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A Stream-Aquifer Three-Dimensional Groundwater Flow Model

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ABSTRACT. This paper discusses the proper way to describe the important stream-aquifer interaction boundary condition, to represent the effect of a stream on an aquifer. It also shows the incorporation of the stream-aquifer boundary or third type boundary condition in a curvilinear coordinate three-dimensional groundwater flow model.

The model is tested by a comparison with the simple problem of a pumping well near a perennial stream fully penetrating the aquifer. The numerical model shows a good agreement with the analytical solution for both transient and steady state condition.

Additionally, the model is then applied to the same system but without the assumption of fully penetrating well and stream by using both the reach transmissivity concept and the three-dimensional representation of the aquifer system. The comparison shows a large deviation between the analytical solution and the numerical representation especially for the calculation of the return flow, which can be attributed to the effects of the boundary conditions on the results of the numerical model.

The model proved to be working accurately in comparison with the analytical solution, but the estimation of the reach transmissivity needs to be investigated further to show its effect on the calculation of the return flow.

Introduction

Surface and groundwater bodies interact with each other in nature under a variety of boundary conditions. The interaction between the surface and groundwater provides favorable conditions for conjunctive use management policies. Efficient management of the system can be achieved during excess surface water availability that can be stored in the aquifer. During rainy seasons or flooding periods, excess water may be diverted into the field to recharge the groundwater reservoir, or the delivery system can serve directly as a recharge facility. In the dry season water may be pumped from the aquifer as an alternative source of water for meeting various demands. The previously mentioned technique can be used for high flood years where water can be stored in the ground reservoir and be used in low flood year, when the surface water is not enough to meet all the water demands.

The management of stream-aquifer interaction system requires modelling procedures that account for the appropriate boundary conditions.

In nature three main types of boundary conditions occur in groundwater system :

The first is when an aquifer is in hydraulic connection with a major body of water such as a large size lake or reservoir. In that case the reservoir imposes its head on the aquifer. The boundary condition is thus one of a prescribed head at the interface between the lake and the aquifer. If the lake level remains constant in time the prescribed, head is constant.

The second is where the aquifer terminates as when a permeable alluvium encounters solid bedrock, the boundary condition at the interface is one of no flow. This natural boundary condition is a particular case of the more general mathematical boundary condition which stipulates a prescribed flux as the boundary. An example of a prescribed flux as a boundary condition is that of injection of water from high pressure wells or through a recharge trench with very good permeability.

The third type is encountered when a river intersects the aquifer and this occurs under different geographic and geologic conditions. The river may fully or partially penetrate the aquifer thickness resulting in surface-groundwater interaction. The most common case encountered in nature is when the stream partially penetrating the aquifer and the stream stage is either lower or higher than the groundwater table. The exchange of water between the aquifer and the stream can be estimated by using an integrated form of Darcy's Law as described by Morel-Seytoux (1985).

$$Q_r = \Gamma\,(h - H) \tag{1}$$

where Q_r is the return flow between the stream and the aquifer. Q_r is algebraically defined as positive when the direction of flow is from the aquifer towards river and negative otherwise, h is the water table elevation in the aquifer, H is the stage in the river (both measured from a common datum), and Γ, the coefficient of proportionality, is the river or reach transmissivity which is a function of the fluid, aquifer geometry, hydraulic properties and river cross section. This type of boundary condition is typical in the presence of a stream and is therefore called a stream-aquifer boundary condition.

Typically in groundwater studies one is interested in only a part of an aquifer. Thus the boundary for the study is not a natural boundary but an artificial one which separates the part of the aquifer of interest from the rest. Since the division is artificial, the boundary condition cannot be one of a prescribed head or prescribed flux. The flux at this artificial interface will depend on what happened internally in the aquifer described by the hydraulic head h and externally in the aquifer as described by the stage height H.

Typically in groundwater models which consider the presence of a stream, the whole cell containing the stream is given the same hydraulic head as that of the stream which necessitate, the use of a small grid system in the stream cell in order for the assumption to be valid.

In the following sections, the stream-aquifer boundary condition is introduced to a three-dimensional curvilinear groundwater flow model, followed by case studies to show the superiority of the stream-aquifer boundary condition over the typically used constant head boundary condition for the cases where a river interacts an aquifer.

Theoretical Background

The general differential equation for the three-dimensional groundwater flow in a curvilinear coordinate system (Khadr 1988) is written as :

$$\frac{1}{e_\xi \, e_\eta \, e_\zeta} \left[\frac{\partial}{\partial_\zeta} \left\{ \frac{e_\eta e_\zeta}{e_\xi} K_\xi \frac{\partial h}{\partial_\xi} \right\} + \frac{\partial}{\partial_\eta} \left\{ \frac{e_\xi e_\zeta}{e_\eta} K_\eta \frac{\partial h}{\partial_\eta} \right\} + \frac{\partial}{\partial_\zeta} \left\{ \frac{e_\xi e_\eta}{e_\zeta} K_\zeta \frac{\partial h}{\partial_\zeta} \right\} \right] = S_s \frac{\partial h}{\partial t} + q \quad (2)$$

where ξ, η and ζ are the curvilinear axes, $e_\xi \, e_\eta$ and e_ζ are the coordinate scale factors, K is the hydraulic conductivity, S_s is the specific storage and q is a sink term representing the external excitation to the system (discharge per unit horizontal area).

