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Using of the Adaptive Simulated Annealing (*ASA*) for Quantitative Interpretation of Self-Potential Anomalies due to Simple Geometrical Structures

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Abstract. A quantitative interpretation method of self-potential field anomalies has been proposed in this work. The method is designed and implemented for determining the center depth, electric dipole moment or magnitude of polarization, polarization angle, and geometric shape factor of an underground buried body from field data related to simple geometric structures such as cylinders, spheres and sheet-like bodies. The method is based on mathematical modeling by using the global optimization method of which known as an adaptive simulated annealing (*ASA*). The utility and validity of this method have been first demonstrated on a theoretical example using simulated data generated from a known model with different random errors and a known statistical distribution, where a very close agreement was obtained between assumed and evaluated parameters of the model. Subsequently, field data from Germany, India and Turkey have been considered for which the interpreted results by other interpretation methods are available for comparison. The agreement between the results obtained by our proposed method and other methods is good and comparable.

Keywords: Self-potential anomaly, Mathematical modeling, Exponential penalty function, Global optimization algorithm.

Introduction

The self-potential (SP) method is one of the oldest methods and enjoys wide applications in sulphides and graphites exploration and in geophysical groundwater investigations. The interpretation methods of SP anomalies are not yet very well developed. However, the quantitative interpretation of *SP* anomalies is usually carried out by approximating the causative source by simple geometrically shaped models (*viz*, sheet, cylinder, sphere…*etc*). According to this simplified concept, different interpretation techniques are available in the literature for the quantitative interpretation of *SP* anomalies. The methods proposed by (Yungul, 1950), (Paul, 1965) and (Bhattacharya and Roy, 1981) use certain characteristic points of the anomaly and hence they turned out to be less reliable in most cases. The curve-matching method proposed by (Meiser, 1962) is cumbersome, especially when there are many parameters to be determined. The methods of least-squares (Shalivahan *et al.,* 1998), (Abdelrahman *et al.,* 1997) and (Abdelrahman and Sharafeldin, 1997) involve a series of trials in minimizing the difference between the observed and the calculated values. The interpretation made by the methods based on Fourier and Hilbert transforms (Atchuta Rao *et al.,* 1982) and (Sundararajan *et al.,* 1990; 1996; 1998) is not straightforward and subjects to some inevitable errors in the estimation of the parameters, due to the inaccurate location of origin. Further, these methods are reliable only for very long profiles. Derivative analysis methods (Abdelrahman *et al.,* 1998; 2003) involve higher derivative anomaly.

We describe here a practical method of a nonlinear inversion technique for interpreting self-potential anomalies due to simple geometrical structures using a global optimization method known as the adaptive simulated annealing (Ingber, 1996).

Using the adaptive simulated annealing (*ASA*) - a variant of simulated annealing (*SA*), a constrained, nonlinear programming problem (*CNPP*) has been solved in an attempt to estimate the geophysical parameters related to spheres, cylinders or sheet-like structures.

The (*CNPP*) has been mathematically formulated to describe the geophysical problems. This (*CNPP*) is consisting of a mathematical objective or target function $f(v)$ to be minimized on an unbounded subset X contained in the real space of parameters. Where v is the vector of geophysical parameters and X is a subset defined by mathematical inequalities constraints of the form $g_i(v) \geq 0 \quad (i = 1, ..., m)$ where the geophysical parameters are supposed to satisfy. Ignoring these inequalities constraints probably yields to error estimations of parameters in the general case. The objective function $f(v)$ is taken, in this research, as the statistical likelihood function, which depends on the deviations between observed points and synthetic potentials and also on the number of observations.

The (*CNPP*) is very difficult to be solved in the domain of convex programming because the feasible subset X is not bounded in the parameters space. To avoid this difficulty, the (*CNPP*) is transformed into an unconstrained, nonlinear programming problem (*UNPP*), by introducing an exponential penalty function. The goal of using the penalty function is to eliminate the constraints $g_i(v) \geq 0 \quad (i = 1, \ldots, m)$ of (*CNPP*) and reactive them anew in the target function of (*UNPP*). The target function of (*UNPP*) is then consisting of both, the objective function of (*CNPP*) and the suggested exponential penalty function. The penalty function concept has been previously used, in logarithmic form, for the interpretation of *SP* and magnetic anomalies (Asfahani and Tlas, 2004; 2005).

In the present work, the (*UNPP*) is solved by the adaptive simulated annealing (*ASA*) random search algorithm (stochastic method), known for optimizing numerical functions of several real variables. The obtained solution of the (*UNPP*) includes the geophysical parameters of the studied structure such as: depth, amplitude coefficient, index parameter, and shape factor.

The validity of this method is tested on a synthetic example with different random errors of 5% and 10% and through practical field data from Germany, India and Turkey.

The (*ASA*) stochastic algorithm is one of the most well developed and widely used iterative techniques for solving optimisation problems. The method uses an analogy between the process of physical annealing and the mathematical problem of obtaining the global minimum of a function (assimilated to energy) which may have local minima (metastable states). This algorithm can be easily coded, robust, and does not require differentiation of the target function with respect to the decision variables (Ingber and Rosen, 1992) and (Ingber, 1989; 1996). The appendix explains in more detail the steps of this algorithm and provides sufficient information to allow its implementation and coding.

