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Transient Analysis of a Telecommunication System Using State Dependent Markovian Queue under Bi-Level Control Policy

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Abstact. In this paper, we develop a finite queueing model having single and batch service modes for telecommunication system, where two types of traffic, *i.e.* voice and data arrive in Poisson fashion. The server starts service only when N packets are accumulated in the system. The server performs service singly until there are C packets in the system. After then type 2 packets are discarded and all type 1 packets are served in a batch. The arrival rates of packets depend upon the server's status. The transient state probabilities of system states are obtained by solving a set of linear equations with the help of Laplace Transform technique. Performance indices such as average queue length, expected idle time, and expected busy period are determined. We also investigate the optimal value of threshold parameter N and C after which the server changes the mode of service in order to minimize the expected cost. The numerical illustrations are provided to visualize the effect of various parameters on system performance.

Keywords: Bi-level, N-Policy, Integrated traffic, Priority, State-dependent rates, Optimal control, Queueing models, Markovian analysis.

1. Introduction

In the recent years, analytical tools based on queue-theoretic approaches are becoming of great importance in performance modelling of traffic control in congestion situations of complex communication networks. In packet-switched environment, buffering could be done at the input and output ports. In many real situations a pre-assigned number of packets are accumulated in the input buffer before starting the transmission. In queueing theory, this is known as N-policy and plays a very important role for the performance prediction of communication problems. Several authors studied the characteristic of N-policy queues in different frameworks. In this direction, the contribution of Yadin and Naror[1], Heyman[2], Kella and Chaudhary[3] are worth-mentioning. Boham and Mohanty[4] gave transient solution of Markovian queue under (M, N) policy. Time dependent solution for discrete time M/M/1 queue was provided by Boham and Mohanty[5]. Lee *et al.*[6] considered batch arrival queue with N-policy. Choudhary[7] analysed N-policy queue with general setup time. Artalejo[8] showed some applications of stochastic decomposition properties for the queue size and waiting time distribution in M/G/1 queue with N-policy. Various optimal control policies were studied by Lillo and Martin[9]. Response time for general service time queue with Bernoulli feedback was investigated by Medhi[10]. Jain and Rakhee[11] discussed a optimal N-policy for the state dependent $M/E_k/1$ queue with server breakdown. Jain *et al.*[12] developed M/M/R machine interference model with balking, reneging, spares and two modes of failure by using birth-death process. Jain *et al*. (2004) gave a numerical solution for machine repair system with spares and reneging under N-policy. Arumuganathan and Jeyakumar[13] analysed a M^X/ G(a, b)/1 queueing system with multiple vacation, setup time with N-policy and closedown times. Choudhury and Madan[14] considered a batch arrival queueing system, where the server provides two stages of heterogeneous service with a modified Bernoulli schedule under N-policy.

In many situations, packets are served in batch with reduced rate when queue size become large. It reduces the delay and release the conjection. A comprehensive survey on bulk queues can be found in Neuts[15] and Chaudhary and Templeton[16]. Expressions for probability density function (pdf) of the busy period for single server bulk queue were obtained by Kambo and Chaudhary[17]. Infinite markovian queue with two mode of service was studied by Raj and Manoharan[18]. Jain and Rakhee[11] obtain explicit results for finite capacity queue with batch service of two type of traffic. They minimize expected cost by evaluating the optimal value of threshold to start the batch service. Ke[19] considered a batch arrival queue under bi-level control policy. Optimal control of batch arrival and batch service queueing system under N-policy was discussed. Dshalalow *et al.*[20] obtained a bi-level hysteretic control policy for a stochastic hybrid system with compound poisson input general batch service and two vacation modes.

The present investigation suggests transient analysis of finite capacity Markovian queueing model under bi-level control policy. We consider two type of traffic (*i.e.*, voice/data). Server turn on and serves one packet at a time when N-packets are accumulated in the system. If the number of packets in the system exceed the threshold value C, then it switches to batch service mode and serve all the type 1 packets in a batch whereas type 2 packets are lost. We obtain the set of algebraic linear equations by taking Laplace transform of differential difference equations governing the model. The set of equations is solved by Cramer's rule and invert the solution by well known partial fraction technique to get the probabilities. The transient state queue size distribution and expected busy period, expected idle period have been obtained. Optimal value of N and C, namely N* and C* are computed by minimizing the cost function. Numerical illustration provides the validity of proposed model.

