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Vadose and Groundwater Interactive Model to Study the Soil Water Flow in Dry Condition

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Abstract. In order to estimate the efficiency of biological drainage in lowering groundwater levels, a numerical interactive link between the SWATR and FMAQ models is developed. SWATR simulates water flow in the vadose zone, while FMAQ simulates the flow in groundwater. In the vadose zone, one-dimensional vertical flow is assumed, while it is simulated as two-dimensional horizontal flow for the groundwater. The main assumption of this interactive link is that the actual evapotranspiration is considered equal to the negative recharge rate from the groundwater table, which is the only excitation of the groundwater FMAQ model. The effect of meteorological conditions, sink term description of plants and soil properties of vadose and groundwater zones are considered. The water table fluctuation is the main output of the model.

For the climate, soil and plant characteristics considered, and to drain biologically a groundwater table at depth 100 cm below ground surface, the conducted sensitivity analysis shows that best root depth of a plant should be between 90 and 100 cm. For the same water table in a clayey soil, plants of root depth of 60 cm do not need irrigation (under the studied conditions) since they are capable of extracting water from the groundwater for a period of 20 days. For root depth of 90 cm with no precipitation, the overall actual transpiration increases with the increase of temperature or wind speed. No change in overall actual transpiration was noticed until the value of precipitation reaches 5 mm/day. After this value, the trend of increase is exponential although it is very small. The overall transpiration after 28 days decreases in the following order: clay, silt, and finally sand.

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1. Introduction

It has been observed that, in the last two decades, the groundwater level has risen considerably in the agricultural lands due to the increase of water use, the lack of adequate drainage and the presence of the High Dam. For example, the average groundwater level in Siwa Oasis rose from 150 cm below ground surface in 1962 (Parsons, 1962), to 75 cm in 1987, according to the "First Annual Report of the Biological Drainage Project in Siwa Oasis" undertaken by the Desert Research Institute (1989). This rise in water levels has damaging effects on plant growth since it reduces the roots' capability of extracting water even from the vadose or the unsaturated zone.

This shallow groundwater level could affect the evapotranspiration that can be predicted while modeling the flow in the vadose zone, taking water uptake into consideration, which is essentially a one-dimensional vertical flow. To compute the change in groundwater the in-plan two-dimensional horizontal flow in the saturated zone has to be considered. For the complete analysis of the biological drainage problem, which is an interaction between the two above described systems, a three dimensional flow model for the unsaturated-saturated zones would be needed (Neuman, 1973). This may turn to be expensive, time consuming and of quite an elaborate mathematical formulation.

To bypass solving the three-dimensional problem, a numerical interactive link between two existing models is proposed in this research. The models are:

- SWATR (Feddes *et al.,* 1978), which is a one-dimensional vertical model of unsaturated flow that computes the actual evapotranspiration, and
- FMAQ (El-Didy and Contractor, 1986), which is a multi-aquifer model for the flow in the saturated zone, that computes the groundwater level for a given stimulation.

This suggested numerical link permits predicting the actual temporal evapotranspiration and the drop in groundwater level, given the type of plant, soil characteristics, and hydrogeological and meteorological data. The aim of this paper is first, to describe this link procedure, and second, to present the results of a parametric study of the response of the complete model to changes in the above-mentioned data. We begin first by describing briefly the two already existing models SWATR and FMAQ.

The SWATR Model

The numerical model SWATR (Feddes *et al.,* 1978) is a transient one-dimensional finite-difference model for the vadose zone. The model, which has a possibility of simulating water uptake by roots, was developed at the Institute for Land and Water Management Research, Wageningen, the Netherlands, and is programmed in FORTRAN 77.

The basic flow equation of this model (Equation 1) is based on Darcy's law, the continuity equation, and a volumetric sink term, describing the root water uptake. This equation is solved by a finite difference scheme, as proposed by Haverkamp *et al.,* 1978. The derivation of this equation and the main idea of the description of the sink term S and its calculations can be found in Feddes *et al.,* 1978.

$$\frac{\partial_\psi}{\partial_t} = \frac{1}{C(\psi)} \cdot \frac{\partial}{\partial_z}\left[K(\psi)\cdot\left(\frac{\partial\psi}{\partial z} - 1\right)\right] - \frac{S(\psi)}{C(\psi)} \tag{1}$$

Where

- ψ Soil water pressure head (negative in the vadose zone), [cm];
- C Differential moisture capacity $d\theta/dh$, with θ being the volumetric soil water content, [cm^{-1}];
- K Hydraulic conductivity of soil, [cm/day];
- S Water uptake by roots (sink term considered positive from the soil to the roots), [day^{-1}]. The integral of this sink term (S), over the rooting depth L_r, gives the actual transpiration, E_{pl}
- z Vertical distance, positive downwards, [cm]; and
- t Time, [days].

$$E_{pl} = \int_{z=0}^{z=L_r} S\,dz \tag{2}$$

and

$$E_{pl} = E - E_s \tag{3}$$

where E is the actual evaporation from soil and transpiration by plants, and E_s is the bare soil actual evaporation.