Geophysical Problem Formulation for the Case of a Two-Dimensional Inclined Sheet Model

The expression for potential (V) for a sheet like body, *i.e.,* for a pair of line poles at any point on the free surface in a Cartesian coordinate system (Fig.1) is given by (Roy and Chowdhury, 1959) as follows:

$$V\,(r_1, r_2) = \frac{\rho I}{2\pi} Ln \frac{r_1^2}{r_2^2}$$

Where r_1 and r_2 are the distance of the edges of the sheet from the point of observation, I is the current per units length and ρ is the resistivity of the surrounding medium.

From Fig.1, it is shown that:

$$r_1^2 = x^2 + \left(h - \frac{a}{2}\sin\theta\right)^2 \text{ and } r_2^2 = \left(x - a\cos\theta\right)^2 + \left(h + \frac{a}{2}\sin\theta\right)^2$$

Where, h is the depth centre of the sheet, θ is the inclination, a is the width of the sheet and x is the distance of the observation point from the origin.

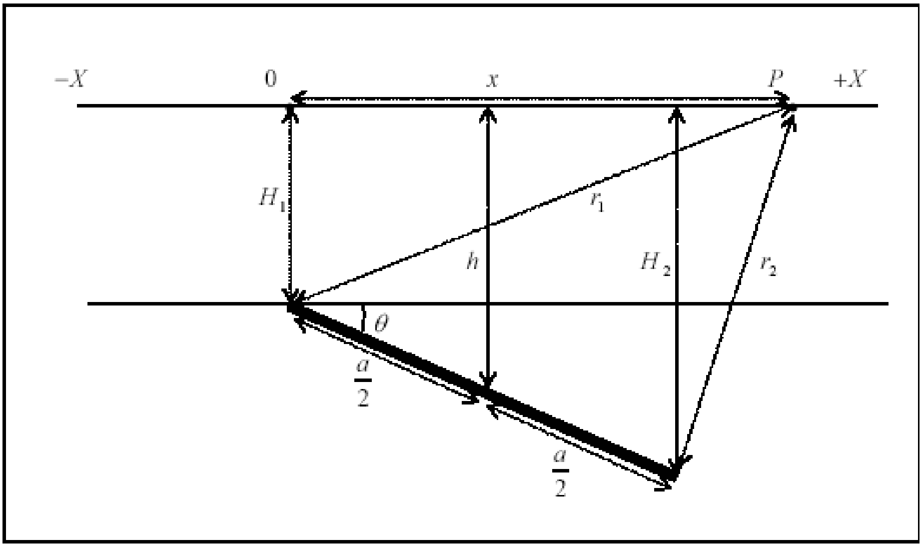

Fig. 1. The diagram for a sheet-like structure.

Replacing the expressions of r_1, r_2 and $k = \frac{\rho I}{2\pi}$ (electric dipole moment) into the equation of $V\,(r_1, r_2)$, the following equation could be obtained:

$$V\,(x_i, h, a, \theta, k) = kLn\left(\frac{x_i^2 + \left(h - \frac{a}{2}\sin\theta\right)^2}{\left(x_i - a\cos\theta\right)^2 + \left(h + \frac{a}{2}\sin\theta\right)^2}\right) \quad (i = 1, ..., N)$$

The evaluation of the geophysical parameters (h, a, θ, k) related to the sheet-like structure could be obtained by solving the following constrained, nonlinear programming problem (*CNPP*):

$$Maximize\ LH\,(h, a, \theta, k, \sigma) = \prod_{i=1}^{N} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{e_i - \bar{e}}{\sigma}\right)^2}$$

$$\begin{aligned} &Subject\ to && h - \frac{a}{2}\sin\theta \geq 0, \\ &&& h \geq 0, \\ &&& a \geq 0, \\ &&& 0 \leq \theta \leq 180^0 \quad and \\ &&& -\infty < k < +\infty. \end{aligned} \qquad (CNPP)_1$$

Where N is the number of observation points and $e_i = L(x_i) - V\,(x_i, h, a, \theta, k)\ (i = 1, ..., N)$ are the deviations between the observed points $L(x_i)\ (i = 1, ..., N)$ and the synthetic potentials $V\,(x_i, h, a, \theta, k)\ (i = 1, ..., N)$ at the discrete points $x_i\ (i = 1, ..., N)$.

The objective function $LH\,(h, a, \theta, k, \sigma)$ of the mathematical model $(CNPP)_1$ is known as the statistical likelihood function.

The variable $e = (e_1, e_2 ...., e_N)^T$ is a random variable and accords to the Gaussian (normal) distribution. The variables $\bar{e}$ and σ are the arithmetic mean and the standard deviation of the residuals $e_i\ (i = 1, ..., N)$ (standard error) respectively. For a best estimation of parameters, $\bar{e}$ must be equal to zero and σ must be a minimal. Taking these criteria in consideration, the mathematical model $(CNPP)_1$ becomes:

$$Maximize\ LH(h,a,\theta,k,\sigma)=\prod_{i=1}^{N}\frac{1}{\sqrt{2\pi}\,\sigma}\,e^{-\frac{1}{2}\left(\frac{L(x_i)-V(x_i,h,a,\theta,k)}{\sigma}\right)^2}$$

$$\begin{aligned} Subject\ to\qquad & h-\frac{a}{2}\sin\theta\geq 0,\\ & h\geq 0,\\ & a\geq 0,\\ & 0\leq\theta\leq 180^0\quad and\\ & -\infty<k<+\infty. \end{aligned}\qquad (CNPP)_2$$

The statistical likelihood function $LH(h,a,\theta,k,\sigma)$ is strictly positive. To insure obtaining more accurate and more precise parameters estimation, it is preferable to take the natural logarithm of the statistical likelihood function, which gives the following equivalent mathematical model.

$$Maximize\ g(h,a,\theta,k,\sigma)=-N\,Ln\left(\sqrt{2\pi}\,\sigma\right)-\frac{1}{2\sigma^2}\sum_{i=1}^{N}\left(L(x_i)-V(x_i,h,a,\theta,k)\right)^2$$

$$\begin{aligned} Subject\ to\qquad & h-\frac{a}{2}\sin\theta\geq 0,\\ & h\geq 0,\\ & a\geq 0,\\ & 0\leq\theta\leq 180^0\quad and\\ & -\infty<k<+\infty. \end{aligned}\qquad (CNPP)_3$$

Where, $g(h,a,\theta,k,\sigma)=Ln\ \left(LH(h,a,\theta,k,\sigma)\right)$.

