

## Deterministic Modelling of Al-Madinah Al-Munawarah Groundwater Quality Using Lumped Parameter Approach

ASEM Y. BOKHARI\* and M.Z. ALI KHAN\*\*

\**Faculty of Earth Sciences, and \*\*Civil Engineering Dept., Faculty of Engineering King Abdulaziz University, Jeddah, Saudi Arabia.*

**ABSTRACT.** Groundwater sampling and analysis of 105 wells in the Al-Madinah Al-Munawarah region, Saudi Arabia was carried out using the "lumped parameter" approach. Regression equations are developed for electrical conductivity versus other ions,  $\text{Na}^+ + \text{K}^+$  versus other ions, electrical conductivity versus distance (km) from upstream and  $\text{SO}_4^{2-}/\text{Cl}^-$  ratio versus distance from upstream. The equations developed are easy to use in determining the groundwater quality of upstream or downstream wells from a known electrical conductivity value at a specific well. The present groundwater quality is mostly sulfates and chlorides of calcium and magnesium, and is suitable for domestic use.

The analysis indicates that the ratio of  $\text{SO}_4^{2-}/\text{Cl}^-$  decreases with the distance from upstream and electrical conductivity increases with the distance from upstream. The relationship of number of tributaries and hydraulic gradient with the distance from upstream was poor.

**KEY WORDS.** Deterministic Modelling; Regression Analysis; Al-Madinah Al-Munawarah Region Groundwater Quality; Electrical Conductivity; Tributary and Hydraulic Gradient Effect.

### Introduction

The physical, chemical and trace elements characteristics of water determine its usefulness for industry, agriculture especially the potability of drinking water. The study of water chemistry gives important indications of the geologic history of the presence of hidden ore deposits. Waters which are very high in dissolved solids may yield lithium, potassium, sodium chloride and other chemicals in commercial quantities. Natural water contains impurities due to its contact with the air, soil, reservoir circulations, and wastewater from communities and treatment plants, agricultural activity and industrial waste.

Monitoring of groundwater quality for the Madinah region, Saudi Arabia, has shown that the quality has been deteriorating in the last five years and also, in general, upstream reaches have relatively better water quality than the downstream ones. At present easier and simpler relationships are not available or developed, which would be useful in determining the quality (chemical compositions) along various reaches of a groundwater region as well as at a single well, if one or more parameters are known.

The main objectives of this paper are:

- (1) To review some basic geochemical changes occurring in the underground water;
- (2) To review some groundwater classification tools used;
- (3) To develop easier deterministic empirical relationships for use in determining the groundwater composition, quality and its suitability for domestic purposes.

### **Study Area Description**

#### **Location**

The study area (Al-Madinah Al-Munawarah Region) lies between 24° 00'-25° 00' longitudes and 39° 00'-39° 45' latitudes.

#### **Geomorphology**

Al-Madinah Al-Munawarah (Madinah Region) is bound by the Harrat Rahat in the west with deep valleys and in the east penclains of Cambrian age. The drainage pattern east of the Madinah Quadrangle is controlled by basaltic lenses. Most of the wadis in the north flow in the southeast direction and have a dendritic drainage pattern. This region consists of several catchment areas, which drain to the Red Sea. In contrast to the wadis in the other parts of Saudi Arabia, these wadis are wide at the upstream end, partly covered by basalts, the downstream reaches however are evolved into the Precambrian basement rocks as passages. The region has an arid climate with hot dry summers and cold winters, and evaporation rates are very high.

#### **Hydrogeology**

Hydrogeological studies were carried out throughout the region, but especially in the areas around the edge of the basalt cover. Some zones seem to have a good groundwater potential and there rapid expansion of the agricultural activity is seen. The main aquifers of the Al-Madinah Al-Munawarah region are primarily Tertiary and Quaternary basalts, sub-basaltic alluvium, and recent quaternary alluvial deposits. Hydrogeologically the area can be divided under the following headings, Eastern Harrat at Khaybar area, Wadi Al-Hamd area, Eastern Harrat Rahat area, Yatima plane area, and Wadi Mastrah area.

## **Groundwater System Chemical Changes and Classification, Literature Review**

### **General**

A groundwater system can be characterized or modelled as deterministic or probabilistic. A deterministic system is one that is defined by definite cause and effect relations. Cause is generally referred to as "excitation", while the effect refers to "reaction" or "response". A system is thought to be well understood if the cause and effect relations within are understood.

For groundwater systems, the factor relating to cause and effect, invariably the geology of the system, is missing. For example, the abstract nature of a map shows pumpage and its distribution (a cause) related to the water level change (an effect) or water level change (a cause) related to the amount and distribution of land subsidence (an effect). These cause and effect relations will remain abstract in their meanings until they are related to the geology of the system and, unless of course, the geology is of the simplest type conceivable.

In deterministic models effects can be related directly to causes. A stochastic or probabilistic system allows prediction with uncertainty, but many provide expected values within the limitations of the probability terms which define its behavior. A random or stochastic variable is a variable quantity with definite values, each one of which, depending on chance, can be attained with a definite probability. An expected value is the mean value of a random variable.