Two factors control the rate at which water is returned from the soil to the atmosphere by evapotranspiration E (evaporation from soil and transpiration by plants): atmospheric demand and soil-water availability. If soil water at the surface or in the root zone is not limited then, E^*, defined as the potential evapotranspiration, is equal to the potential rate as determined by air temperature, wind speed, relative humidity, solar radiation and other meteorological conditions.

Since in our research only biological drainage is considered, the evaporation from bare soil (E_s) is negligible and the whole area is cultivated with plants.

Therefore, the actual evapotranspiration and the actual transpiration by plants are equal and the two terms could be used interchangeably.

Assumptions

For the formulation of the equation and its solution procedure, the following assumptions are made:

1. One-dimensional vertical flow in the vadose zone,
2. Soil is homogeneous and isotropic,
3. Only desorption curve is considered in $K(\psi)$ and $\psi(\theta)$ (relationships (*i.e.,* no hysteresis),
4. Pneumatic potential is neglected ($\psi_{gas} = 0$):
   – Water flow through very dry soils can be satisfactory described by Darcy's law, considering liquid flow only.
   – Soil evaporation can be calculated as a Darcian flux into the atmosphere, where the soil matric potential head at soil surface ψ is in equilibrium with the humidity of the surrounding atmosphere.
5. Effect of salinity of the soil water on water flow is neglected,
6. Crop is supposed optimally supplied with nutrients, and
7. Flow of water from roots into soil is neglected.

Initial and Boundary Conditions

As initial condition for the SWATR model, one has to prescribe for each nodal point either the water content or the suction. For the boundary conditions, the case is that of a soil profile with a shallow groundwater level fluctuating with time. The processes in the vadose zone are governed both by the meteorological conditions at the soil surface and the conditions in the groundwater zone of the soil. Here one has a mixed type of boundary conditions: Specification of the dependent variable, *e.g.,* the groundwater level where the pressure head is zero at the bottom and specification of the flow through the boundaries at the soil surface, *e.g.,* the precipitation and the actual evapotranspiration as determined by the meteorological conditions and the water availability. The input and output of the SWATR program is summarized in Fig. 1.

The FMAQ Model

The FMAQ model (El-Didy and Contractor, 1986) is a two-dimensional finite element model, which has been developed for the simulation of water flow in a multi-aquifer system consisting of a number of aquifers separated by aquitards. Two-dimensional (x, y) flow is assumed to occur in the aquifers, with one-dimensional vertical flow occurring through the aquitards without any storage effect. The transient groundwater equation for horizontal flow in a single aquifer is expanded to include terms representing its interaction with adjacent aquifers. This equation is solved numerically using the finite element theory and the Galerkin weighted-residual method. The output of this program gives the piezometric head and velocity components at each node in all aquifers.