To solve the mathematical model $(CNPP)_3$, it is sufficient to solve the following equivalent mathematical model

$$Minimize\, f(h,a,\theta,k,\sigma) = N\,Ln\left(\sqrt{2\pi}\,\sigma\right) + \frac{1}{2\sigma^2}\sum_{i=1}^{N}\left(L(x_i) - V(x_i,h,a,\theta,k)\right)^2$$

$$\begin{aligned} Subject\ to \quad & h - \frac{a}{2}\sin\theta \ge 0, \\ & h \ge 0, \\ & a \ge 0, \\ & 0 \le \theta \le 180^0 \quad and \\ & -\infty < k < +\infty. \end{aligned} \qquad (CNPP)_4$$

Where

$$Maximize\ g(h,a,\theta,k,\sigma) = -Minimize\left(-g(h,a,\theta,k,\sigma)\right) = -Minimize\ f(h,a,\theta,k,\sigma)$$

The problem $(CNPP)_4$ is very difficult to be solved in the domain of convex nonlinear programming because the feasible region:

$$X = \left\{(h,a,\theta,k) \in \mathbb{R}^4 / h \ge 0, a \ge 0, h - \frac{a}{2}\sin\theta \ge 0, 0 \le \theta \le 180^0, and -\infty < k < +\infty\right\}$$

is not bounded in the geophysical parameters space $\mathbb{R}^4$. To avoid this difficulty, the mathematical problem $(CNPP)_4$ is converted into an unconstrained, nonlinear minimization ($UNPP$) one by introducing a new objective function $\phi(h,a,\theta,k,\sigma)$, which considers both the objective function $f(h,a,\theta,k,\sigma)$ of $(CNPP)_4$ and a suggested exponential penalty function. The penalty function is defined by the bounded constraints of the studied problem $(CNPP)_4$. This new objective function is defined as follows:

$$\phi(h,a,\theta,k,\sigma) = f(h,a,\theta,k,\sigma) + r\sum_{i=1}^{m} e^{-\frac{g_i}{r}}$$

Where: $g_1 = h - \frac{a}{2}\sin\theta, g_2 = h, g_3 = a, g_4 = \theta,\ g_5 = 180^0 - \theta$, m is the number of constraints $g_i\ (i = 1,...,m = 5)$ and r (penalty factor) is an arbitrary positive real number chosen to be close to zero, and is taken as equal to $\frac{1}{N}$ in the interpretation related to a sheet-like structure. Using this new function, the problem $(CNPP)_4$ becomes as follows:

$$\begin{aligned} &Minimize\ \phi(h,a,\theta,k,\sigma) \\ &subject\ to\quad (h,a,\theta,k,\sigma)\in\mathbb{R}^5. \end{aligned} \qquad (UNPP)$$

Where the objective function of (*UNPP*) is:

$$\phi(h,a,\theta,k,\sigma)=N\,Ln\left(\sqrt{2\pi}\sigma\right)+\frac{1}{2\sigma^2}\sum_{i=1}^{N}\left(L(x_i)-kLn\left(\frac{x_i^2+\left(h-\frac{a}{2}\sin\theta\right)^2}{\left(x_i-a\cos\theta\right)^2+\left(h+\frac{a}{2}\sin\theta\right)^2}\right)\right)^2+$$

$$r\times\left(e^{-\frac{h-\frac{a}{2}\sin\theta}{r}}+e^{-\frac{h}{r}}+e^{-\frac{a}{r}}+e^{-\frac{\theta}{r}}+e^{-\frac{180-\theta}{r}}\right)$$

The unconstrained, nonlinear programming problem (*UNPP*) is thereafter solved by the adaptive simulated annealing random search algorithm, which is implemented for solving unconstrained, nonlinear mathematical problems. The new interpretative technique is firstly tested on a theoretical synthetic example with different random errors in order to demonstrate its efficiency and stability. The method is secondly tested on field examples from India and Germany.

Interpretation of a Theoretical Synthetic SP Anomaly due to a Sheet Model with Random Errors

A synthetic *SP* anomaly $V(x_i,h,a,\theta,k)$ $(i=1,...,N)$ due to a sheet-like body is generated using the following assumed parameters: width of sheet $a=14$ unit length, depth centre of sheet $h=40$ unit length, inclination angle $\theta=50^0$ and electric dipole moment $k=70\ mV$. Two new theoretical *SP* anomalies are randomly regenerated, depending on the synthetic *SP* anomaly, by using the continuous uniform distribution with maximum random errors of 5% and 10%, respectively. The continuous uniform distribution is purposely used in order to randomly regenerate two *SP* anomalies resembling to real field data.

Both regenerated theoretical *SP* anomalies are thereafter interpreted by the proposed interpretative method, where the *SP* evaluated parameters are presented in Table 1.