A lumped parameter system "black box" is a system in which inputs and outputs can be measured or estimated although the process which interrelates them are not often observable. Detailed knowledge of the internal mechanism relating input and output is not necessary in the so called "black box" analyses. In this system a space coordinate system is not required in problem formulation and solution. For example, a rise in water levels in wells over a certain time interval (a response variable) may be converted to recharge without any regard to the location of wells in the field or their spacing (or to the manner in which recharge reaches the water table or even to the amount of rainfall). In such a system, parameters are expressed in lumped form, when the total system is regarded as located at a single point in space.

The lumped parameter, deterministic performance systems always require pertinent variables represented by "averages" over the space of interest. A variety of instruments, techniques and empirical formulas (Linsley *et al.* 1975; Kazmann 1972; Thornthwaite 1948; Blaney 1952; Penman 1956) have been developed to measure or estimate the quantity of water in various components of hydrological cycle.

### **Groundwater Geochemical Changes**

The geochemical cycle (prepared in this study) closely follows the hydrological cycle and can be illustrated schematically by a diagram that lump the atmosphere,

land and ocean areas into a single hydrogeochemical component (Fig. 1). The

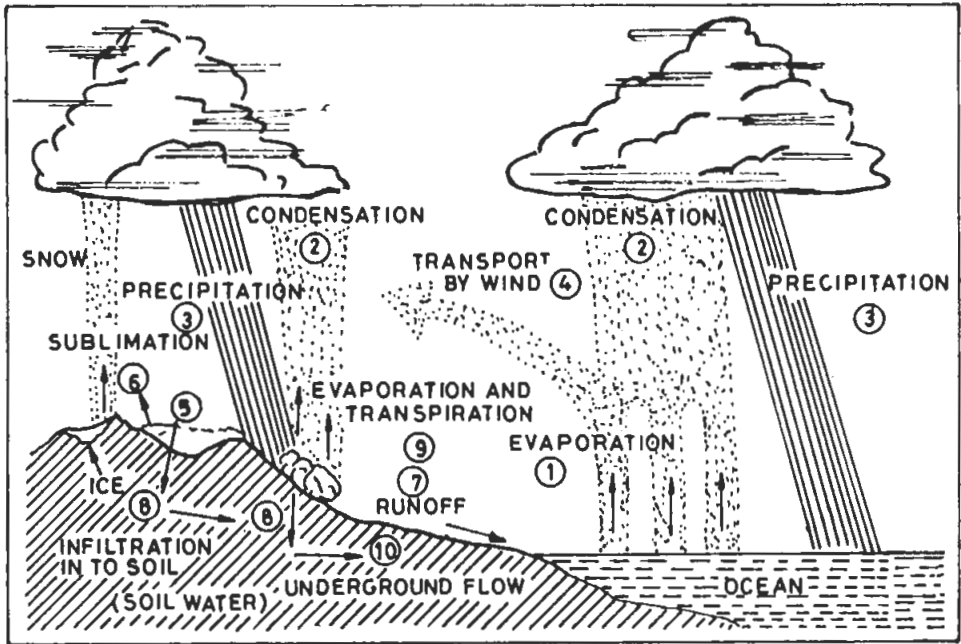


FIG. 1. Schematic representation of Geochemo hydrologic cycle.

- (1) Transfer of chlorides and sulfates of sodium, magnesium, calcium and potassium carried by water vapors to atmosphere.
- (2) Condensation of evaporated chemicals.
- (3) Precipitation of dissolved oxygen, nitrogen and carbon dioxide.
- (4) Transport of evaporated chemicals.
- (5) Temporary retention of water with dissolved carbon dioxide, dissolution of Ca, Mg, Na bicarbonates and sulfates due to the downward movement through soils.
- (6) Evaporation of chemicals from water impoundments, lakes and reservoirs with water vapors.
- (7) Run-off of precipitation to lakes, streams, rivers and oceans.
- (8) Infiltration and sub-surface movement of groundwater dissolving soil minerals with carbonic acid to form soluble bicarbonates; precipitation of colloidal iron, aluminum and silica carbonates as solubility limits are reached, cation exchange and other chemical changes.
- (9) Evapo-Transpiration (ET) of mineral matter largely retained in soil and partly carried off in crop plants.
- (10) Groundwater movement towards oceans and seas with physical, chemical and biological changes in water quality, depending on the rock geology, residence time and hydraulic gradients.

geochemical cycle represents quantities and types of mineral matter in solution. The role of each component is that of chemical or physio-chemical process and transformation. The transformation is mainly controlled by the movement of natural groundwaters and the mobility of chemical elements within the rock units.

The chemical evolution and interpretation of groundwater with position in the groundwater flow has been studied by various researchers (Chebotarev 1955;

Schoeller 1962; Back 1961, 1966; Gorrell 1958; Hem 1959, 1970; White 1963; Goldschmidt 1937; Piper's 1944; Back and Hanshaw 1965). These studies highlight the following fundamental premises:

(1) The concentration of the dissolved mineral matter is directly proportional to the length of flow path and to the residence time of water;

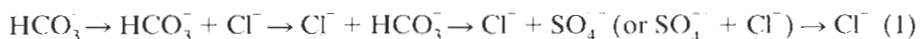
(2) The chemical type of groundwater at any point in the system is a function of both of the chemical composition of rocks at that point and of the antecedent water quality. Very few concepts, methods and empirical findings are available which may help relate the chemical character of the water to the geological environment and prevailing flow pattern.