The remainder of this paper is organized as follows: In Section 2, we give a detailed description of the model. In Section 3, we present queue size distribution which is useful in solving the model. System characteristics are given in Section 4. Cost analysis for obtaining the performance measures is presented in Section 5. Section 6 presents numerical results. Finally the conclusions of the paper are given in Section 7.

2. Model Description

We consider a Bi-level (N, C) policy for single server Poisson queue with two modes of service and finite capacity. For system modeling purpose, the following characteristics are assumed:

The two types of packets originate according to Poisson distribution with state dependent arrival rates as:

$$\lambda_{i,j} = \begin{cases} \lambda_{0,j} & \text{server is idle and j packets are in the system} \\ \lambda_{1,j} & \text{server is in single service mode and j packets are in the system} \\ \lambda_{b,j} & \text{server is in batch service mode and j packets are in the system} \end{cases}$$

where j = 1 and 2 denote the types 1 and 2 jobs respectively.

The server follows (N, C) policy, *i.e.*, server turns on when N (N ≥ 1) packets are accumulated and then serves the packets one by one with rate μ_1 according to FIFO discipline upto a threshold queue level C. After reaching the size of queue as C, type 2 packets are lost and the server switches to bulk service mode and serves all the queued type 1 packets in a batch with reduced rate μ_b. We denote the server state as follows:

(a) (0, n) server is idle and n packets in the queue where n = 0, 1,…., N – 1.

(b) (1, n) server is on and n packets in the queue where n = 0, 1, …., C – 1.

(c) (b, n) server is in batch mode and n packets in the queue where:

n = C, C+1, …., K.

The system state space are mutually exclusive.

Denote $\Lambda_0 = \lambda_{0,1} + \lambda_{0,2}$

$\Lambda_1 = \lambda_{1,1} + \lambda_{1,2}$

$P_{0,n}(t)$ Prob. that server is off and n packets are in the queue at time t

$P_{1,n}(t)$ Prob. that server is on and n packets are in the queue at time t

3. The Analysis

The Chapman-Kolmogorov equations governing the model are as follows:

$$\frac{dP_{0,0}(t)}{dt} = -\Lambda_0 P_{0,0}(t) + \mu_1 P_{1,1}(t) + \mu_b \sum_{i=C+1}^{K} P_{1,i}(t) \tag{1}$$

$$\frac{dP_{0,n}(t)}{dt} = -\Lambda_0 P_{0,n}(t) + \Lambda_0 P_{0,n-1}(t) \qquad 1 \le n \le N-1 \tag{2}$$

$$\frac{dP_{1,1}(t)}{dt} = -(\Lambda_1 + \mu_1) P_{1,1}(t) + \mu_1 P_{1,2}(t) \tag{3}$$

$$\frac{dP_{1,n}(t)}{dt} = -(\Lambda_1 + \mu_1)P_{1,n}(t) + \Lambda_1 P_{1,n-1}(t) + \mu_1 P_{1,n+1}(t) \quad 2 \le n \le N-1 \tag{4}$$

$$\frac{dP_{1,N}(t)}{dt} = -(\Lambda_1 + \mu_1)P_{1,N}(t) + \Lambda_1 P_{1,N-1}(t) + \mu_1 P_{1,N+1}(t) + \Lambda_0 P_{0,N-1}(t) \tag{5}$$

$$\frac{dP_{1,n}(t)}{dt} = -(\Lambda_1 + \mu_1)P_{1,n}(t) + \Lambda_1 P_{1,n-1}(t) + \mu_1 P_{1,n+1}(t), \quad N+1 \le n \le C-1 \tag{6}$$

$$\frac{dP_{1,C}(t)}{dt} = -(\lambda_{b,1} + \mu_1)P_{1,C}(t) + \Lambda_1 P_{1,C-1}(t) \tag{7}$$

$$\frac{dP_{1,n}(t)}{dt} = -(\lambda_{b,1} + \mu_b)P_{1,n}(t) + \lambda_{b,1} P_{1,n-1}(t) \quad C+1 \le n \le K-1 \tag{8}$$

$$\frac{dP_{1,K}(t)}{dt} = -\mu_b P_{1,K}(t) + \lambda_{b,1} P_{1,K-1}(t) \tag{9}$$