Table 1. Interpretation of a *SP* synthetic anomaly due to a sheet model with 5% and 10% random errors.

SP geophysical parameters	*SP* assumed parameters	*SP* evaluated parameters with 5% random error	*SP* evaluated parameters with 10% random error
a (unit length)	14	14.08	14.67
h (unit length)	40	39.68	39.47
θ^0	50	51.09	51.28
k (mV)	70	70.14	70.82
σ (mV)	-	0.011	0.020

The results of Table 1 show a good agreement between assumed and evaluated *SP* parameters, which consequently indicate the efficiency of the proposed interpretative method.

Interpretation of SP Field Anomalies Due to a Sheet-Like Structure

Two *SP* field anomalies due to a sheet model from India and Germany have been reinterpreted by the proposed method.

1. The first field anomaly is presented in Fig.2; this anomaly of a 255 *m* long profile is taken across a mineralized belt in Kalava fault zone, 52 *km* south of Karnool in Cuddapah baisn, Andhra, Pradesh, India (Atchuta *et al.,* 1982).

The evaluated *SP* parameters of this anomaly obtained by the present interpretative inversion are:

$$a = 26.58\,m\,, h = 34.68\,m\,, \theta^0 = 102.32, k = 69.75\,mV\,, and\ \sigma = 8.01\,mV\,.$$

The obtained results are shown in Table 2, which includes also the interpretation results obtained by (Atchuta *et al.,* 1982) who used a Fourier transform method and by (Asfahani and Tlas, 2005) who used a constrained and penalized nonlinear least squares optimization method for the same *SP* field anomaly.

It is to notice that the standard error σ of the proposed method (8.01) is approximately the same of that of (Asfahani and Tlas, 2005) and is less than that of (Atchuta *et al*., 1982), which obviously attests the accuracy of the proposed inversion.

Table 2. Interpretation of *SP* anomaly over a sulfide body in the Kalava fault zone, Cuddapah basin, India.

SP geophysical parameters	*SP* evaluated Parameters (Atchuta *et al.* , 1982)	*SP* evaluated parameters (Asfahani and Tlas, 2005)	*SP* evaluated parameters (present method)
a (m)	26.92	26.73	26.58
h (m)	28.55	34.74	34.68
θ^0	110	102.59	102.32
k (mV)	63.68	69.50	69.75
σ (mV)	10.97	8.09	8.01

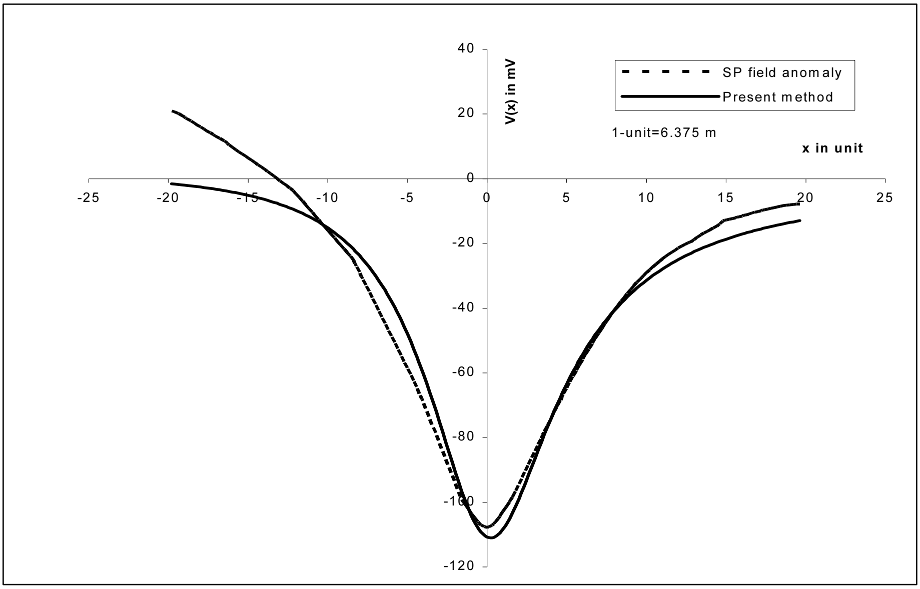

Fig. 2. SP **field anomaly over a sulfide body in the Kalava fault zone (Cuddapah basin, India). The theoretical curve for our method is shown.**

2. The second field anomaly is presented in Fig.3; this anomaly of a 520.5 *m* long profile is taken over a graphite deposit in the south Bavarian woods, Germany (Meiser, 1962) and (Abdelrahman *et al.,* 1999).

The sheet evaluated parameters obtained by the interpretative method are:

$a = 51.84\,m, h = 48.72\,m, \theta^0 = 51.10, k = 269.97\,mV, and\ \sigma = 7.51\,mV.$

The obtained results are shown in Table 3, which includes also the interpretation results obtained by (Abdelrahman *et al.,* 1999) who used an unconstrained least squares approach and by (Asfahani and Tlas, 2005) for the same *SP* field anomaly. The depth to the centre of the sheet obtained by the present method also agrees with the depth obtained by (Meiser, 1962), who used a double logarithmic net method (53 m).

Table 3. Geophysical parameters interpretation of *SP* anomaly over a graphite ore body, southern Bavarian woods, Germany.

SP geophysical parameters	*SP* evaluated Parameters (Abdelrahman et al. , 1999)	*SP* evaluated parameters (Asfahani and Tlas, 2005)	*SP* evaluated parameters (present method)
a (m)	40.11	51.85	51.84
h (m)	51.05	48.73	48.72
θ^0	49.5	47.83	51.10
k (mV)	363.6	269.88	269.97
σ (mV)	12.27	7.68	7.51

It is to notice that the standard error σ of our method (7.51) is less than that of (Abdelrahman *et al.,* 1999) and that of (Asfahani and Tlas, 2005), which indicates clearly the accuracy of the proposed inversion.