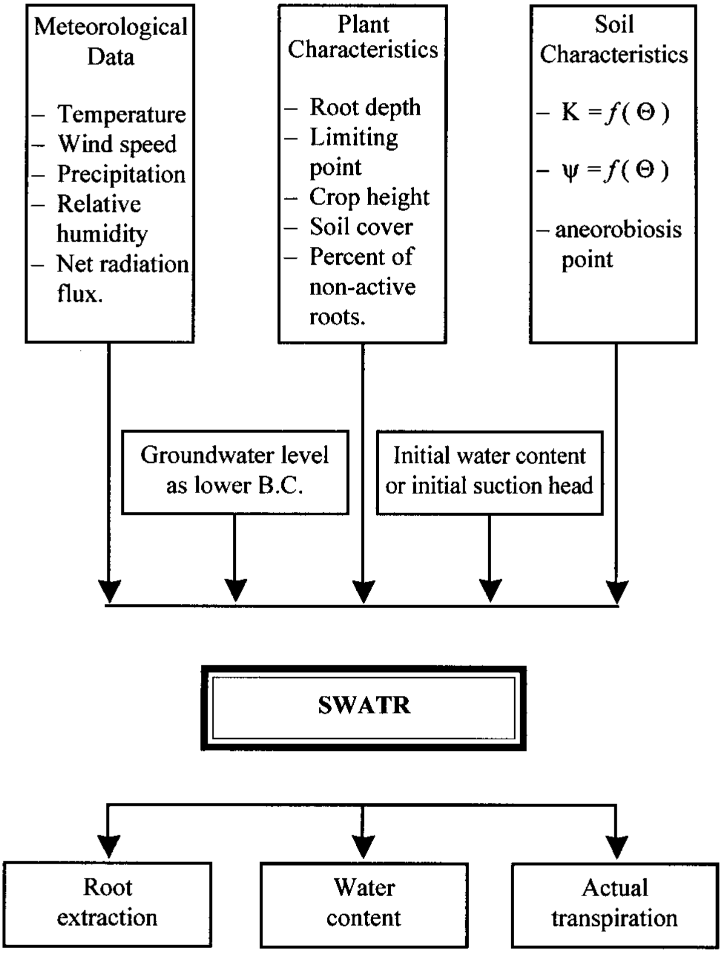

FIG. 1. Input-output description for the SWATR model.

The governing differential Equation (4) for two-dimensional groundwater flow in a non homogeneous, anisotropic, with point and distributed sources or sinks is:

$$\frac{\partial}{\partial x}\left(K_{xx}\cdot B\cdot\frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial y}\left(K_{yy}\cdot B\cdot\frac{\partial h}{\partial y}\right)+r_c=S_t\cdot\frac{\partial h}{\partial t} \tag{4}$$

Where:

h = $h(x, y, \mathrm{t})$ = depth-averaged piezometric head, [L];

K_{xx} = $K_{xx}(x, y)$ = aquifer conductivity in the x-direction,[L/T];

K_{yy} = $K_{yy}(x, y)$ = aquifer conductivity in the y-direction,[L/T];

B = $B(x, y)$ = saturated thickness;

r_c = $r_c(x, y, t)$ = distributed flux of water due to evapotranspiration or due to recharge,[L/T] ;

S_t = $S_t(x, y)$ = aquifer storage coefficient,[L^0];

x, y = Cartesian coordinates (principal axes of the hydraulic conductivity/transmissivity tensor), [L];and

t = time,[T].

Assumptions

For this simulation, the following assumptions are made:

1 – Water is considered to be a homogeneous fluid with constant density,
2 – Darcy's law is applicable,
3 – The Depuit approximation is assumed valid; therefore the flow is essentially horizontal in each aquifer,
4 – The aquifer storage is attributable only to specific yield if phreatic, or to storativity, if confined,
5 – The off-diagonal terms of the conductivity tensor, $K_{xy} = K_{yx} = 0$ (x and y are principal axes) for all aquifers, and
6 – The aquitard storage coefficient is neglected, since its effect is generally short lived and the concern is for long term effects.

Initial and Boundary Conditions

For transient flow problems, Equation (4) requires an initial condition, which is the specified piezometric head over the entire domain. Two types of boundary conditions for Equation (4) are considered. The first type requires the piezometric head prescription as

$$h = h_q\,(x, y, t) \tag{5}$$

The second type of boundary condition is prescription of the water flux q_s, across the aquifer boundary.

$$\left(K_x \cdot B \frac{\partial h}{\partial x} \cdot n_{-x} + K_y \cdot B \frac{\partial h}{\partial y} \cdot n_{-y} \right) = q_s(x, y, t) \tag{6}$$

Where n_x and n_y are the x and y components of the unit inward-pointing normal vector n on the boundary. The input and output of the FMAQ program is summarized in Fig. 2.

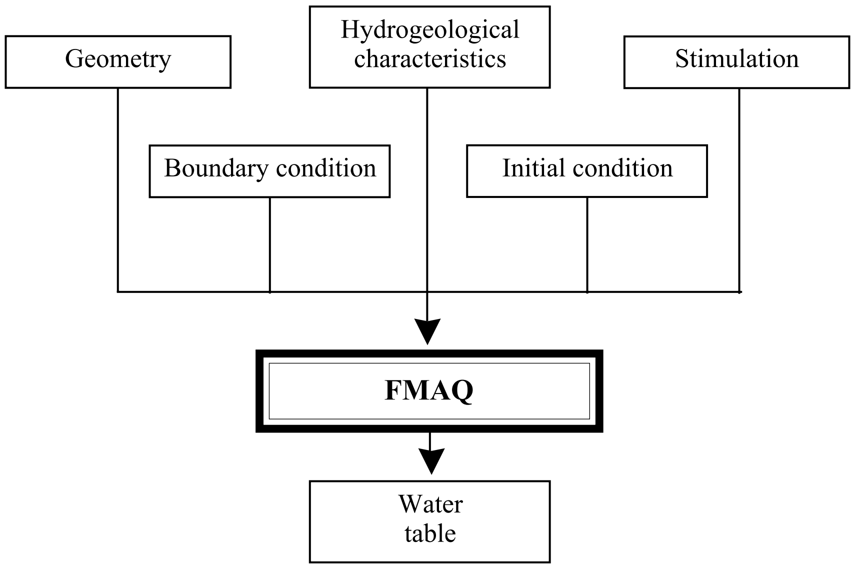

FIG. 2. Input/output description for the FMAQ model.