Taking the Laplace transform of (1)–(9) and using the initial condition $P_{0,0}(t) = 1$; $P_{i,j}(t) = 0$; (i =0,1 and j=1,……K), we obtain

$$(s + \Lambda_0)\, P^*_{0,0}(s) - 1 = \mu_1\, P^*_{1,1}(s) + \mu_2 \sum_{i=C+1}^{K} P^*_{1,i}(s) \tag{10}$$

$$(s + \Lambda_0)\, P^*_{0,n}(s) = \Lambda_0\, P^*_{0,n-1}(s) \quad 1 \le n \le N-1 \tag{11}$$

$$(s + \Lambda_1 + \mu_1)\, P^*_{1,1}(s) = \mu_1\, P^*_{1,2}(s) \tag{12}$$

$$(s + \Lambda_1 + \mu_1)\, P^*_{1,n}(s) = \Lambda_1\, P^*_{1,n-1}(s) + \mu_1\, P^*_{1,n+1}(s), \; 2 \le n \le N-1 \tag{13}$$

$$(s + \Lambda_1 + \mu_1)\, P^*_{1,N}(s) = \Lambda_0\, P^*_{0,N-1}(s) + \Lambda_1\, P^*_{1,N-1}(s) + \mu_1\, P^*_{1,N+1}(s) \tag{14}$$

$$(s + \Lambda_1 + \mu_1)\, P^*_{1,n}(s) = \Lambda_1\, P^*_{1,n-1}(s) + \mu_1\, P^*_{1,n+1}(s), \; N+2 \le n \le C-1 \tag{15}$$

$$(s + \lambda_{b,1} + \mu_1)\, P^*_{1,C}(s) = \Lambda_1\, P^*_{1,C-1}(s) \tag{16}$$

$$(s + \lambda_{b,1} + \mu_2)\, P^*_{1,n}(s) = \lambda_{b,1}\, P^*_{1,n-1}(s), \quad C+1 \le n \le K-1 \tag{17}$$

$$(s + \mu_2)\, P^*_{1,K}(s) = \lambda_{b,1}\, P^*_{1,K-1}(s) \tag{18}$$

The equations (10)-(18) can be written in matrix form as

$$\mathbf{A}(s)\ \mathbf{P}^*(s) = \mathbf{P}(0) \tag{19}$$

where $\mathbf{P}^*(s) = [P_{0,0}(s), P_{0,1}(s)\ P_{0,2}(s) \ldots P_{0,N-1}(s), P_{1,1}(s), \ldots P_{1,C-1}(s), P_{1,C}(s), P_{1,C+1}(s), \ldots P_{1,K}(s)]$ (20)

and $\mathbf{P}^*(0) = [P_{0,0}(0), P_{0,1}(0)\ P_{0,2}(0) \ldots P_{0,N-1}(0), P_{1,1}(0), \ldots P_{1,C-1}(0), P_{1,C}(0), P_{1,C+1}(0), \ldots P_{1,K}(0)]$ (21)

Where:

$\mathbf{A}(s)$ is an $(N+K)\times(N+K)$ matrix (22)

As given in Fig. 1.

By using Cramer's rule, we can easily solve equation (19) and get $P_{i,j}(s)$ as

$$P^*_{i,j} = \frac{|\boldsymbol{A}_j(\boldsymbol{s})|}{|\boldsymbol{A}(\boldsymbol{s})|} \quad ;\ i=0,\ 1;\ j=0,\ 1,\ 2,\ \ldots,\ K \tag{23}$$

Here $|\boldsymbol{A}(s)|$ is a determinant of the matrix $\boldsymbol{A}(s)$ and $|\boldsymbol{A}_j(s)|$ is the determinant of matrix obtained by replacing j^{th} column of matrix $\boldsymbol{A}(s)$ by initial vector $\boldsymbol{P}(0) = [1, 0, 0, \ldots 0]^T$.

$$|\boldsymbol{A}(s)| = s\left[\prod_{k=1}^{i}(s+r_k)\right]\left[\prod_{k=1}^{j}\left\{s^2+\left(r_{j+k}+\bar{r}_{j+k}\right)s+r_{j+k}\bar{r}_{j+k}\right\}\right] \tag{24}$$

