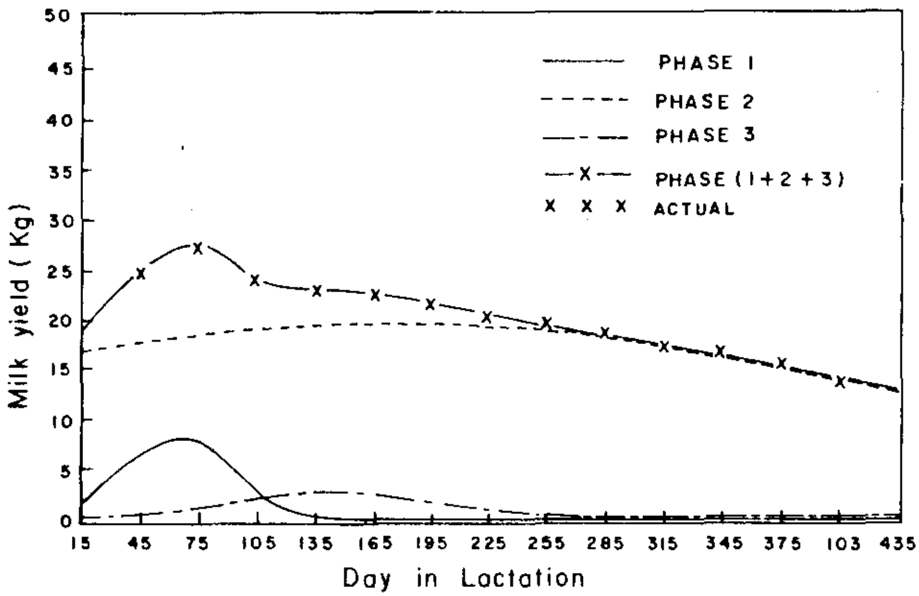

Fig. 4. Triphasic lactation curve and first lactation actual data.

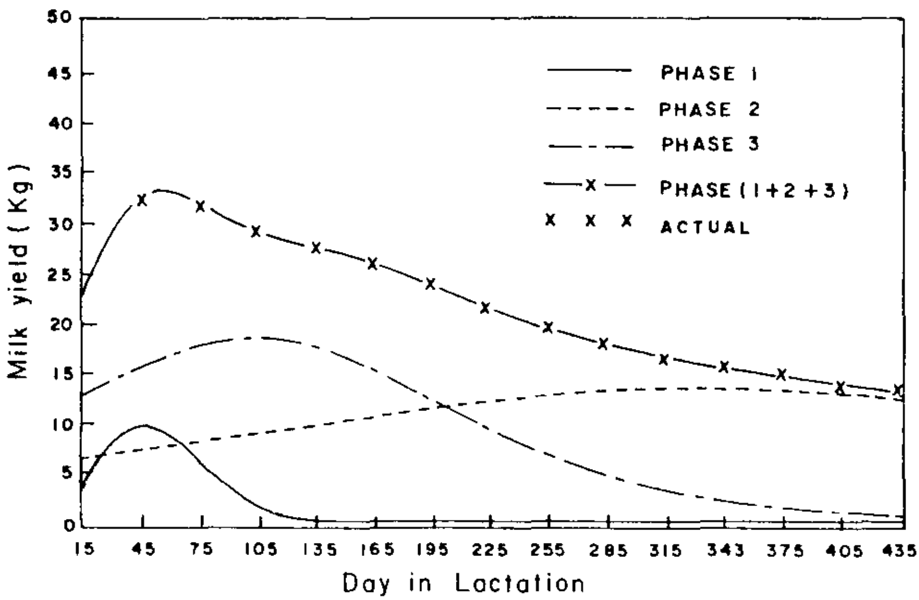

Fig. 5. Triphasic lactation curve and second lactation actual data.