The Interface between the Two Models

As was previously mentioned, the SWATR program needs as an input the *a-priori* prescribed groundwater level as the lower boundary condition for the unsaturated zone, which prevents the estimation of the drop in water level due to biological drainage. On the other hand, the FMAQ program requires the distributed flux of water r_c as the excitation to compute the new water level. It is assumed that the actual evapotranspiration is equal to this distributed intake from groundwater table. This is true if no change of storage is considered in the vadose zone in each time increment. Therefore, an interaction is needed between the two programs since both depend on each other in a non-linear manner. For this an effective procedure for the transfer of parameters such as the water content in the unsaturated zone, the groundwater level, ... was implemented between the two codes. Thus, to determine the groundwater level drop due to evapotranspiration, the following steps are carried out:

- The study area is discretized into small elements. Cultivated areas (dashed elements) are identified. Actual evapotranspiration will be calculated as negative vertical recharge on each element. For each element, the iteration described in Fig. 3 is carried out for the four corner nodes as presented in Fig. 4.

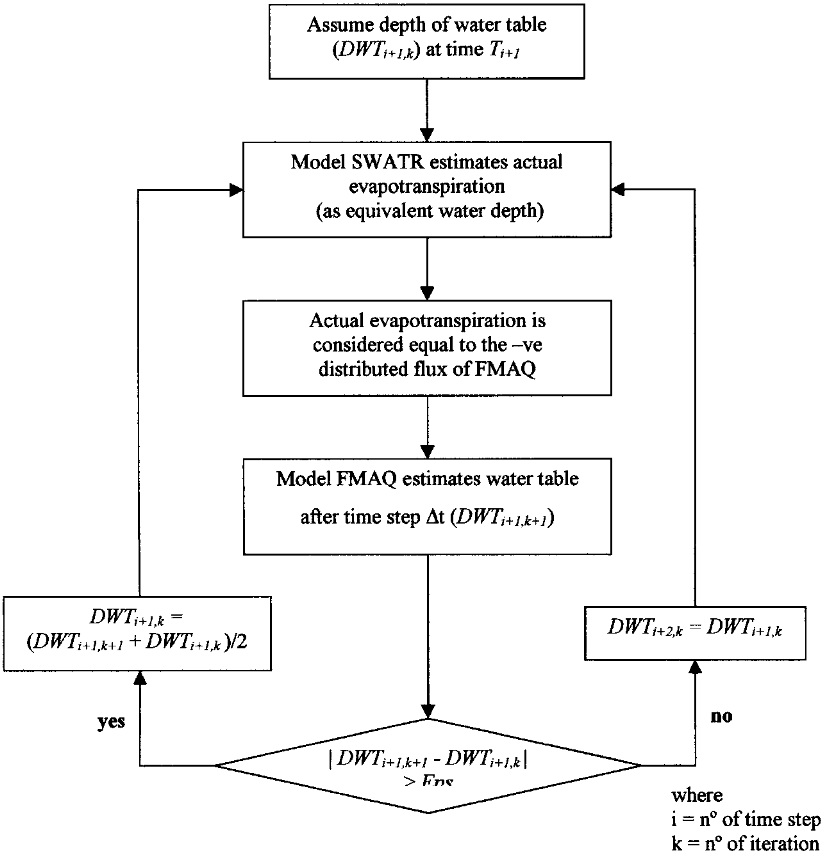

FIG. 3. Flow chart of the suggested numerical interactive link between SWATR and FMAQ.

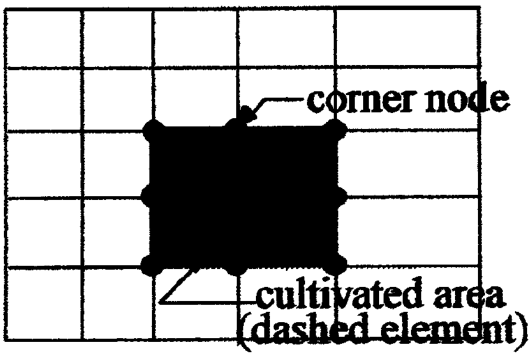

FIG. 4. Area discretization.