(3) It is not uncommon for the ratio of sulfate to chlorides to decrease in the direction of flow.

(4) Within a given system, a major recharge area is characterized by water lower in dissolved solids than in an area where recharge is minor. Similarly recharge areas may be leaked of all soluble mineral matter, and the absence of chemical reactions would prevent the pH of water from becoming higher than that of the rainwater. Chemical zonations are controlled by depth of circulation, sedimentological zones and climatic factors (Schoeller 1959, 1962; Chebotarev 1955). Vertical zonations is influenced by dynamic aspects of water (velocity of movement, recharge-discharge relations, intensity of movement with depth and residence time). Salinity increases as the movement deteriorate with depth, with distance from recharge area, with nearness to the sea, and with the duration of contact.

Constituents in solution in groundwater may be viewed as a chemical system, with cations and anions in equilibrium with each other (Piper 1944). Studies attempting to explain the genesis of the chemical character of groundwater make use of diagrams (Back 1966; Piper 1953) in combination with the knowledge of the mineralogy of the environment, the transmissivity of its important component parts, and the distribution of piezometric head. Back (1966) identified a calcium-magnesium facies in recharge areas underlain by calcareous clays, and a sodium facies in discharge areas, the latter a result of ion-exchange and saltwater intrusion. Bicarbonate content is low in recharge areas and high in discharge areas. Toth (1966) reported a general tendency for a shift from pure bicarbonate facies in recharge areas to a sulfate facies in discharge areas, due to utilization of carbon dioxide in the root zone to form bicarbonates in downward moving groundwaters and little addition of bicarbonates thereafter. With passage into the system, the sulfate ion, which is available in the rocks of the area, becomes a dominant constituent. Hydrochemical data (Feth *et al.* 1962, 1966) suggested a change from calcium-magnesium-bicarbonate to sodium bicarbonate with distance from recharge area. Sulfate content decreased in the direction of flow and bicarbonate increased, because of the sulfate reduction.

Chebotarev (1955), based upon 10,000 chemical analysis, indicated that all groundwater tend towards the composition of sea-water.



This can be described in three vertical zones:

- (1) upper most zone (high circulation,  $\text{HCO}_3^-$  type and low mineralization);
- (2) intermediate zone (less circulation, high mineralization and water is sulfate type);
- (3) lower most zone (stagnant condition, water is chloride type).

The least soluble salts are precipitated first and the most soluble last, with the order being bicarbonate (calcite), sulfate (gypsum) and chloride (halite).

Maxey and Mifflin (1966) and Mifflin (1969) showed an increase in sodium, potassium, sulfate and chloride with length of flow path.

### Groundwater Classification

Geochemical studies often involve synthesis and interpretation of a mass of analytical data. The objectives of interpretation may be to avoid any difficulties in the classification of waters of different geochemical characteristics for utilization purposes, solving problems of saline water intrusion or ascertaining various factors on which the chemical characteristics of water depend.

The classification of hydrochemical facies (Back 1966) can be carried out using Table 1.

TABLE 1. Classification of Hydrochemical Facies (After Back 1966).

	Percentage of Constituents, epm			
	$\text{Ca}^{++} + \text{Mg}^{++}$	$\text{Na}^+ + \text{K}^+$	$\text{HCO}_3^- + \text{CO}_3^{--}$	$\text{Cl}^- + \text{SO}_4^{--}$
Cation facies :				
Calcium-magnesium	90-100	0    10		
Calcium-sodium	50-90	10   50		
Sodium-calcium	10-50	50   90		
Sodium-potassium	0-10	90-100		
Anion facies :				
Bicarbonate			90-100	0    10
Bicarbonate-chloride-sulfate			50-90	10   50
Chloride-sulfate-bicarbonate			10-50	50   90
Chloride-sulfate			0-10	90-100

Several systems of groundwater classification were used based upon the chemical composition (Ivanov *et al.* 1968; Altoviski 1962; Alkein 1984; Jetel *et al.* 1978) of groundwaters.

The residual sodium carbonate, RSC, influences the suitability of a certain groundwater for irrigation.

$$\text{RSC} = (\text{HCO}_3^- + \text{CO}_3^{--}) - (\text{Ca}^{++} + \text{Mg}^{++}) \quad (2)$$

where concentrations are expressed in epm.

If RSC exceeds 2.5 epm, the water is unsuitable for irrigation. If the value is between 1.25 to 2.5 epm the water is of marginal quality while less than 1.25 epm indicates that the water is probably safe.