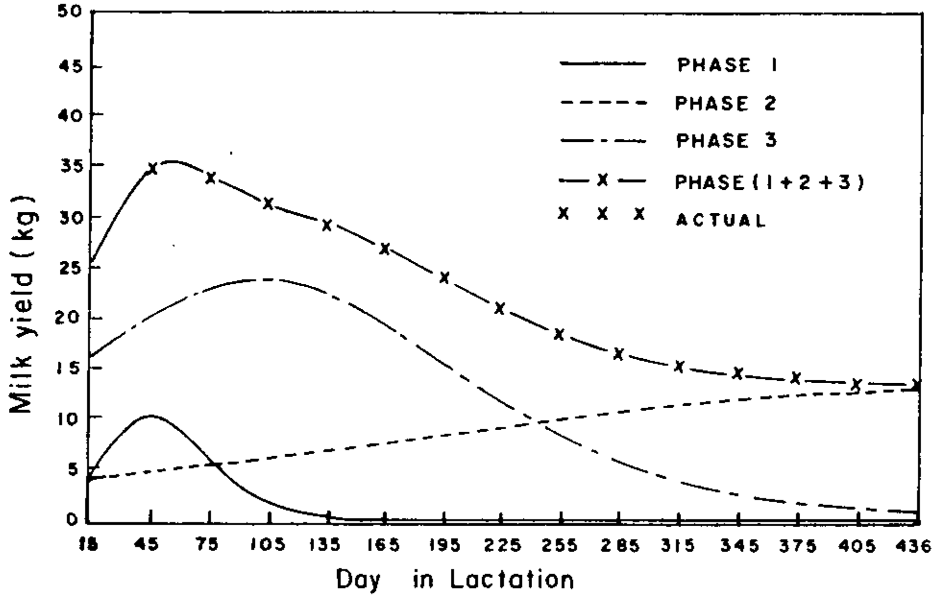

Fig. 6. Triphasic lactation curve and third lactation actual data.

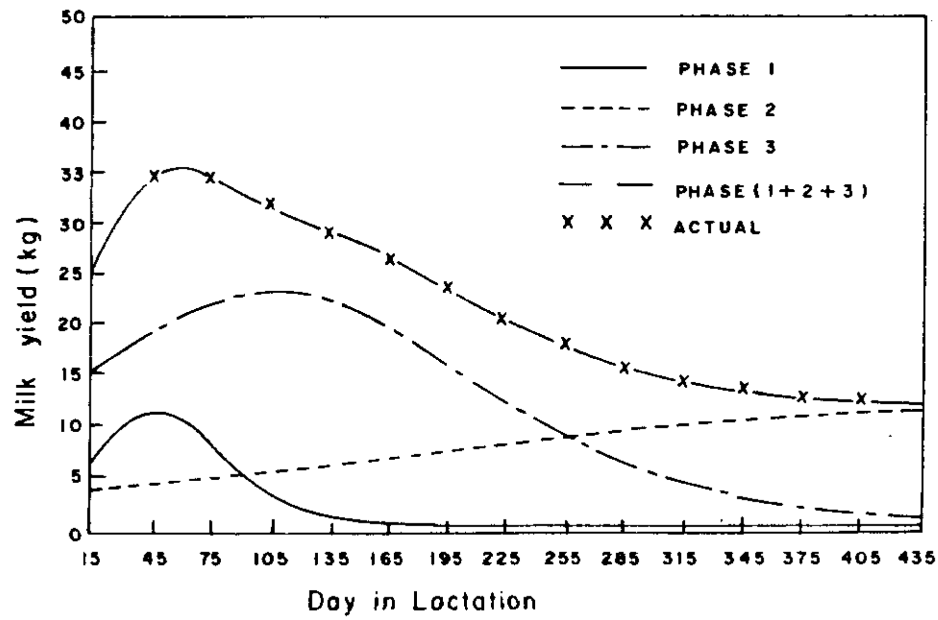

Fig. 7. Triphasic lactation curve and fourth lactation actual data.