The regular finite difference simulation technique uses the known initial conditions at the beginning of the time step to compute the unknown hydraulic heads at the end of the time step under the effect of the external excitation to the system.

External excitation to a stream-aquifer system can be divided into two types. The first is the pumping or recharge excitation out of a cell and is donated by q. The second is the loss of water as a return flow from the aquifer to the river, q_Γ. From the point of view of the mathematical formulation using the finite difference solution, both have the same effect on the draw down of the cell. In order to account for these two types of excitation, then equation (2) will take the following form :

$$\frac{1}{e_\xi \, e_\eta \, e_\zeta} \left[\frac{\partial}{\partial_\zeta} \left\{ \frac{e_\eta e_\zeta}{e_\xi} K_\xi \frac{\partial h}{\partial_\xi} \right\} + \frac{\partial}{\partial_\eta} \left\{ \frac{e_\xi e_\zeta}{e_\eta} K_\eta \frac{\partial h}{\partial_\eta} \right\} + \frac{\partial}{\partial_\zeta} \left\{ \frac{e_\xi e_\eta}{e_\zeta} K_\zeta \frac{\partial h}{\partial_\zeta} \right\} \right] = S_s \frac{\partial h}{\partial t} + q + q_\Gamma \quad (3)$$

Substituting the term q_Γ by its equivalent value from equation (1) gives

$$\frac{1}{e_\xi\, e_\eta\, e_\zeta}\left[\frac{\partial}{\partial_\zeta}\left\{\frac{e_\eta\, e_\zeta}{e_\xi} K_\xi \frac{\partial h}{\partial_\xi}\right\} + \frac{\partial}{\partial_\eta}\left\{\frac{e_\xi\, e_\zeta}{e_\eta} K_\eta \frac{\partial h}{\partial_\eta}\right\} + \frac{\partial}{\partial_\zeta}\left\{\frac{e_\xi\, e_\eta}{e_\zeta} K_\zeta \frac{\partial h}{\partial_\zeta}\right\}\right] = S_s \frac{\partial h}{\partial t} + q + \Gamma\,(h - H) \qquad (4)$$

where Γ is the reach transmissivity per unit area of the excitation cell.

It is to be noted that equation (1) represents the system steady state condition. It is assumed that the river stage is constant during the time step. The reach transmissivity (Γ) is time dependent since it is a function of the stage in the river and the saturated thickness of the aquifer, which are assumed to have small fluctuation. The reach transmissivity values are calculated explicitly at the beginning of the time period and held constant during the duration of the time step.

Equation (4) represents the general differential Equation for a stream-aquifer system. The Integration of equation (4) over a subregion of the flow domain, using the three dimensions *i, j,* and *k* induces, leads to the general implicit equation shows as follows :

$$\begin{aligned} &a_{ijk}\ [\lambda_i^{o}\,(h^{o}_{i+ijk} - h^{o}_{ijk}) + \lambda_i^{\nu}\,(h^{\nu}_{i+ijk} - h^{\nu}_{ijk})] + b_{ijk}\ [\lambda_i^{o}\,(h^{o}_{i-1jk} - h^{o}_{ijk}) + \\ &\lambda_i^{\nu}\ (h^{\nu}_{i-ijk} - h^{\nu}_{ijk})] \quad + c_{ijk}\ [\lambda_j^{o}\,(h^{o}_{ij+1k} - h^{o}_{ijk}) + \lambda_i^{\nu}\,(h^{\nu}_{ij+1k} - h^{\nu}_{ijk})] + \\ &d_{ijk}\ [\lambda_i^{o}\,(h^{o}_{ij-1k} - h^{o}_{ijk}) + \lambda_i^{\nu}\,(h^{\nu}_{ij-1k} - h^{\nu}_{ijk})] + e_{ijk}\ [\lambda_k^{o}\,(h^{o}_{ijk+1} - h^{o}_{ijk}) + \\ &\lambda_k^{\nu}\ (h^{\nu}_{ijk+1} - \mathrm{h}^{\nu}_{ijk})] + f_{ij,\,k}\,[\lambda_k^{o}\,(h^{o}_{ijk-1} - h^{o}_{ijk}) + \lambda_k^{\nu}\,(h^{\nu}_{ijk1} - h^{\nu}_{ijk})] \\ &= [\,(S_s - \frac{h^{\nu} - h^{o}}{\Delta\, t} + q)]_{ijk} + \Gamma_{ijk}\ (\lambda^{o}\, h^{o}_{ij,\,k} + \lambda^{\nu}\, h_{i\bar{j},\,k} - H^{\nu}_{ijk}) \end{aligned} \qquad (5)$$