Gorrell (1958), used a simple classification system based on "ranges" in total dissolved solids: (1) fresh water, 0 to 1,000 ppm; (2) brackish water, 1,000 to 10,000 ppm; (3) salty water, 10,000 to 100,000 ppm; (4) brine, greater than 100,000 ppm. Hem (1959), however, recognized that an immediate purpose of routine chemical quality data is to determine whether water is satisfactory for a proposed use. Whether a given quality is acceptable for a particular use depends on arbitrary standards of acceptability for that use. Three main uses are recognized: domestic (drinking water), agriculture (irrigation), and industrial. Drinking water standards are described by upper limits for bacterial, physical (turbidity, color, etc.) and mineralogical constituents. Agricultural standards are made contingent upon the effects of chemical constituents in water on soils and the plant life they support. Industrial standards vary with the particular industry. In all cases, emphasis is placed on the fact that the prescribed standards are not applicable to every situation.

Genetic classification systems have been proposed by White (1957a, 1957b) and encounter various problems concerning the chemical criteria which may be used to identify a given origin. Genetic terms have been defined as follows:

1. Meteoric water: Water that has recently been involved in atmospheric circulation;
2. Marine water: Water that has recently invaded rocks along coastlines;
3. Connate water: Water that was buried with sedimentation;
4. Metamorphic water: Water associated with rocks during their metamorphism;
5. Magmatic water: Water derived from a magma;
6. Volcanic water: Water derived from a magma at shallow depth;
7. Plutonic water: Water derived from a magma at considerable depth;
8. Juvenile water: Water that has never been part of the hydrosphere.

Meteoric water has an isotopic composition similar to surface water; marine water has an isotopic composition similar to sea water. Of the other types, connate, metamorphic, and magmatic waters are isotopically examined on the basis of their  $\text{H}^2/\text{H}^1$  and  $\text{O}^{18}/\text{O}^{16}$  ratios.

For flow-system studies, the most useful chemical classification would attempt to relate water to rock types on the basis of dissolved mineral matter. Ivanov *et al.* (1968) classified the mineral waters of earth's crust into two main groups:

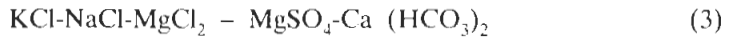
- (1) Infiltrating meteoric waters which are characterized by high Na/Cl ratios usually greater than one;
- (2) Waters of the marine origin which are characterized by low Na/Cl ratios usu-

ally less than one. These waters are further classified into two subgroups:

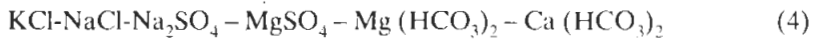
- (i) The normal marine water of the sedimentary environment;
- (ii) The residual strong brines of salt bearing basins.

Chemical composition of groundwater reflects either the primary marine origin or the meteoric genesis of continental conditions. The gradual removal, mixing and dilution of the original marine water between rock pores by the invading meteoric one, could be traced from one zone to another, or from one area to another by the identification of the water types.

Water of "marine" origin, entrapped between the rock pores is magnesium chloride type water. Such water has the following chemical composition.



Dilution of the original marine water between the rock pores, by the invading meteoric water, changes the chemical composition of the water to sodium sulphate type water "mixed water". Such water has the following composition:



With complete removal of the original marine water by the invading meteoric water, the chemical composition changes to sodium bicarbonate water. Such water has the following composition:



Sodium-sulphate type water (mixed), can change to sodiumbicarbonate type water (meteoric), in the presence of carbon and water, the chemical reaction is as follows:



Alkein (1984), classified water types according to the predominant cations and anions together with certain proportions to the concentration of ions.

The above mentioned literature indicates that most of the groundwater classification was carried out using typical analytical procedures but very few empirical relationships existed or were developed about how the water quality changed with the travel distance downwards or the residence time or the natural gradient. The main objective of this paper is to present empirical relationships in determining the chemical nature (quality) of Al-Madinah Al-Munawarah region groundwaters.

### Data Sampling and Data Collection

For groundwater quality sampling (Rainwater & Thatcher 1960) and analysis 105 wells were sampled in Al-Madinah Al-Munawarah Region (Fig. 2).

The main constituents monitored in the groundwater (105 wells) were calcium, magnesium, sodium, potassium, sulfates, chlorides and bicarbonates using Standard