Table 3. Components of lactation curve as estimated by using parameters of multiphasic function of overall data

Function	Phase no	initial yield	Peak yield	305-day yield	Time of peak	Duration of phase
Monophasic		27.7	27.9	7518	38.4	769.2
	1	10.5	16.3	2549	67.9	199.7
Diphasic	2	11.4	17.9	4887	160.0	709.0
	Total	21.9		7436		
	1	2.4	9.2	641	48.5	74.9
Triphasic	2	5.8	13.6	3062	337.0	680.4
	3	10.6	17.8	3737	102.5	274.0
	Total	18.8		7440		

Table 4 . Functions of estimates of parameters for triphasic functions for different parities

Lactation No	Phase	Initial yield	Peak yield	305-day yield	Duration yield	Time of yeak
	1	.8	8.8	580	64.0	67.8
1st	2	16.4	19.9	5761	172.3	774.1
	3	.1	2.9	370	143.9	138.7
	Total	17.3		6711		
	1	1.9	10.0	644	50.0	67.7
2nd	2	6.0	13.5	3096	336.0	695.0
	3	11.1	18.5	3924	103.3	277.8
	Total	19.0		7664		
	1	2.4	10.2	674	48.0	70.6
3rd	2	3.9	12.6	2241	433.0	723.9
	3	14.2	23.6	4978	102.0	274.0
	Total	20.5		7892		
	1	4.1	11.3	891	48.0	88.1
4th	2	3.4	11.2	1987	435.0	728.7
	3	13.6	23.2	4975	107.0	281.7
	Total	21.1		7853		

For the overall data : (a) the monophasic function (Table 3) gave initial yield of 27.7 kg and peak yield of 27.9 kg which indicated that the initial yield was actually a peak yield while no distinct inclining phase was shown by the curve of monophasic function. Time of peak was 38.4 days, duration phase was 769.2 day and 305- milk yield was 7518 kg of milk. (b) In diphasic function, comparison between the two phases showed that about 48% of the initial yield was contributed by the first phase(10.5 kg) and about 52% by the second phase(11.4 kg). For the first phase, initial yield was about 64.4 % of the peak yield (16.3) while for the second phase it was 63.7% (17.9 kg).

Total 305-day yield for the first phase accounted for 34% (2549kg) of total yield, while that for second phase accounted for 66% (4887kg). First phase was characterized by an early peak (68 day) whereas the second phase exhibited the late peak (160 day). Duration of the first phase attained at 200 days .Yet,duration of second phase included the entire 305-days of lactation(709 day). Discrepancy between the results of Grossman and Koops [3] and those of the present study can be explained by differences in the data, their study analyzed mean milk yields from pure bred Dutch Friesian Black and white cows for 15 test-day with 20-day intervals starting from 10 day in milk, whereas this study analyzed means from Holstein Friesian cows for 30-day intervals beginning with day 15 after calving. (c) In triphasic function, the contribution of the first, second and third phases for each function of the overall data (Table 3) was as follows: initial milk yield was 12.8%, 30.9% and 56.3%, respectively, peak yield was 9.2, 13.6 and 17.8 (kg), respectively, time of peak was 48.5, 337.0 and 102 day, respectively.

The yield of each phase as a percentage of 305 day yield was 8.6%,41.2% and 50.2% for the three phases respectively. Finally, duration for three phases was 74.9,680.4 and 274.0 day, respectively.

Functions estimated from the parameters of triphasic functions across lactations are given in Table 4 . For the first lactation, second phase was the most important phase in determining initial milk yield, peak yield, 305 milk yield and duration. The percentages of second phase contributions on initial milk yield (16.4 kg) and 305-day yield (19.9kg) were 95% and 86%. Initial yield (16.4 kg) accounted for 82% of peak yield. Moreover, second phase showed a late peak (172 day) and a long duration (774.1 day).

For second, third and fourth lactations, more contribution resulted from the third compared with the first and second, phaes. The contribution of the third phase in initial milk yield was 58% (11.1 kg), 69% (14.2 kg) and 64 % (13.6 kg) for second,third and fourth lactations respectively. Initial yield accounted for 60%, 60% and 59 % of the peak yield of the third phase for second,third and fourth lactations, respectively. Peak yields for the third phase were 18.5, 23.6 and 23.2 kg for the three lactations, respectively. The contribution of the third phase for 305-day yield was 51% (3924 kg), 63% (4978 kg) and 63% (4975) for second, third and fourth or later lactations, respectively.