Then

$$P^*_{i,j}(s) = \frac{|\boldsymbol{A}_j(s)|}{s\left[\prod_{k=1}^{i}(s+r_k)\right]\left[\prod_{k=1}^{j}\left\{s^2+\left(r_{j+k}+\bar{r}_{j+k}\right)s+r_{j+k}\bar{r}_{j+k}\right\}\right]} \tag{25}$$

$$P^*_{i,j}(s) = \frac{a_0}{s} + \sum_{l=1}^{i}\frac{a_l}{s+r_l} + \sum_{l=1}^{j}\frac{b_l s + c_l}{s^2+\left(r_{i+1}+\bar{r}_{i+1}\right)s + r_{i+1}\bar{r}_{i+1}} \tag{26}$$

where

$$a_0 = \frac{A_j(0)}{\left[\prod_{k=1}^{i} r_k\right]\left[\prod_{k=1}^{j}\left(r_{i+k}\bar{r}_{i+k}\right)\right]} \tag{27}$$

$$a_l = \frac{\left|A_j(-\alpha_l)\right|}{(-r_l)\left[\prod_{\substack{k=1\\k\neq l}}^{i}(r_k - r_l)\right]\left[\prod_{k=1}^{j}\left\{r_l^2 + \left(r_{i+k} + \bar{r}_{i+k}\right)(-r_l) + r_{i+k}\bar{r}_{i+k}\right\}\right]}$$

$$; l = 1, 2, ..., I \tag{28}$$

and

$$b_l(-r_{i+l}) + c_l = \frac{\left|\mathrm{A}_j(-r_{i+l})\right|}{(-r_{i+l})\left[\prod_{k=1}^{i}(r_k - r_{i+l})\right]\left[\prod_{\substack{k=1\\k\neq l}}^{j}\left\{(-r_{i+l})^2 + \left(r_{i+k} + \bar{r}_{i+k}\right)(-r_{i+l}) + r_{i+k}\bar{r}_{i+k}\right\}\right]}$$

$$l = 1, 2, ..., j \tag{29}$$

Taking the inverse Laplace transform of equation (26), we obtain

$$P_{i,j}(t) = a_0 + \sum_{l=1}^{i} a_l e^{-r_l t} + \sum_{l=1}^{j}\left[b_l e^{-u_l t}\cos(v_l t) + \frac{c_l - b_l u_l}{v_l} e^{-u_l t}\sin(v_l t)\right] \tag{30}$$

where u_l and v_l denote the real and imaginary part of complex eigen value r_{i+l} *and* a_0, a_l, b_l, c_l *are all real numbers*.

4. System Characteristics

We obtain various performance indices for N-policy controllable finite capacity queue with the help of transient state probabilities derived in previous section as follows:

The expected number of packets in the system at any instance t is obtained by

$$ES(t) = \sum_{j=0}^{N-1} jP_{0,j}(t) + \sum_{j=1}^{K} jP_{1,j}(t) \tag{31}$$

The expected number of packets in the queue at any instance t is given by (*i.e.*, average queue length)

$$EQ(t)=\sum_{j=0}^{N-1}(j-1)P_{0,j}(t)+\sum_{j=1}^{C}(j-1)P_{1,j}(t) \tag{32}$$

The probability that the server being idle at time *t*, is

$$P[I(t)]=\sum_{j=0}^{N-1}P_{0,j}(t)=NP_{0,0}(t) \tag{33}$$

The probability that the server being busy

$$P[B(t)]=\sum_{j=1}^{K}P_{0,j}(t) \tag{34}$$

Using the memoryless property of the Poisson Process, the length of the idle period is sum of *N* exponential random variables, each having mean rate $1/\Lambda_0$ so that

$$E[I(t)]=\frac{N}{\Lambda_0} \tag{35}$$

According to this model busy period and idle period generate an alternative renewal process, hence

$$\frac{E[T(t)]}{E[I(t)]}=\frac{1-P[I(t)]}{P[I(t)]} \tag{36}$$

substituting the value of *P[I(t)]* and *E[I(t)]* from equation (33) and (35), we get:

$$E[T(t)]=\left(\frac{1-NP_{0,0}(t)}{NP_{0,0}(t)}\right)\left(\frac{N}{\Lambda_0}\right)=\left(\frac{1-NP_{0,0}(t)}{\Lambda_0 P_{0,0}(t)}\right) \tag{37}$$

The expected busy cycle, is the sum of expected idle period and expected busy period, hence

$$E[B(t)]=E[I(t)]+E[T(t)] \tag{38}$$

5. Cost Analysis

To construct the cost function for proposed model, we assume the following cost components:

C_0 Setup cost

C_1 Holding cost of one job in the system either server is idle or in single service or in batch service mode

C_2 Cost incurred when server is in single service mode

C_3 Cost incurred when server is in batch service mode

C_4 Cost per unit time for turning the server on

C_5 Cost per unit time for turning the server off

The expected cost at time t is given by

$$EC(t) = \frac{C_0}{E[T(t)]} + C_1 ES(t) + C_2 \sum_{j=1}^{C-1} jP_{1,j} + C_3 \sum_{j=C}^{K} P_{1,j} + (C_4 + C_5)\frac{1}{E[B(t)]}$$

We have solved the transient state probabilities by using the numerical method, therefore it is very difficult to get exact results for expected total cost function and optimal value of *N* and *C*, which minimize the expected total cost function, which is highly non-linear. Thus we apply heuristic approach based on discrete allocation to achieve optimal value of *N* and *C*.