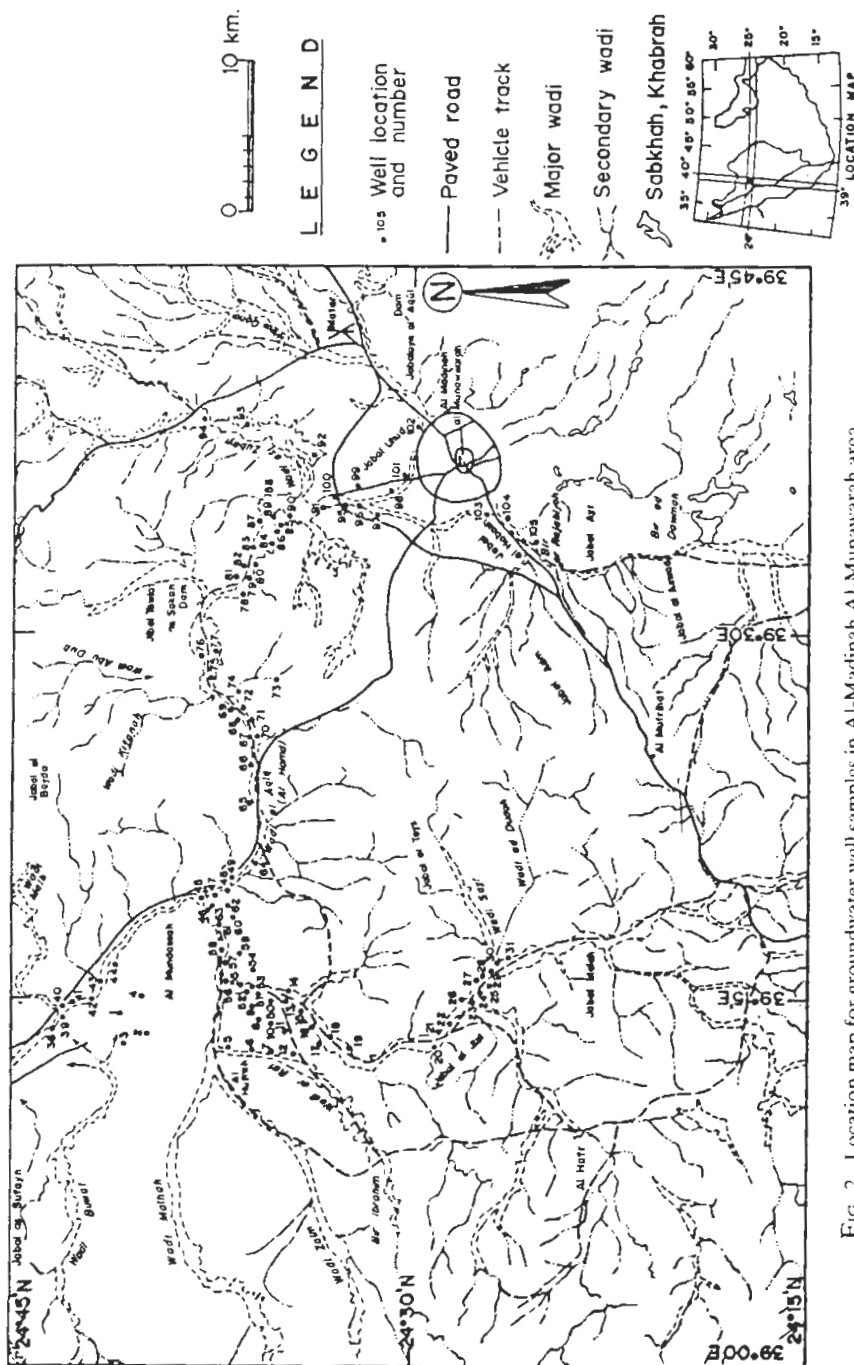


FIG. 2. Location map for groundwater well samples in Al-Madinah Al-Munawwarah area.

Methods (1989); the physical parameters such as pH, temperature and electrical conductivity measured in-situ with a portable kit; and the trace elements such as cadmium, iron and aluminum were measured by Atomic Absorption Spectroscopy. The physical parameters such as distance from upstream and groundwater slopes (Table 2) were used in developing relationships and the general groundwater classification was carried out using Piper diagram.

TABLE 2. Chemical Analysis of Major Ions and Physical Properties (for 105 Groundwater Samples) of Al-Madinah Al-Munawarah.

Sample Number	Water Table Gradient Slope or at Well (m/km)	No. of Tributaries or Wadis	EC	SO <sub>4</sub> /Cl	Distance (km) from Upstream
1	3.0	1	609	1.01	3.75
2	3.0	1	919	0.38	3.00
3	3.0	1	875	0.71	1.75
4	3.0	1	663	0.63	5.50
5	3.5	1	951	0.58	14.50
6	3.5	1	686	0.54	18.75
7	3.5	1	504	0.66	11.75
8	3.5	1	187	0.39	13.50
9	3.5	1	634	0.36	14.50
10	3.5	1	513	0.71	12.50
11	3.5	1	275	0.53	11.50
12	3.5	1	555	0.52	10.00
13	6.0	4	259	0.78	27.50
14	6.0	4	133	0.33	28.00
15	4.0	4	309	0.63	26.25
16	4.0	3	270	0.48	25.00
17	4.0	3	307	0.52	29.00
18	4.0	3	207	0.36	23.50
19	4.0	3	196	0.25	22.00
20	5.0	3	129	0.25	14.50
21	5.0	3	135	0.26	13.25
22	5.0	3	118	0.75	12.50
23	3.0	3	265	2.74	10.25
24	8.0	2	144	0.24	9.50
25	8.0	2	213	0.31	8.75
26	8.0	2	217	0.41	10.75
27	8.0	3	164	0.31	9.00
28	8.0	2	133	0.28	8.00
29	8.0	2	125	0.25	7.75
30	2.7	2	134	0.27	7.00
31	3.7	2	149	0.22	6.00
32	-	-	934	0.40	-
33	-	-	206	0.41	-
34	-	-	659	0.57	-
35	-	-	659	1.14	-
36	-	-	473	1.05	-
37	-	-	550	0.60	-
38	1.3	19	1052	0.45	93.50

TABLE 2. (cont.)