Where

$$\mathrm{H}_{\mathrm{ijk}} = \frac{h^{\nu} - h^{o}}{\Delta\, t}$$

Where H_{ijk} is the river stage height, and for the simplicity in notation the hydraulic conductivity factors a_{ijk}, b_{ijk}, c_{ijk}, d_{ijk}, e_{ijk} and f_{ijk} are defined by :

$$a_{ijk} = \frac{1}{(e_\xi\, e_\eta\, e_\zeta\, \Delta\, \xi)_{ij,\,k}}\ (e_\eta\ e_\zeta\ K_\xi)_{i+1/2j,\,k}\ \frac{1}{(e_\xi\, \Delta\, \xi)_{i+1/2j,\,k}}$$

$$b_{ijk} = \frac{1}{(e_\xi\, e_\eta\, e_\zeta\, \Delta\, \xi)_{ij,\,k}}\ (e_\eta\ e_\zeta\ K_\eta)_{ij,\,k}\ \frac{1}{(e_\xi\, \Delta\, \xi)_{i-1/2j,\,k}}$$

$$c_{ijk} = \frac{1}{(e_\xi\, e_\eta\, e_\zeta\, \Delta\, \eta)_{ij,\,k}}\ (e_\xi\ e_\zeta\ K_\eta)_{ij,\,+1/2k}\ \frac{1}{(e_\eta\, \Delta\, \eta)_{ij-1/2,\,k}}$$

$$d_{ijk} = \frac{1}{(e_\xi\, e_\eta\, e_\zeta\, \Delta\, \eta)_{ij,\,k}}\ (e_\xi\ e_\zeta\ K_\xi)_{ij,\,k-1/2,\,k}\ \frac{1}{(e_\eta\, \Delta\, \eta)_{ij+1/2,\,k}}$$

$$e_{ijk} = \frac{1}{(e_\xi\, e_\eta\, e_\zeta\, \Delta\, \xi)_{ij,\,k}}\, (e_\xi\, e_\eta\, K_\zeta)_{ij,\,k\,+\,1/2}\, \frac{1}{(e_\zeta\, \Delta\, \xi)_{ij,\,k\,+\,1/2}}$$

$$f_{ijk} = \frac{1}{(e_\xi\, e_\eta\, e_\zeta\, \Delta\, \zeta)_{ij,\,k}}\, (e_\xi\, e_\eta\, K_\zeta)_{ij,\,k\,-\,1/2}\, \frac{1}{(e_\zeta\, \Delta\, \xi)_{ij,\,k\,-\,1/2}}$$

The ν denotes the new values of hydraulic heads at the end of the time step, where o denotes the old values at the beginning of the time step. λ is the weight factors to weigh the old and new head values. The weight factors should add to one :

$$\lambda^{o} + \lambda^{\nu} = 1 \tag{7}$$

using $\lambda^{o} = 0$, the system is a fully implicit scheme, $\lambda^{o} = 1$, the system is a fully explicit scheme and $\lambda^{o} = 1/2$ the system is a central or a Crank-Nicolson scheme. Whether the explicit or implicit schemes are used (Khadr 1988), the resulting system of algebraic equation is solved using the finite difference approximation.

Simulation Results and Discussion

1. Model Verification

To check the accuracy of the developed model, the numerical solution will be compared with the analytical solution for a simple problem. The test problem assumes a fully penetrating pumping well near a perennial stream that fully penetrates the aquifer and intersects it along an infinitely long straight line. Initially the hydraulic head in the stream and the aquifer are assumed to be at the same level.

Under the assumption of a fully penetrating stream, it is a recognized fact that the stream constitutes an equipotential line and that the cone of depression cannot spread beyond the stream.

The analytical solution of the problem is obtained by applying the principle of superposition. An imaginary recharge well is placed directly on the opposite side and at equal distance from the stream as the real well. The recharge image well operates simultaneously and at the same rate as the real well. The resultant real cone of depression is the arithmetic summation of the components of the real well cone of depression and the image well cone of impression which is given by :

$$\left.\begin{aligned} s &= S_r - S_i \\ s &= \frac{Q}{4\,\pi\,T}\,[W(U_r) - W(U_i)] \end{aligned}\right] \tag{8}$$

Where s is the drawdown in an observation point e is the drawdown due to pumping of the well, S_i is the build up due to image well, Q is the discharge, T is the transmissivity of the aquifer, and

$$U_r = \frac{r_r^2\, S}{4\, T\, t}$$

$$U_i = \frac{r_i^2 S}{4 T t}$$

where r_r is the distance from observation point to pumped well, r_i is the distance from the observation point to the image well, W is the well function and S storage coefficient.

For the purpose of simulation, the pumping rate from the well is assumed to be 10,000 m^3/period, the hydraulic conductivity, 10 m/period, the saturated thickness 100 m and the storage coefficient 0.2. The shortest distance between the pumping well and the stream is 50.00 m. A variable cartesian grid system is used to represent the aquifer system.

The distance-drawdown result of the analytical and numerical simulation for the transient condition after a pumping period is presented by Fig. 1. It is clear that the analytical solution matches well with the one dimensional numerical solution except near the well.