6. Numerical Results

To check the optimal values of threshold parameters N and C in order to minimize the expected cost, we provide numerical results using MATLAB. We compute the expected number of packets in the queue EQ(t) and system ES(t) and expected cost as summarized in the graphs.

Figures 1 and 2 display the EQ(t) and ES(t) for different and same values of λ_0, λ_1, λ_b respectively. Other parameters are fixed as $\mu_1 = 0.35$, $\mu_b = 0.2$, $C = 7$ and $N = 3$ $K = 10$, $C_0 = 500$ $C_1 = 2$, $C_2 = 5$, $C_3 = 1000$, $C_4 = 500$, $C_5 = 200$. It is observed from the figures that queue length increases with the increase in λ_1 and time (t) but after a long time becomes steady.

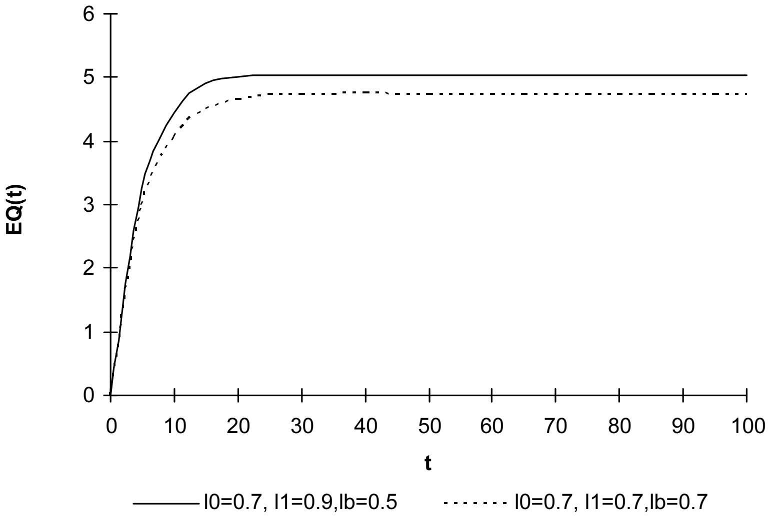


Fig. 1. Expected number of packets in the queue.

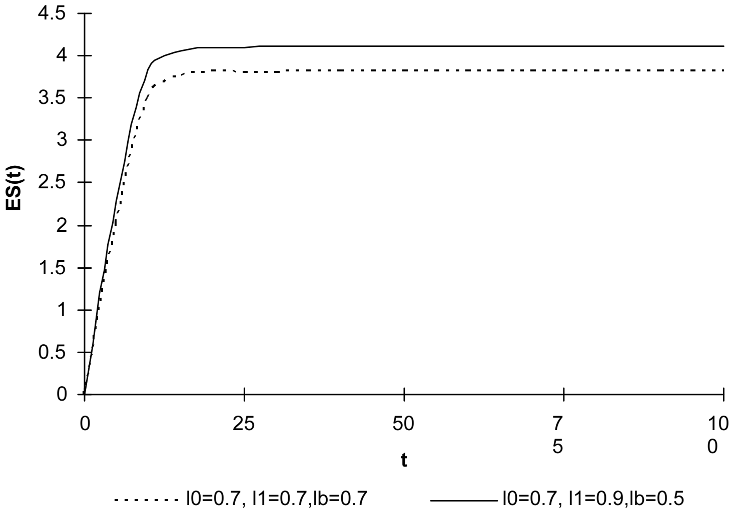
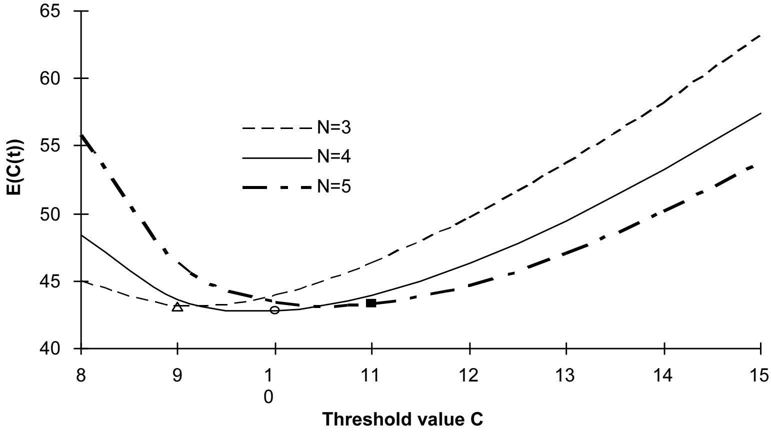