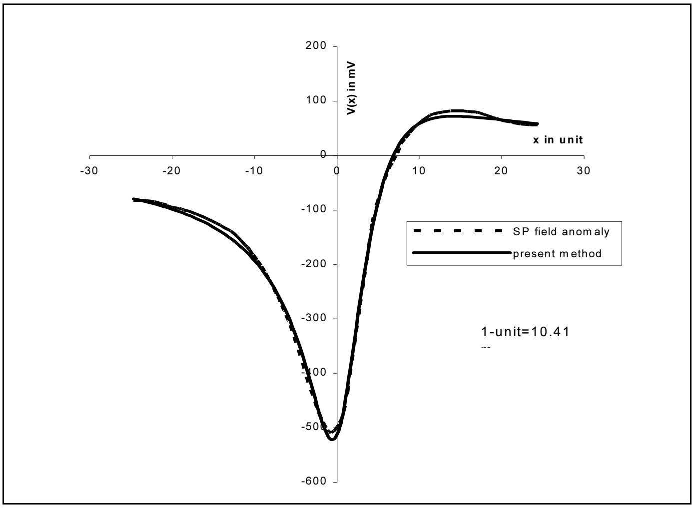

Fig. 3. ***SP*** **field anomaly over a graphite ore body, southern Bavarian woods, Germany. The theoretical curve for our method is shown.**

Geophysical Problem Formulation for the Case of a Cylinder or a Sphere Model

The general mathematical expression for potential (T) for a cylinder or a sphere-like structure at any point on the free surface in a Cartesian coordinate system (Fig.4) is given by (Bhattacharya and Roy, 1981) as follows:

$$T(x_i, z, \varphi, p, q) = p\,\frac{x_i \cos\varphi + z \sin\varphi}{\left(x_i^2 + z^2\right)^q} \qquad \left(i = 1, \dots, N\right)$$

Where z is the depth from the surface to the centre of the body, φ is the polarization angle, p is the electric dipole moment or the magnitude of polarization, x_i is the position coordinate, and q is the geometric shape factor which takes values as follows:

$$q = \begin{cases} 0.5 & \textit{for a vertical cylinder} \\ 1 & \textit{for a horizontal cylinder} \\ 1.5 & \textit{for a sphere} \end{cases}$$

Following the same procedure presented in the case of the sheet model, the evaluation of the geophysical parameters (z, φ, p, q) related to a cylinder or a sphere-like structure could be obtained by solving the following constrained, nonlinear programming problem:

$$\textit{Minimize } U(z, \varphi, p, q, \sigma) = N\,Ln\left(\sqrt{2\pi}\,\sigma\right) + \frac{1}{2\sigma^2}\sum_{i=1}^{N}\left(L(x_i) - T(x_i, z, \varphi, p, q)\right)^2$$

$$\begin{aligned} \textit{Subject to} \quad & z \geq 0 \\ & 0.5 \leq q \leq 1.5 \\ & -90^0 \leq \varphi \leq 90^0 \\ & -\infty < p < +\infty. \end{aligned} \qquad (CNPPS)$$

The (*CNPPS*) model is converted to an unconstrained, nonlinear programming problem using the exponential penalty function, where the new model becomes as follows:

$$\begin{aligned} & \textit{Minimize } \psi(z, \varphi, p, q, \sigma) \\ & \textit{Subject to } \quad (z, \varphi, p, q, \sigma) \in \mathbb{R}^5 \end{aligned} \qquad (UNPPS)$$

Where the objective function of (*UNPPS*) is:

$$\psi(z,\varphi,p,q,\sigma)=U(z,\varphi,p,q,\sigma)+r\times\left(e^{-\frac{z}{r}}+e^{-\frac{90+\varphi}{r}}+e^{-\frac{90-\varphi}{r}}+e^{-\frac{1.5-q}{r}}+e^{-\frac{q-0.5}{r}}\right)$$

The adaptive simulated annealing algorithm is then used to solve the (*UNPPS*) model in order to directly obtain the *SP* parameters (z,φ,p,q) of the structure.