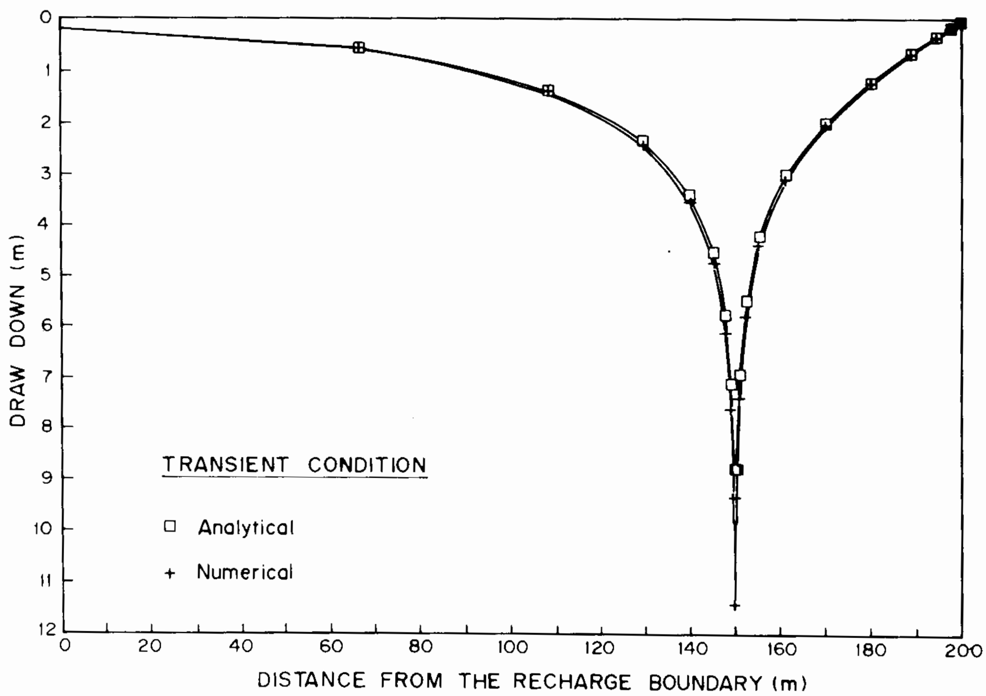

FIG. 1. Comparison of hydraulic head versus distance for transient condition.

Figure 2 represents the drawdown calculated by the one dimensional numerical and the analytical solutions for the steady state pumping rates. Both the analytical and the numerical solutions are seen to be in good agreement. However there is a slight deviation between the analytical and numerical solutions near the pumping well, especially near the well face that have reached 2 meter. This can be attributed to the fact that the analytical solution assumes a line sink, whilst for the numerical solution a point sink is used and a diameter of 0.25 m has been specified to the pumping well.

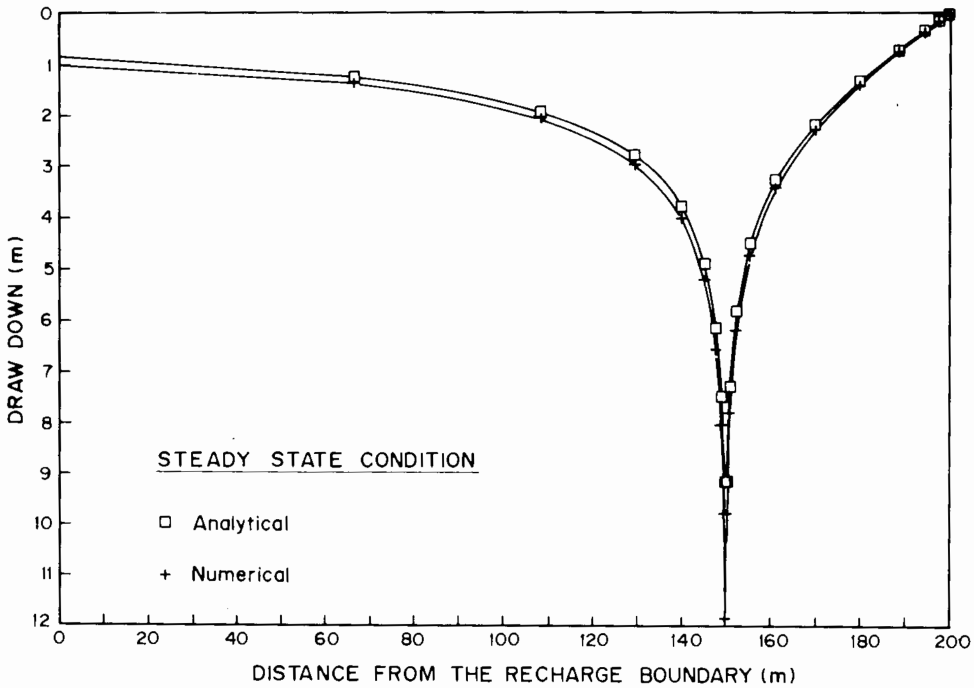

FIG. 2. Comparison of hydraulic head versus distance for steady state condition.

It is concluded from the previous test that the numerical model works accurately and represents the simulation result under the assumptions of both fully penetrating well and stream and an infinitely long stream.

2. Boundary Conditions Test

It was demonstrated from the previous section that the numerical model represents well the aquifer system, the second step then is to eliminate the analytical solution assumptions in order to show the effect on the aquifer system behavior.