- Assume the groundwater level after time increment ΔT for this first iteration to be the original $DWT(i, k)$; *i.e.,* let $DWT(i + 1, k) = DWT(i, k)$, where i is the time step and k is the iteration number.
- Run SWATR once for each corner node of the cultivated (gray) elements (Fig. 4). SWATR is to be run for a time period ΔT with initial depth of groundwater level $DWT(i, k)$ and a final depth equal to $DWT(i + 1, k)$. SWATR's output is the actual evapotranspiration at the nodes in $[LT^{-1}]$ units.
- For each element, the algebraic mean of evapotranspiration at the four corner nodes (in adjacent elements) is the distributed flux of water from the element due to evapotranspiration (negative recharge).
- FMAQ is, then, run for the whole area having the negative recharge of evapotranspiration equal to the negative distributed flux of water, which is the only load on the system with a time increment ΔT and an initial groundwater level depth equal $DWT(i, k)$.
- The output of FMAQ is the new depth of the groundwater level after the time increment and equal to $DWT(i + 1, k + 1)$.
- $DWT(i + 1, k + 1)$ is compared to $DWT(i + 1, k)$.
- If $| DWT(i + 1, k + 1) - DWT(i +1, k) | = \varepsilon$ which determines the acceptable accuracy), then the assumption of $DWT(i + 1, k)$ is correct and $DWT(i + 2, k)$ is calculated similarly. It should be noted that when a new value of DWT is assigned, the following changes are introduced:
    - A new discretization of the vadose zone is made due to the fact that the groundwater level depth is the boundary of this zone.
    - New values of soil moisture content are assigned with a new time step, while soil moisture content values remain unchanged within the same time step.

Capabilities and Limitations of the Proposed Model

The proposed model can handle

- *Various soil types with two-layer soil profile in the vadose zones;*
- *Different types of vegetations,* the user specifies plant characteristics (*e.g.,* root depth, limiting point, crop height, soil cover fraction and non-active percentage);
- *Daily input of meteorological data,* in order to calculate potential evapotranspiration and soil potential evapotranspiration (*e.g.,* temperature, wind speed, precipitation, relative humidity and net radiation flux);
- *Shallow level of groundwater,* described in depth in the previous sections; and
- *Presence of wells, canals and ditches.*

On the other hand, the model can not handle:

1 – *Daily variation in meteorological data,* because the SWATR program offers only the opportunity for *one* 24-hour input of (mean) climatological data.

2 – *Vapor displacement inside soil* ($\psi_{gas\ is\ not\ zero}$): The liquid flow from the soil profile into the atmosphere is calculated with Darcy's Law. Thus, it is not taking into consideration, in the topsoil, of a liquid-vapor interface.

3 – *Osmotic potential gradients:* In saline soils, soil water does not have the same physical properties as in non-saline soils; *e.g.* density, viscosity and surface tension between air and water will be different in saline soil water. This problem is not considered.

Parametric Study of the Proposed Model

The second objective of this paper is to make a parametric study of factors affecting actual evapotranspiration. This sensitivity analysis is applied to evaluate the relative importance of almost all parameters and boundary conditions on the output of the model. In this study, the red cabbage on sticky clay case (as presented by Feddes, 1971) is taken as reference case, and input parameters are varied one at a time. This reference case is described later in detail in the next sections.

Assumptions of Parametric Study

For our numerical experiments, some values and empirical relations are kept the same as in the reference case described in the next section. These are:

1 – Initial Depth of the Groundwater

Since it is not possible to change, in the same time, both the initial depth of the groundwater and other plant and weather conditions, the initial depth of the groundwater was chosen to be at 100 cm below ground surface in all numerical experiments, to take advantage of the capabilities of the suggested numerical link previously described.

2 - Plant Characteristics

Because of lack of data on specific crops and their characteristics, it is assumed that the soil cover is 85% and that the crop height is 42 cm. Both of these factors influence the potential evapotranspiration. The empirical relations governing the potential evapotranspiration were kept constant in all our numerical experiments. These relations are:

- The leaf area index as a function of soil cover (Feddes, 1971);
- The aerodynamic resistance of a crop as a function of crop height (Rijtema, 1965); and

❍ The flux of intercepted precipitation as a function of precipitation rate (Rijtema, 1965).

3 – Properties of the Groundwater Zone

In our numerical experiments, no change was made to the properties of the groundwater zone.

The Reference Case: Red Cabbage on Sticky Clay

The input parameters describing the red cabbage case are as follows:

1 – Plant Characteristics

- Sink term description is shown in Fig. 5.
- Root depth = 90 cm.