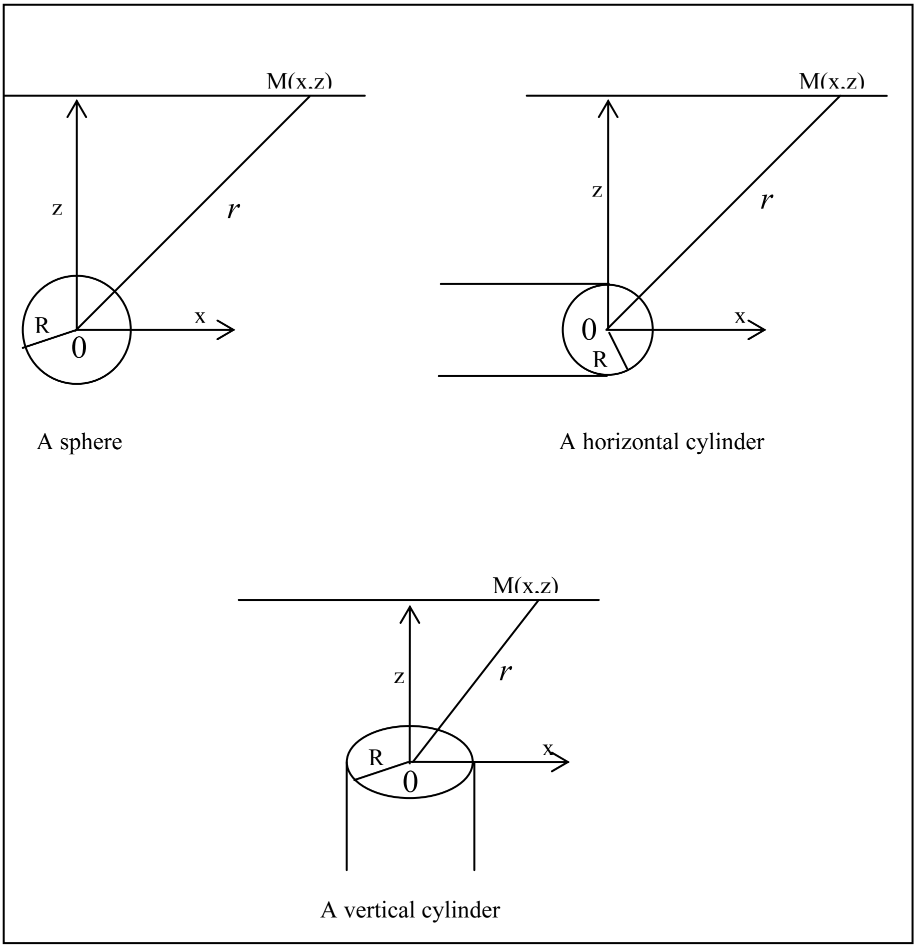

Fig. 4. The diagrams for simple geometrical structures (sphere and cylinder).

Interpretation of SP Field Anomalies Due to a Cylinder or a Sphere-Like Structure

A *SP* field anomaly due to a cylinder or a sphere model from Turkey has been reinterpreted by the proposed method.

The field anomaly is presented in Fig.5, this anomaly of a 262 *m* long profile is taken over a polarized copper ore body in Ergani district, 65 *km* SE of Elazig in eastern Turkey (Yungul, 1950) and (Bhattacharya and Roy, 1981).

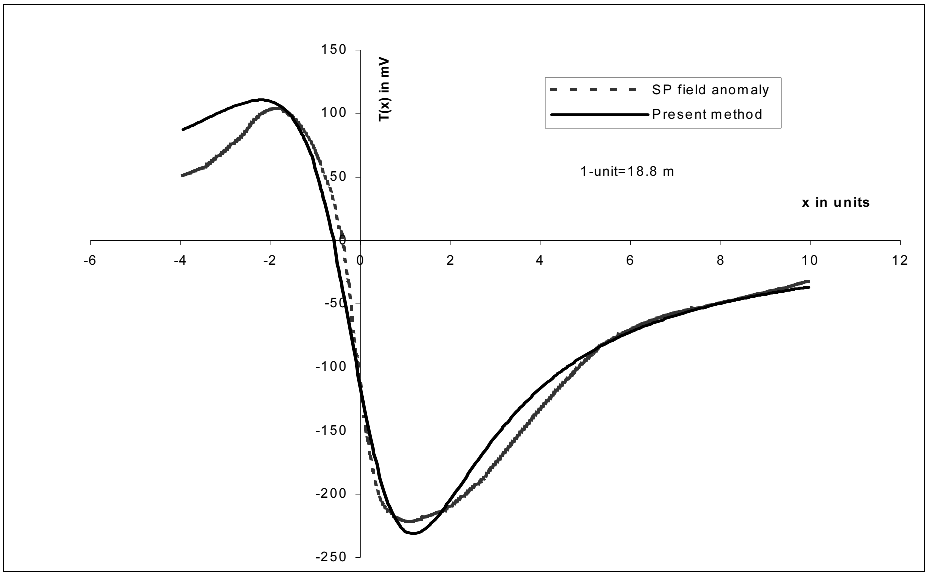

Fig. 5. ***SP*** **anomaly over a polarized copper ore body formation in Ergani district,Turkey. The theoretical curve for our method is shown.**

The evaluated *SP* parameters of this anomaly obtained by the present interpretative inversion are:

$$z = 35.41\,m, \varphi^0 = 17.76, p = -904.03\,mV, q = 1.19, and\ \sigma = 15.47\,mV.$$

The obtained results are shown in Table 4, which includes also the interpretation results obtained by (Abdelrahman *et al.,* 1997), who used an unconstrained least squares method, and by (Asfahani and Tlas, 2005) who used a constrained and penalized nonlinear inversion approach for the same *SP* field anomaly.

Table 4. Interpretation of *SP* anomaly over a polarized copper ore body, Turkey.

SP geophysical parameters	*SP* evaluated Parameters (Abdelrahman et al., 1997)	*SP* evaluated parameters (Asfahani and Tlas,2005)	*SP* evaluated parameters (present method)
z (m)	38.78	35.69	35.41
φ^0	14.67	17.66	17.76
p (mV)	-1549.36	-928.21	-904.03
q	1.36	1.19	1.19
σ (mV)	18.27	15.62	15.47

It is to notice that the standard error σ of our method (15.47) is less than that of (Abdelrahman *et al*., 1997) and also less than that of (Asfahani and Tlas, 2005), which attests the accuracy of the proposed inversion.