Using the same grid system and data sets, except the stream will be enlarged in width to represent a real stream and this became no longer assumed as a line sink. The cone of depression of the pumping well is also expected to expand beyond the stream cross section. Rectangular stream with a 10 m width and 5 m depth was chosen in the simulation. Under the given condition of the reach transmissivity developed by Morel-Seytoux and Zhang (1988) can be used to account for the interaction between stream with large width and undertaken aquifer as follows :

$$\Gamma = \frac{T}{e} \; L \; \left(\frac{W_p + 2e}{e + 10\,Wp} \right) \tag{9}$$

where W_p is the wetted perimeter of the reach, e is the saturated thickness of the aquifer near the reach, T is the transmissivity of the aquifer near the reach and L is the length of the reach. The calculated value of Γ is 7.33 L m^2/period for the given set of data.

Figure 3 shows a comparison between the analytical and the numerical two-dimensional simulation of the hydraulic head in the aquifer along a line perpendicular to the stream and passing through the pumping well. It can be seen that the analytical solution over-estimates the hydraulic head in the aquifer near the recharge boundary as a result of approximation of conversion of stream lines represented by the equation 9. This is because it forces a potential line along the stream which does not exist in reality. The simulation results indicate that a misleading result can be deduced by the assumption of fully penetrating stream treated as an equipotential line as was suggested in Fig. 1 and 2.

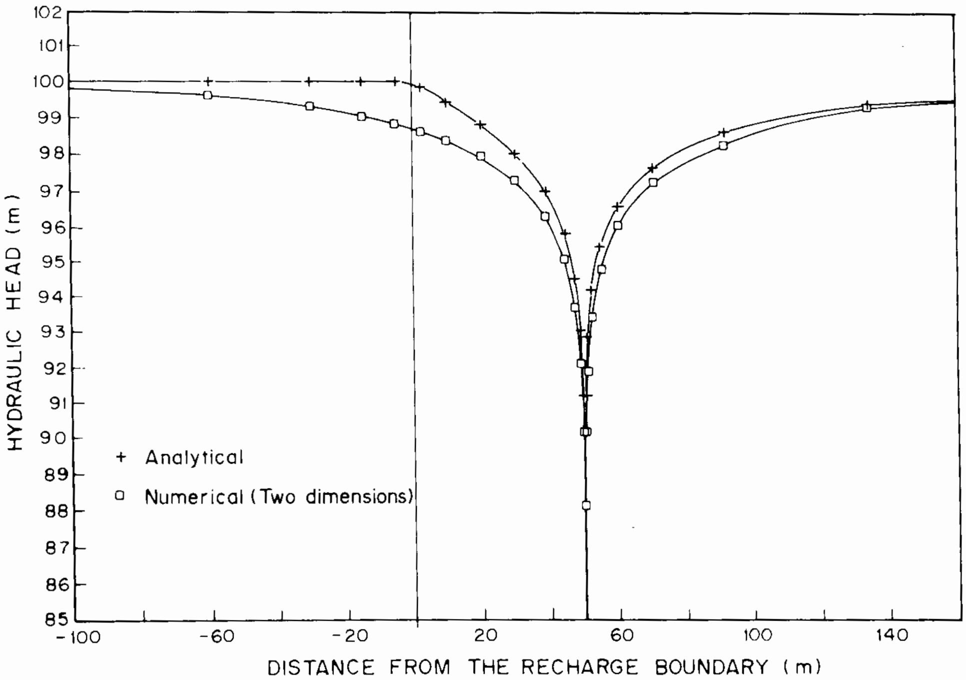

FIG. 3. Comparison of hydraulic head versus distance for two-dimensional cases.

The total flow from the river to the aquifer can be deduced from (Walton 1970) the analytical solution and is given as under :

$$Q_r = \frac{2\,Q}{\pi} \int_0^{\infty} e^{-F} \frac{(1 + Z^2)}{1 + Z^2}\, dz$$

$$z = \frac{x}{a} \quad , \quad dz = \frac{1}{a}\,dx$$

$$F_t = \frac{a^2\,S_y}{4\,Tt}$$

where a is the distance from the pumped well to the recharge boundary and S_y is a storage coefficient.

The return flow (volume over the first period) calculated from the analytical solution is 6170 m^3, while the return flow calculated by the two-dimensional model is 64.5 m^3. It can be observed that large deviation resulted using both approaches due to the large head deviation at the recharge boundary.

The return flow estimated by the numerical model is a function of the reach transmissivity which in turn a function of the stream cross section. The effect of the estimation of the return flow is then affected by the shape of the stream and aquifer geometry which is still under both analytical and numerical investigations to come out with an accurate estimation of the reach transmissivity.

To represent the effect of the three-dimensional flow that is expected near the stream, the three-dimensional option of the model was used to simulate the same case. The same grid system is used, except that the saturated thickness of the aquifer was represented by a four grid vertical system. The grids in the vertical direction are 5, 15, 20 and 60 m. The upper grid has the same depth as the stream, on the other hand the well pumps only from the lower grid to represent the real practical case where perforations in the casing are usually located in the deep portion of the aquifer.