Sample Number	Water Table Gradient Slope or at Well (m/km)	No. of Tributaries or Wadis	EC	SO <sub>4</sub> <sup>-</sup> /Cl <sup>-</sup>	Distance (km) from Upstream
39	1.5	19	921	0.51	92.00
40	1.5	18	346	0.50	91.00
41	1.5	18	173	0.51	90.00
42	1.5	18	834	0.46	88.00
43	1.6	17	784	0.69	88.50
44	1.8	19	786	0.38	86.00
45	0.5	7	1392	0.39	79.00
46	4.4	10	986	0.41	36.00
47	0.5	10	990	0.40	78.00
48	0.5	10	1061	0.38	77.00
49	0.5	4	851	0.35	76.00
50	1.9	4	460	0.69	28.00
51	1.9	4	462	0.60	29.75
52	1.9	4	498	0.55	30.75
53	1.9	4	528	0.73	30.00
54	1.9	4	713	0.17	31.36
55	4.4	7	540	0.52	31.50
56	1.9	7	436	0.42	32.25
57	4.4	7	843	0.33	32.50
58	4.4	4	701	0.51	33.25
59	4.4	4	651	0.55	33.11
60	4.4	7	618	0.59	38.66
61	4.4	4	763	0.54	35.00
62	4.4	7	1083	0.44	36.80
63	4.4	10	1094	0.36	36.00
64	1.0	10	827	0.36	74.00
65	1.0	10	960	0.38	69.00
66	1.0	10	683	0.33	66.00
67	1.0	10	1202	0.42	64.00
68	1.0	10	1052	0.42	62.55
69	1.0	10	888	0.46	61.20
70	1.0	1	771	0.43	7.00
71	1.0	1	674	0.42	5.75
72	1.0	1	722	0.44	5.00
73	1.0	1	668	0.45	1.25
74	4.0	9	971	0.42	3.75
75	4.5	9	992	0.51	5.80
76	0.9	9	1346	0.69	56.00
77	0.9	8	1157	0.53	56.00
78	3.5	8	1020	0.55	49.70
79	7.5	8	206	0.45	48.50
80	3.5	8	364	0.37	47.75
81	1.5	8	1334	0.17	49.75
82	1.5	8	222	0.43	49.50
83	3.5	8	1863	0.46	47.25
84	0.1	8	1426	0.52	46.00
85	0.1	8	627	0.37	45.00

TABLE 2. (cont.)

Sample Number	Water Table Gradient Slope or at Well (m/km)	No. of Tributaries or Wadis	EC	SO <sub>4</sub> <sup>2-</sup> /Cl <sup>-</sup>	Distance (km) from Upstream
86	0.1	8	196	0.22	44.50
87	3.5	8	225	0.52	46.75
88	0.1	8	1550	0.57	45.50
89	0.1	8	668	0.38	44.00
90	0.5	5	194	0.21	43.50
91	2.0	8	1128	0.38	41.00
92	8.0	8	680	0.68	16.00
93	8.0	8	257	0.52	10.50
94	8.0	5	694	1.86	7.50
95	13.0	5	1457	0.53	39.00
96	4.0	3	938	0.46	38.00
97	4.0	2	657	0.80	36.00
98	2.5	5	1104	0.40	20.00
99	4.0	10	810	0.31	22.50
100	10.0	2	199	0.34	40.00
101	3.0	2	1062	0.46	19.00
102	7.0	2	984	0.30	15.00
103	5.0	2	539	1.40	28.00
104	6.0	2	107	0.38	26.50
105	7.0	1	137	0.42	24.00

### Data Analysis

The experimental data was analyzed using the regression analysis for the following two categories:

1. The relationship of (Na<sup>+</sup> + K<sup>+</sup>) versus other ions (Table 3).

TABLE 3. Regression Analysis Between (Na<sup>+</sup> + K<sup>+</sup>) and Other Parameters (Cl<sup>-</sup>, SO<sub>4</sub><sup>2-</sup>, Ca<sup>++</sup>, Mg<sup>++</sup>, EC).

Description	Na <sup>+</sup> + K <sup>+</sup> Versus				
	Cl <sup>-</sup>	SO <sub>4</sub> <sup>2-</sup>	Ca <sup>++</sup>	Mg <sup>++</sup>	EC
No. of Data Points	94	93	74	80	96
Standard Error of Estimate	.477	20.39	.849	19.76	.583
Correlation Coefficient (R)	.867	.796	.540	.772	.807
Coefficient of Determination, R <sup>2</sup>	.757	.634	.292	.596	.652
Intercept* (Log a)	-1.088	-	1.183	-	-3.228
Intercept** (St. Line)	-	-.082	-	-11.3836	-
Slope	1.174	2.164	.889	3.474	1.045