The geometric shape factor obtained by the proposed method ($q = 1.19$) suggests that the shape of the buried structure resembles a 2D horizontal cylinder model buried at a depth of 35.41 m. The depth obtained by the proposed method agrees well with that obtained by (Yungul, 1950)$(z = 38.8m)$, (Bhattacharya and Roy, 1981) $(z = 40m)$ and (Abdelrahman and Sharafeldin, 1997)$(z = 42m)$, a priori assuming the buried body resembles a spherical target$(q = 1.5)$. It also agrees well with the depth obtained by (Sundararajan and Srinivas, 1996) $(z = 36m)$ and (Shalivahan *et al*., 1998) $(z = 45m)$, assuming a priori the buried body resembles a horizontal cylinder target$(q = 1)$.

Discussion and Conclusion

A practical interpretative inversion has been presented in order to interpret self-potential(SP) field data due to simple geometrical structures (sheets, cylinders, spheres…etc). The method is proposed to determine the center depth, amplitude coefficient, and shape factor of buried structures. The inversion has been first tested on a theoretical model with different random errors of 5% and 10%, where a very close agreement was obtained between assumed and evaluated parameters.

Subsequently, field data from Germany, India and Turkey have been considered for reinterpretation. The agreement between the results obtained by the proposed inversion and other interpretative methods is good and comparable. The present inversion is based on the mathematical modeling concept and also on the stochastic optimization. It consists of three main phases: the first phase is to formulate the conventional nonlinear mathematical model under constraints imposed on the geophysical parameters, which describes the geophysical problem related to the studied structure. The second phase is to convert the constrained, nonlinear model to an unconstrained, nonlinear one (penalized model) by introducing an exponential penalty function. The third phase is to solve the penalized model by the adaptive simulated annealing, stochastic algorithm to determine the *SP* geophysical parameters due to the studied structure. In addition, most of the interpretative methods developed to interpret *SP* field anomalies assume a fixed simple geometrical model as a sphere, a horizontal cylinder, a vertical cylinder, or a sheet. In most cases, these methods consider the geometric shape factor of the buried body to be a priori assumed or predetermined. The problem of determining the geometric shape factor of a buried structure can be solved by using the present inversion, where it is considered as an additional free parameter to be evaluated. Moreover, a statistical criterion of preference (σ) is used and applied on the interpretative results of three *SP* field data. This criterion indicates clearly the robustness and the efficiency of the proposed inversion. This inversion can be also easily generalized for interpreting *SP* field data of various geometries, knowing the suitable mathematical formula related to the data. Therefore, this easy and accurate inversion technique can be employed for routine analysis and inversion of self-potential field data.

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References

Abdelrahman, E.M., Ammar, A.A., Hassanein, H.I. and **Hafez, M.A.** (1998) Derivative analysis of *SP* anomalies, *Geophys*, **63**: 890-897.

Abdelrahman, E.M., El-Arabi, H.M., Hassaneen, A.R. and **Hafez, M.A.** (2003) New methods for shape and depth determinations from *SP* data, *Geophys*, **68**: 1202-1210.

Abdelrahman, E.S.M., El-Araby, T.M., Ammar, A.A. and **Hassanein, H.I.** (1997) A least-squares approach to shape determination from residual self-potential anomalies, *Pure and Appl Geophys*, **150**: 121-128.

Abdelrahman, E.S.M. and **Sharafeldin, S.M.** (1997) A least-squares approach to depth determination from self-potential anomalies caused by horizontal cylinders and spheres, *Geophys*, **62**: 44-48.

Abdelrahman, E.S.M., Hassaneen, E.G. and **Hafez, M.A.** (1999) A least-squares approach for interpretation of self-potential anomaly over a two-dimensional inclined sheet, *Arabian Journal for Science and Engineering*, **24**(1A): 35-42.

Asfahani, J. and **Tlas, M.** (2004) Nonlinearly constrained optimization theory to interpret magnetic anomalies due to vertical faults and thin dikes, *Pure and Appl Geophys*, **161**: 203-219.

Asfahani, J. and **Tlas, M.** (2005) A constrained nonlinear inversion approach to quantitative interpretation of self-potential anomalies caused by cylinders, spheres and sheet-like structures, *Pure and Appl Geophys*, **162**: 609-624.

Atchuta, R., Ram, B. and **Sivakumar Sinha, G.D.J.** (1982) A Fourier transform method for the interpretation of self-potential anomalies due to two to two-dimensional inclined sheets of finite depth extent, *Pure and Appl Geophys*, **120**: 365-374.

Bhattacharya, B.B. and **Roy, N.** (1981) A note on use of a Nomogram for self-potential anomalies, *Geophys Prosp*, **29**: 102-107.

Ingber, L. (1989) Very fast simulated re-annealing, *Mathematical and Computer Modelling*, **12**(8): 967-973.

Ingber, L. (1996) Adaptive simulated annealing (ASA): lessons learned, *J. Control and Cybernetics*, **25**: 33-54.

Ingber, L. and **Rosen, B.** (1992) Genetic algorithms and very fast simulated re-annealing: A comparison, *Mathematical and Computer Modelling*, **16**(11): 87-100.

Meiser, P. (1962) A method of quantitative interpretation of self- potential measurements, *Geophys Prosp*, **10**: 203-218.

Paul, M.K. (1965) Direct interpretation of self –potential anomalies caused by inclined sheets of infinite extension, *Geophys*, **30**: 418-423.

Roy, A. and **Chowdhury, D.K.** (1959) Interpretation of self-potential data for tabular bodies, *J. Sci. Engng Res*, **3**: 35-54.

Shalivahan, B., Bhattacharya, B. and **Mrinal, K.S.** (1998) Interpretation of self-potential anomalies by nonlinear inversion, *Journal of Geophysics*, **19**(4):219-224.