Figure 4 shows a comparison of the hydraulic head in the pumped layer of the aquifer versus the analytical solution. It can be noticed that the hydraulic head in the three-dimensional case simulation is lower than the ones calculated by the analytical solution. This is due to the fact that the stream lines change direction from the stream to the aquifer going through a longer path, which reduces the contribution of the stream to the return flow. The foregoing effect certainly causes the hydraulic head in the aquifer to decline.

It is common in the simulation of ground water systems to assume that the hydraulic head in the cells containing stream portions to have the same hydraulic head as the stream. A hypothetical case study was used to represent the effect of the concept of the reach transmissivity on an aquifer system versus the above mentioned assumption.

The hypothetical system is a square aquifer of dimension 6000 m represented by a uniform 6 × 6 grid system. Passing by the aquifer is a stream as given in Fig. 5. The aquifer is homogeneous and isotropic with hydraulic conductivity of 4000 m/period, storage coefficient of 0.2 and initial saturated thickness of 100 m. The reach transmissivity for the stream portions passing by a cell was assumed to be constant for all the cells and of a value of 100,000 m^2/period. The hydraulic head in the aquifer is assumed to be the same in the aquifer and the stream at the beginning of the simulation and of a value of 100 m.

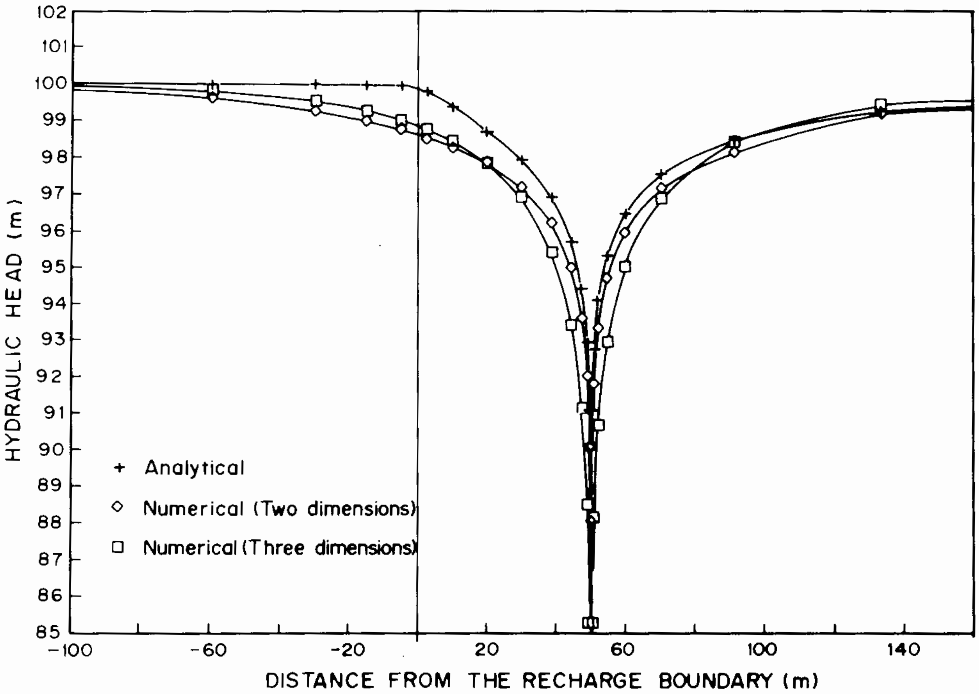

FIG. 4. Comparison of hydraulic head versus distance for two and three-dimensional cases.

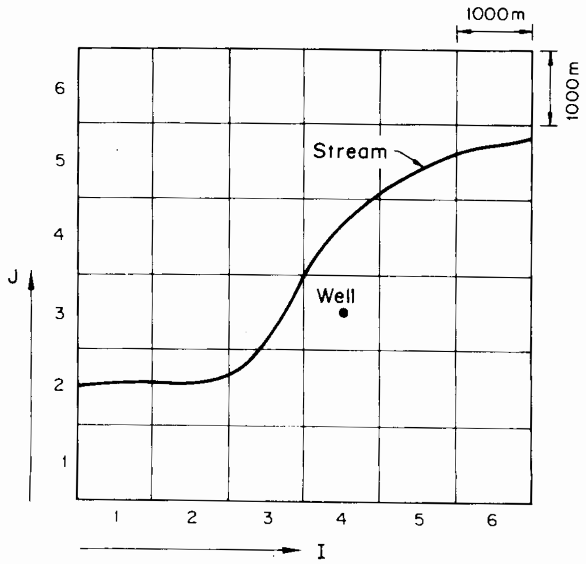

FIG. 5. Grid system (Hypothetical case).

The excitation to the stream aquifer system is a pumping well in cell (4, 3) as shown in Fig. 5. The pumping rate was chosen to be 30,000 m^3/period.