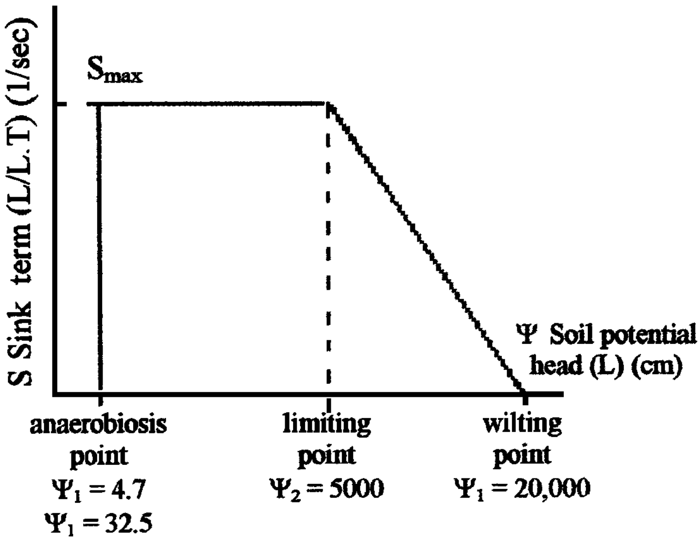

FIG. 5. Sink term description for reference case study (after Feddes *et al.,* 1978).

2 – Meteorological Data

- Tmperature = 25°C
- Wind speed = 3 m/s
- Precipitation rate = 0.0 mm/day

3 – Soil Characteristics

- Upper layer: 42.5 cm of light clay (1.125 gm/cm^3);
- Lower layer: 57.5 cm of dense clay (1.35 gm/cm^3);

- The soil moisture characteristics curve ($\psi = \theta$ relation) was taken from measurements done by Feddes (1971);
- The unsaturated hydraulic conductivity relation for clay is taken as well from the previously mentioned reference; and
- Initial water contents (after Bower, 1988).

Plan of Parametric Study

In the parametric study, the following inputs were changed one at a time with respect to the red cabbage reference case:

- Sink term description,
- Root depth,
- Average daily temperature,
- Precipitation rate at the 4th day of the 7-day time period simulation,
- Wind speed at 2 m from ground surface, and
- Soil type in the vadose zone.

The variation of the actual transpiration against the previous parameters are studied as well as the effect of the variation of each of these parameters on the soil water content and the water uptake by roots.

Conclusions of the Parametric Study

Several conclusions can be drawn out of this parametric study:

1 – Change in anaerobiosis point (ψ_1), for different types of soil, in the upper layer makes no change at all in the system since the range of pressure head ψ is far from the anaerobiosis point in this region, while a small change in anaerobiosis point of the lower layer has the largest effect on cumulative transpiration: the highest cumulative transpiration is obtained at the lowest value of anaerobiosis point = 4.7 cm. Increasing the value of ψ_1 to 40 cm yields to a strong reduction in cumulative transpiration. This is particularly true for the case of shallow groundwater.

2– The effect of changes in limiting point, ψ_2, seems to be of less importance, for root depths of 30, 60, 90 and 120 cm, with groundwater level initially at 100 cm from ground surface.

3 – Changing the wilting point, ψ_3, from 20,000 cm to 15,000 cm yields a bigger decrease in cumulative transpiration. The percentage of decrease is up to 20% in the case of 30 cm root depth. This is particularly true for the case of zero rainfall.

4 – Since it is required to biologically drain a groundwater, that is 100 cm below ground surface, the ideal plant should have a root depth of 90 cm to 100 cm. It is the root depth that gives the maximum transpiration (Fig. 6). It would be also of interest to define the optimum groundwater table that maximizes the transpiration for a fixed root depth, but this will be dealt with in further studies.

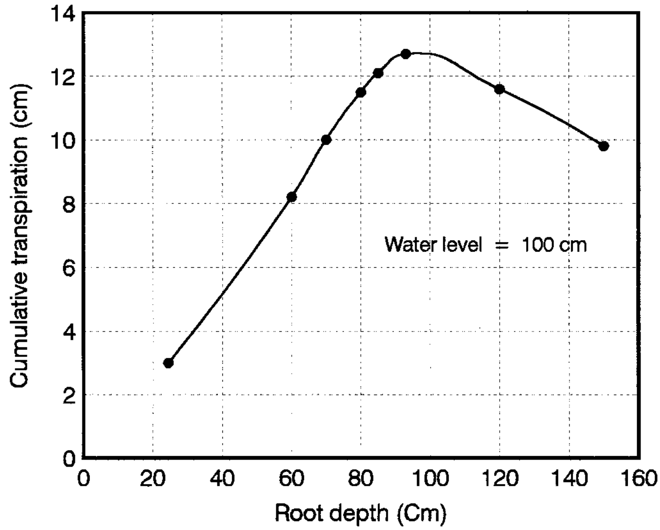

FIG. 6. Overall transpiration after 28 days versus root depths.