\* Log Equation : Y = ax<sup>b</sup>

$$\begin{aligned} \log y &= \log a + b \log x \\ \log a &= \text{intercept} \\ b &= \text{slope} \\ x &= \text{other ions, emp} \\ y &= (\text{Na}^+ + \text{K}^+) \text{ ions, epm} \end{aligned}$$

\*\* Straight Line Equation :

$$\begin{aligned} y &= a + bx \\ y &= (\text{Na}^+ + \text{K}^+) \text{ ions, epm} \\ b &= \text{slope} \\ x &= \text{other ions, epm} \\ a &= \text{intercept} \end{aligned}$$

2. The relationship of electrical conductivity versus other ions (Table 4).

TABLE 4. Regression Analysis Between Electrical Conductivity and Other Parameters (Ca<sup>++</sup>, Mg<sup>++</sup>, Na<sup>+</sup>+K<sup>+</sup>, SO<sub>4</sub><sup>-</sup>, Cl<sup>-</sup>, HCO<sub>3</sub><sup>-</sup>).

Description	Ca <sup>++</sup>	Mg <sup>++</sup>	Na <sup>+</sup> +K <sup>+</sup>	SO <sub>4</sub> <sup>-</sup>	Cl <sup>-</sup>	HCO <sub>3</sub> <sup>-</sup>
No. of Data Points	77	83	96	97	96	92
Standard Error of Estimate	.435	.216	.582	.347	.167	.926
Correlation Coefficient (R)	.704	.906	.807	.917	.972	.560
Coefficient of Determination, R <sup>2</sup>	.496	.820	.652	.840	.944	.313
Log a (Intercept)*	-.706	-1.550	-3.228	-3.934	-2.061	2.931
Slope (b)	.524	.650	1.046	1.052	0.923	.0016

\* Model Equation  $Y = ax^b$  :

$$\begin{aligned} \log y &= \log a + b \log x \\ \log a &= \text{intercept} \\ \log b &= \text{slope} \\ x &= \text{other ions, emp} \\ y &= \text{EC, mg/l.} \end{aligned}$$

Review of the regression analysis coefficients indicate that there is a very strong relationship between (Na<sup>+</sup>+K<sup>+</sup>) and other ions Cl<sup>-</sup>, SO<sub>4</sub><sup>-</sup>, Mg<sup>++</sup>, Electrical Conductivity as evidenced by the high coefficient of determination (R<sup>2</sup>) values 0.6 to 0.8. The coefficient of determination (R<sup>2</sup>) for the Ca<sup>++</sup> is very small, indicating poor relationship. The electrical conductivity seemed to have high positive correlation coefficient (R) ranging from 0.6 to 0.9, with all the ions, except Ca<sup>++</sup> and HCO<sub>3</sub><sup>-</sup> (Table 4) indicating a good relationship. Thus if electrical conductivity value of a certain groundwater well is known, then other ions can be estimated with reasonable accuracy. This fact was previously documented in another study (Bokhari and Khan 1990) also.

In order to determine the water quality changes in a groundwater region (upstream to downstream), if electrical conductivity is related to the travel distance, slope and other number of contributing tributaries (indirectly a function of the area of contribution) for the region, then from a known value of electrical conductivity at one location, other EC values upstream or downstream can easily be estimated. The estimated electrical conductivity can be used to determine other ions. Horizontal distances and slopes from upstream to downstream for each main wadi for its tributary

containing the sampled wells, were measured and listed against the electrical conductivity of each well. The listing of such values is shown in (Table 2).

Regression analysis, electrical conductivity values versus travel distance, hydraulic gradient and the number of tributaries meeting before or at a well, were also carried out.

The coefficient of determination ( $R^2$ ) values of electrical conductivity with the hydraulic gradient and the number of tributaries were 0.12 and 0.18 respectively. However, the  $R^2$  value for the electrical conductivity versus distance (km) from upstream, showed a much higher value of 0.25 and the following linear relationship model (Fig. 3) was applicable for Al-Madinah Al-Munawarah region :