Three types of simulation tests are made. First it is assumed that the aquifer hydraulic head in the cell containing the stream is constant and having the same hydraulic head as the stream. The hydraulic head contour lines simulation is presented in Fig. 6. The second simulation assumes that a two-dimensional aquifer system with the effect of the streams represented by the reach transmissivity. Figure 7 shows the contour lines output for that case. Thirdly the system was assumed to be of three-dimensional with the stream interaction effect. The saturated thickness in the aquifer was divided into 5, 10, 25 and 60 meter layers from top to bottom. The pumping was from the lowest grid. Figure 8 shows the output for the third case of layer aquifer. From Fig. 6, 7 and 8 it is clear that the common assumption of the same hydraulic head in the stream and the aquifer cells underestimate the effect of pumping on the aquifer. The stream for the case when it is represented by an equipotential line causing the portion of the aquifer beyond the stream to feel no effect of the pumping excitation.

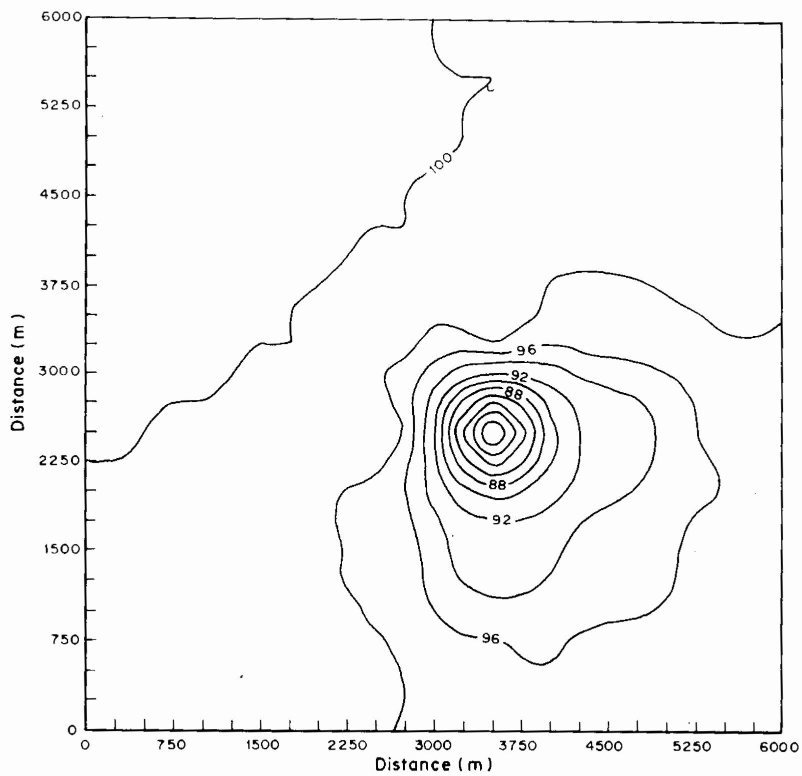

FIG. 6. Hydraulic head contour map for the case of constant hydraulic heads at the stream cells.

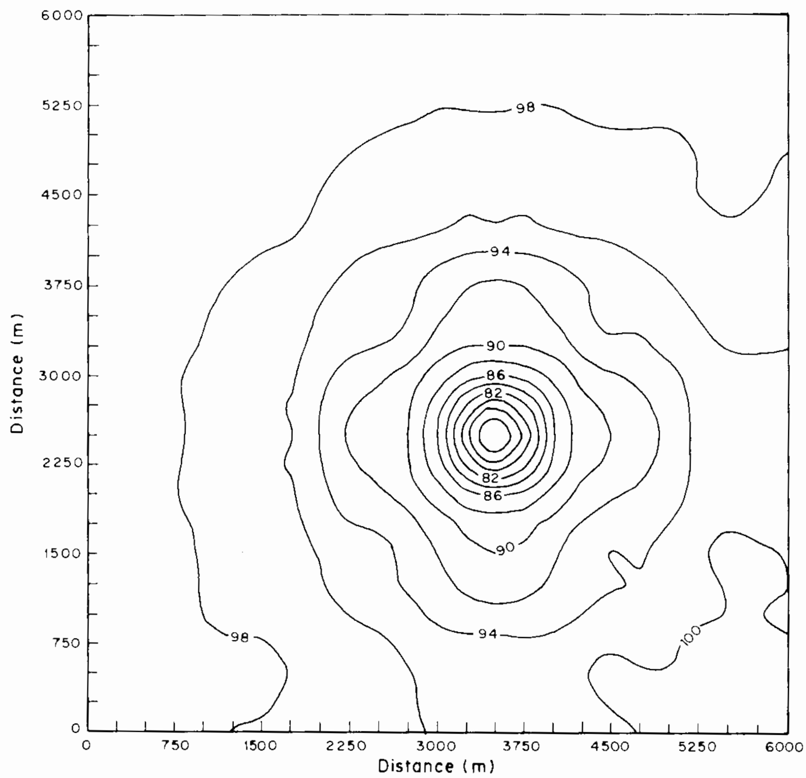

FIG. 7. Hydraulic head contour map for the case of stream-aquifer interaction solution (two dimensions).

The effect of the excitation spreads faster in the aquifer system under the two-dimensional case than the three-dimensional case as shown in Fig. 7 and 8. This is due to the previous mentioned curvature of the stream lines and the longer path of the excitation in the three-dimensional case.

Conclusions

The proposed approach discussed in this paper demonstrate the importance of identifying the appropriate stream-aquifer boundary condition.