5 – For a groundwater level at 100 cm from ground surface in a clayey soil, plants of root depth = 60 cm do not need irrigation, since they are capable of extracting water from the groundwater below for as long as 20 days, even for high temperatures (25°C) or high wind speeds (3m/sec) (Fig. 7). These results could be useful in irrigation scheduling. Nevertheless, it should be remembered that no salt movement was considered in this model.

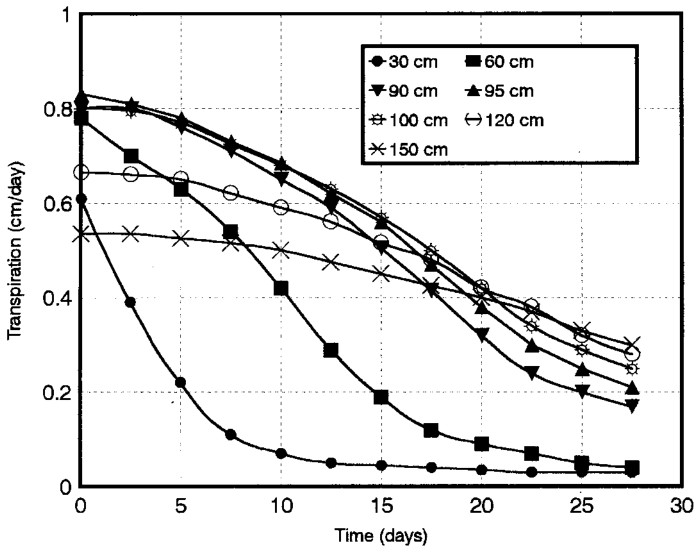

FIG. 7. Transpiration versus time for different root depths.

6 – For root depth of 90 cm and no precipitation condition, the overall actual transpiration increases with the increase of temperature (Fig. 8) or wind speed (Fig. 9.) This increase is not linear and tends to stabilize due to the lack of moisture in the vadose zone.

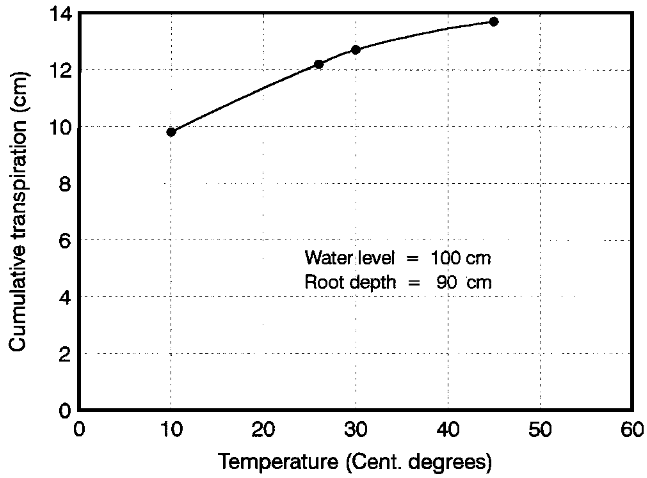

FIG. 8. Overall transpiration after 28 days versus temperature degree.

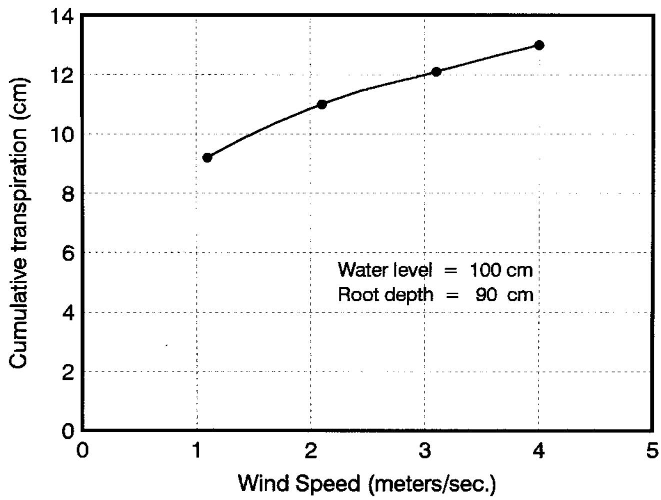

FIG. 9. Overall transpiration after 28 days versus wind speed.

7 – The overall increase in actual transpiration doesn't exceed 0.4%. Thus small values of precipitation have a small effect on water uptake by roots. No change in overall actual transpiration was noticed until the value of pre-

cipitation reaches 5mm/day. After this value, the increase is exponential, although it is very small (Fig. 10).