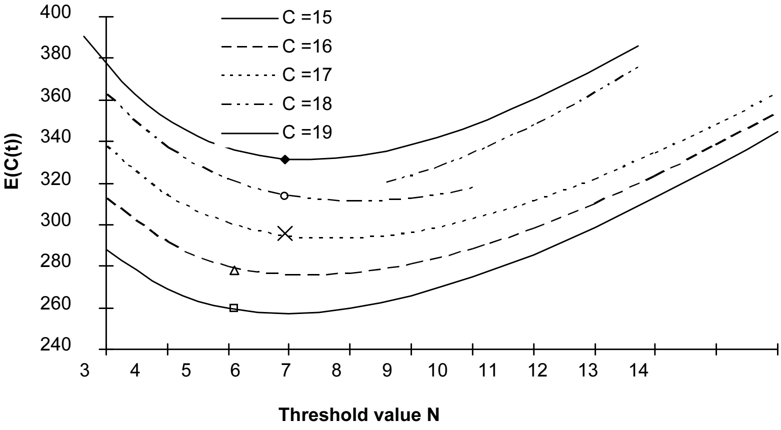
Fig. 2. Expected number of packets in the system.

Effect of N and C is shown in Fig. 3 and 4 respectively. The optimal value of N and C are shown by tick marks in the figures, where the expected cost (EC(t)) is minimum.



(C_0=500, C_1= 5, C_2=10, C_3=20, C_4=30, C_5=20)

Fig. 3. Expected cost vs. C.



(C_0=100, C_1= 50, C_2=10, C_3=200, C_4=300, C_5=200)

Fig. 4. Expected cost vs. N.

From these figures optimal value of N and C for minimum value of EC(t) are obtained and summarized in Table 1.

Table 1. Optimal value of N and C for minimum value of EC(t).

Fig. 3			**Fig. 4**		
N*	C*	EC(t)	N*	C*	EC(t)
3	9	43.10	7	19	329.19
4	10	42.80	7	18	310.87
5	11	43.31	7	17	293.14
			6	16	275.35
			6	15	257.27

7. Conclusion

A finite capacity queueing model having single and batch service modes for telecommunication system was developed. The transient solution of this finite capacity, N-policy queue with two type of service modes has been obtained and the average queue length, expected idle period and expected busy period were derived. Optimal value of N and C are achieved by minimizing the cost function.

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التحليل الانتقالي لإحدى منظومات الاتصالات باستخدام نموذج الصفوف الماركوفية القائمة على سياسة التحكم ثنائية المستوى

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المستخلص . تم في هذا البحث تطوير نموذج لمنظومة اتصالات تتضمن نوعين من المدخلات: صوتية ومعلوماتية، تصل جميعها بطريقة بواسون. حيث يبدأ الخادم في أداء الخدمة عندما يتراكم فى المنظومة عدد محدد سلفاً من الحزم، ويقوم الخادم عندئذ بأداء الخدمة بصورة مفردة حتى يصل عدد الحزم في المنظومة إلى حد معين يتم بعده إهمال النوع الأول من الحزم (أي الحزم الصوتية) وتمرير جميع حزم النوع الثاني (أي الحزم المعلوماتية) على التعاقب. وقد تم الحصول في هذا البحث على الاحتمالات الانتقالية للمنظومة عن طريق حل مجموعة من المعادلات الخطية الناتجة من استخدام تحويل لابلاس، كما تم حساب معايير الأداء بما في ذلك طول صف الانتظار، وزمن الخمول، وفترة الانشغال. وتمت كذلك دراسة القيم المثلى – بغرض تقليل التكلفة– لكل من عدد الحزم المسموح بتراكمها قبل بدء الخدمة, وحد الانتقال الذى يتم بعده تغيير وضعية المنظومة ، كما أضيف توضيح عددي لمدى تأثير المتغيرات المختلفة على أداء هذه المنظومة.