The accuracy and efficiency of the methodology and computer code is tested by the comparison with the analytical solution of the simple problem of pumping near a fully penetrating perennial stream. Through the concept of reach transmissivity, the interaction between the stream and aquifer can be modeled.

The simulation results for different cases indicates that good agreement between the one-dimensional analytical and a numerical solutions when the recharge boundary represented by the stream is assumed as a line sink. However there is a deviation

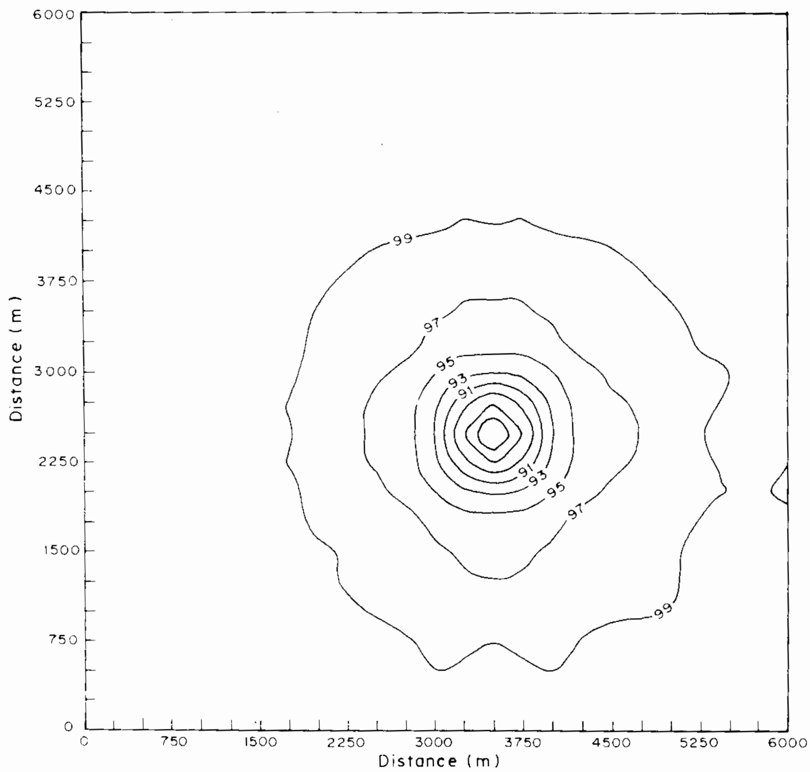

FIG. 8. Hydraulic head contour map for the case of stream-aquifer interaction solution (three dimensions).

between the two approaches occurs as a results of the effect of the stream width and its relation to the values of return flow.

The reach transmissivity equation needs further refinement to account for the physical aquifer-stream interaction flow condition. Reliable estimates of the reach transmissivity are needed for simulation of the system.

Acknowledgement

This paper is being published in honor and memory of Dr. Hassan Khadr, who passed away in 1991. He will always be remembered by us and his colleagues as a good muslim and a fine professor in his field. This paper serves as a final tribute to his work. God bless him.

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نموذج تبادل المياه بين النهر والمياه الجوفية باستخدام النموذج العددي ذي الثلاثة أبعاد

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و قسم علوم وإدارة موارد المياه ، كلية الأرصاد والبيئة وزراعة المناطق الجافة ، جامعة الملك عبد العزيز

جـــــدة – المملكة العربية السعودية

المستخلص . يناقش البحث الطرق النظرية المناسبة لدراسة حركة المياه السطحية وتداخلها مع المياه الجوفية بالتركيز على الشروط الحدية للحالات المختلفة ، حيث تم نمذجة حركة المياه من النهر إلى المياه الجوفية عن طريق المنحنيات الثلاثية في نموذج تعدادي للمياه الجوفية .

وقد تم اختبار النموذج المقترح لحل إحدى المشاكل الممثلة في ضخ بئر اختبارية بالقرب من نهر تخللت مياهه الطبقة الحاملة للمياه الجوفية . وقد أظهرت نتائج النموذج العددي تطابقها مع نتائج الحل التحليلي لكلا الحالتين الثابتة والمؤقتة . كما تم اختبار النموذج المقترح للحالة المفترضة ولكن بدون فرضية النفاذية الكلية للبئر والنهر باستخدام نظرية خواص التربة لحركة المياه في المنطقة بين سطح مجري النهر والمياه الجوفية وبالإضافة إلى النموذج العـددي ذي الثلاثة أبعاد للمياه الجوفية . لوحظ وجود اختلاف كبير بالمقارنة بين الحل التحليلي والتمثيل العـددي للمياه الجـوفية ، خصوصًا بما يتعلق بحسابات كميات المياه المتبادلة بين النهر والمياه الجوفية ، ويرجع هذا الاختلاف نتيجة تأثير الظروف الحدودية على نتائج النموذج العددي . وقد أثبتت نتائج النموذج المقترح ملاءمته لدراسة حركة تبادل مياه النهر والمياه الجوفية للحالات التي تم دراستها ، ولكن يُقترح عمل دراسة نظرية إضافية لمشكلة حساب كمية المياه المتبادلة باستخدام نظرية خواص التربة الخاصة بحركة المياه .