Sundararajan, N., Arun Kumar, I., Mohan, N.L. and **Seshagiri Rao, S.V.** (1990) Use of Hilbert transform to interpret self-potential anomalies due to two dimensional inclined sheets, *Pure and Appl. Geophys*, **133**:117-126.

Sundararajan, N. and **Srinivas, Y.** (1996) A modified Hilbert transform and its application to self-potential interpretation, *J. Appl. Geophys*, **36**: 137-143.

Sundararajan, N., Srinivasa Rao, P. and **Sunitha, V.** (1998) An analytical method to interpret self-potential anomalies caused by 2-D inclined sheets, *Geophys*, **63**(5):1551-1555.

Yungul, S. (1950) Interpretation of spontaneous polarization anomalies caused by spherical ore bodies, *Geophys*, **15**: 237-246.

Appendix

The Adaptive Simulated Annealing Algorithm

Simulated annealing techniques are implemented for finding global minimum (or maximum) of a target function in parameter space. The techniques are adopted from the physical annealing procedure where a liquid is cooled down in order to obtain a minimum energy formation. These techniques where developed due to the fact that stochastic and non linear systems are extremely difficult to be minimized. Stochastic methods such as adaptive simulated annealing (very fast simulated re-annealing) have been found to be extremely useful tools for a wide variety of minimization problems of large non linear systems.

Adaptive simulated annealing is a powerful stochastic optimisation method applicable to a wide range of problems, especially for multi-modal, discrete, non linear and non differentiable target functions. The major advantage of adaptive simulated annealing over other methods is its ability to avoid becoming trapped at local minima. The algorithm employs a random search, which dos not only accept changes that decrease the objective function, but also accepts some changes that increase it, at least temporarily.

The adaptive simulated annealing random search algorithm is here illustrated in solving the following multi-variables unconstrained problem:

$$Minimize \ \ \phi(v)$$
$$Subject \ \ to \ \ v \in \mathbb{R}^n$$

Where the numerical function $\phi(v)$ is called the objective (target) function of the problem and $v = (v_1,...,v_n) \in \mathbb{R}^n$ is the vector of model parameters (decision variables). Using function minimization for illustrative purposes, the algorithm proceeds as follows:

The Algorithm

Initialization: Let

User-defined control parameters: $\alpha_0 > 0$, $r > 0$, $t^0 \in \mathbb{R}^n_+$

A user-defined initial solution: $v^0 \in \mathbb{R}^n$

A user-defined small positive real number close to zero: ε

An initial number of iteration: $i = 0$

Main procedure: Repeat until ($\alpha_i < \varepsilon$)

Step 1: for $j = 1$ **to** n **do**

{

Generate a random number u between 0 and 1: $u = random[0,1]$

Set $\theta = \operatorname{sgn}(u - 0.5) t_j^i \left[\left(1 + \frac{1}{t_j^i}\right)^{|2u-1|} - 1 \right]$

Set $\hat{v}_j = v_j^i + \theta \frac{r}{\sqrt{n}}$

{

Step 2: if $\phi(\hat{v}) < \phi(v^i)$ **then** set $v^{i+1} = \hat{v}$ and go to step 3

else calculate the probability $p = \frac{1}{1 + e^{\frac{\phi(\hat{v}) - \phi(v^i)}{\alpha_i}}}$

if $p > \gamma = random[0,1]$ **then** set $v^{i+1} = \hat{v}$

else set $v^{i+1} = v^i$

Step 3: for $j = 1$ **to** n **do**

{

Set $t_j^{i+1} = \frac{t_j^0}{i+1}$

{

Set $\alpha_{i+1} = \frac{\alpha_0}{i+1}$ **,** $i = i + 1$ and go to step 1**.**

End of the algorithm.

استخدام محاكاة التلدين التلاؤمي للتفسير الكمي لشاذات الكمون الذاتي العائدة لبنى هندسية بسيطة

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المستخلص. تم في هذا العمل اقتراح طريقة تفسير كمية لشاذات الكمون الذاتي، وقد صممت الطريقة بهدف تحديد قيم وسائط جسم مطمور تحت الأرض كعمق مركز والجسم، العزم الكهربائي أو طويلة الاستقطاب، وزاوية الاستقطاب والعامل الذي يحدد شكل الجسم الهندسي، وذلك من خلال معطيات حقلية يرتكز تفسيرها على بنى هندسية بسيطة كالأسطوانة، والكرة، والصفيحة. تعتمد الطريقة على النمذجة الرياضية باستخدام طريقة الأمثلة الشاملة المسماة بخوارزمية محاكاة التلدين التلاؤمي. تمت بدايةً البرهنة على صلاحية و فاعلية الطريقة من خلال تفسير مثال نظري معطياته مولدة عشوائيًا باستخدام توزيع إحصائي معروف مع وجود أخطاء عشوائية مختلفة، حيث تم الحصول على توافق ممتاز بين قيم الوسائط المفترضة والمقدرة للنموذج. تم فيما بعد تطبيق الطريقة المقترحة على معطيات حقلية من ألمانيا و الهند و تركيا، ومن ثم مقارنة نتائج التفسير مع نتائج تفسيرية باستخدام طرق أخرى، وتبين أن التوافق كان جيدًا بين النتائج التي تم الحصول عليها بالطريقة المقترحة ونتائج الطرق الأخرى.

الكلمات المفتاحية: شاذ كمون ذاتي، نمذجة رياضية، تابع الجزاء الأسي، خوارزمية الأمثلة الشاملة.