On the other hand, second phase played a major role in determining the time of peak and the duration for second, third and fourth lactation. The time of peak was 336, 433 and 435 days for second, third and fourth lactations respectively. Corresponding duration of second phase was 695,724 and 729 days respectively.

Third phase gave peaks at 103,102 and 107 days and durations of 278,274 and 282 days for second, third and fourth lactations, respectively. As indicated by Grossman and Koops [3] the duration of a phase can be interpreted as being associated with persistency of lactation, the shorter the duration , the lesser the persistency. These authors stated that the duration of the second phase of lactation was a possible measure of persistency. Second and third lactation cows were 100 to 200 day shorter in duration of the second phase than first lactation cows, hence, cows of second and third lactations were considered to be less persistent than first parity lactation. Similar results were found in the present study, second phase of the second,third and fourth lactation cows being less in duration by almost 50 to 80 days than the second phase of the first lactation. This is an indication of the lesser persistency of the former cows.

The criteria used to test for goodness of fit for different models (Table 5) were autocorrelation, Durbin Watson, coefficient of determination (R^2) and mean square error (MSE). Autocorrelation between residuals after fitting the overall data by non linear regression, was 0.17.-0.14 and -0.44 for mono,di and triphasic functions respectively. Small autocorrelations existed because the data points represented the means of monthly test-day milk which are far apart from each other. Durbin Watson Statistic (D) was1.271,1.996 and 2.785 for the three phases, respectively, where a D value of 2 indicates no autocorrelation ,a lower value indicates more positive autocorrelation and a higher value indicates more negative autocorrelation (Theil [11,pp.199-201]).

Table 5. Autocorrelation,R^2 , MSE and Durbin Watson between residuals after fitting the overall data by different functions

	Monophasic	Diphasic	TRiphasic
Autocorrelation	0.168	-.143	-0.436
Durbin Watson	1.271	1.996	2.785
R^2	0.997	0.999	0.999
MSE	5.036	1.649	1.005

The (R^2) value, which measures the part of variation explained by the model, was high (.99) for the three models, high value being a desired statistic for testing goodness of fit. Yet, a large value of R^2 by itself would not assign the best model. In a study by Koops [1] the values of R^2 of one, two, three and four phases models, on growth curve data were .99 and higher.

Mean square error is an important criterion for testing the adequacy of a certain model to fit the data because it is a measure of the deviation of the predicted value for the actual data. The mean square error equals the square of the bias (the average of deviation between predicted and actual yields) plus the variance of the predictions, the smaller the mean square error the more appropriate the mode l. The mean square value was 5.036, 1.649 and .101 for mono, di and triphasic functions, respectively.

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منحنى الحليب متعدد الأوجه لأبقار الهولشتاين فريزيان السعودية

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ملخص البحث. أستخدمت معادلة الأوجه المتعددة لوصف منحنى الحليب لمتوسط يوم لكل شهر لعدد ١٢٠٢٠ سجل حليب لأبقار الهولشتاين فريزيان في المملكة العربية السعودية. اشتملت البيانات على ٣٧٩٣، ٢٢٥٧، ٢٤٧٠، ٣٥٠٠ سجل لموسم الحليب الأول، الثاني، الثالث والرابع وأكثر على الترتيب. استبعد من التحليل الإحصائي سجلات الحليب التي أقل من ١٠٠ وأكثر من ٤٥٠ يوم. أظهرت المعادلة ذات الوجه الواحد تمثيلا بعيدا عن منحنى البيان الحقيقي حيث كان المنحنى لا يشتمل على مرحلة صعود (في الفترة من ١٥ إلى ١٠٥ يوم) ولا يحتوي على نقطة أكثر إنتاج.

أظهرت المعادلة ذات الوجهين تمثيلا قريبا لمنحنى البيانات الحقيقية ولكن كان هناك فروقا (القيمة الحقيقية ـ القيمة المتنبأة) خصوصا الشهر الأول والثاني من منحنى الحليب بينما كانت المعادلة ذات الأوجه الثلاثة ذات التسعة ثوبت مطابقة للبيانات الحقيقية خلال مرحلة الحليب كلها. وقد ساهم كل وجه من أوجه المعادلة المتعددة بدرجات مختلفة للأربعة مواسم حليب.