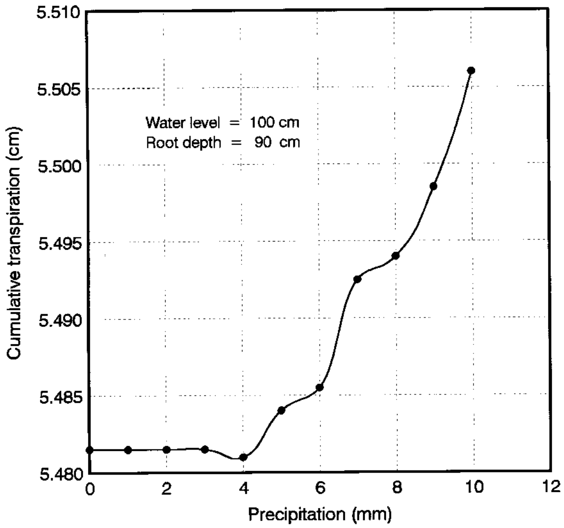

FIG. 10. Overall transpiration versus precipitation.

8 – For what concerns changing the soil type in the parametric studies, overall transpiration after 7 days gives misleading results: It shows that sand gives the second biggest value after clay. This is true only for short time periods, due to high conductivity of sand as was shown (Fig. 11a). From Fig. 11(b), one can notice that overall transpiration after 28 days decreases in the following order: clay, silt, and finally sand.

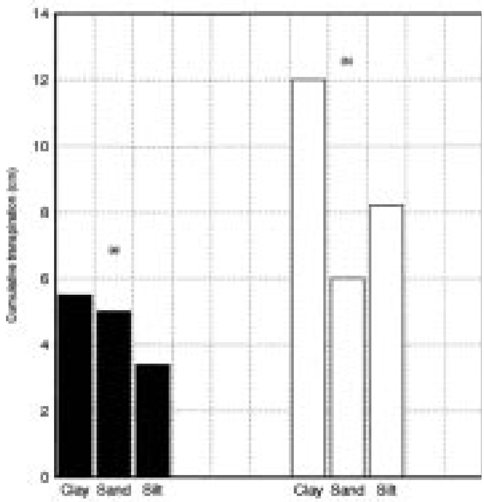

FIG. 11. Overall transpiration for different types of soil: a) After 7 days, b) After 28 days.

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نموذج تفاعلي للطبقة المشبعة وغير المشبعة لدراسة حركة المياه في التربـة تحت ظروف الجفـاف

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كلية الهندسة ، فرع الفيوم ، جامعة القاهرة ، الفيـــوم
و كلية الهندسة ، جامعة القاهرة ، الجـــيزة – جمهورية مصر العربية

المستــخلص . تم بناء نموذج عـددي يربط بين النمـوذجين العــددين (SWATR) ، (FMAQ) وذلك بهـدف دراسة كفـاءة الصرف البيـولوجي للمياه الجوفية ويمثل النموذج (SWATR) الحركة الرأسية للمياه في طبقة التربة غير المشبعة بينما يمثل النمـوذج الآخر (FMAQ) حركة المياه الجوفية المشبعـة في الاتجـاه الأفقي . وقد كـان الفرض الرئيسي في هذه الرابطة العددية هو اعتبار أن البخر – نتح يمثل تغذية سـالبة لسطح المياه الجـوفية ليكون الإثارة الوحيدة للنمـوذج (FMAQ) . تمت الدراسة بأخذ الأرصـاد الجوية وخواص النبات والتربة في الاعتبار ليكون المنتج النهائي هو التغير في مناسيب المياه الجوفية وقد تم عـمل اختبـار حسـاسيـة لأنسب عـمق جذري لـلصرف البيولوجي لمنسوب المياه الجـوفيـة الذي تم فرض القيـمة الابتدائيـة له على عـمق واحد متـر تحت سطح الأرض وكـان أنسب عـمق جـذري بين ٩٠، ١٠٠سم للتربة الرملية عند عدم الري وكـان ٦٠سم في حـالة التربة الطينيـة حيث إن النبات كان قـادراً على سـحب مـياه جوفيـة بدون ري لمدة ٢٠ يوما . وظهـر من الدراسة أيضًا أن النتح من النبات يزيد مع زيادة الحـرارة وسـرعة الرياح وأن تأثير المطر يظهـر حتى قـيـمة ٥ مم والتي بعدها يكون التأثير أسيًا (Exponential) وظهر أيضًا أن النتح بعد ٢٨ يوم يقـل في أنواع التـربة التـاليـة على التـرتيب وهي الطين ثم الطمي ثم الرمل .