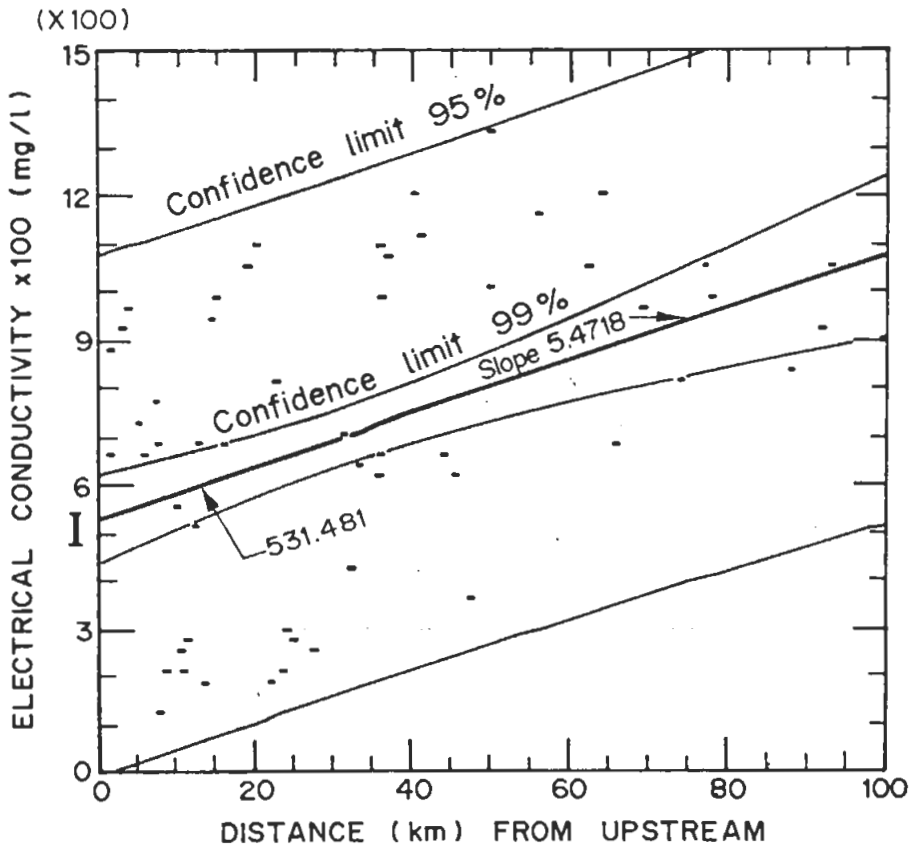


FIG. 3. Regression analysis of changes of EC with distance (km) from upstream. I = intercept.

$$Y = a + bx \quad (7)$$

Where Intercept = 531.481  
Slope = 5.4718

Y = Electrical Conductivity Values  
 X = Distance from upstream in km.

A plot and regression of  $SO_4^-/Cl^-$  ratio (Fig. 4) with distance (km) showed that his ratio decreases with distance as indicated in previous studies. The following linear model was developed with the coefficient of determination ( $R^2$ ) of .25

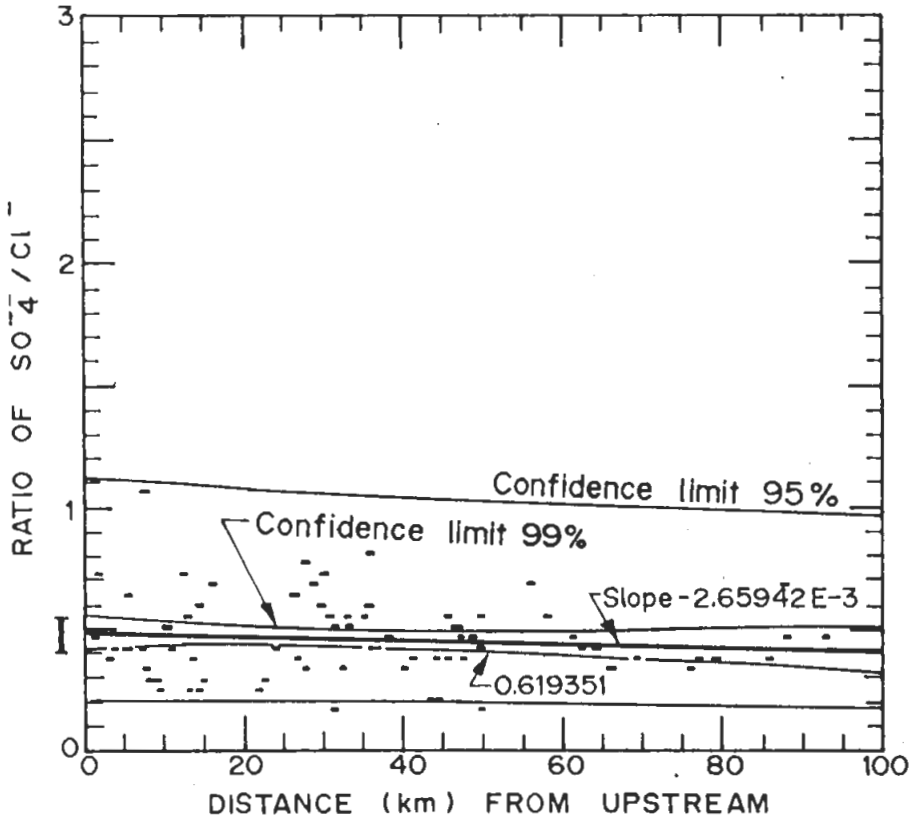


FIG. 4. Regression analysis of changes of  $SO_4^-/Cl^-$  ratio with distance (km) from upstream. I = intercept.

$$Y = a + bx \tag{8}$$

Where Intercept = 0.619351  
 Slope =  $-2.65942E-3$   
 $Y$  = Ratio of  $SO_4^-/Cl^-$   
 $X$  = Distance from upstream in km.

The general classification of the groundwater quality data for Al-Madinah Al-Munawarah Region was carried out using the Piper-diagram plot (Fig. 5). The water quality, in general, is good and contains chloride and sulfates of  $Ca^{++}$  and  $Mg^{++}$ . The

water of upstream and wadis away from the human habitants and barren region are of good quality and useful for both residential and irrigation purposes.

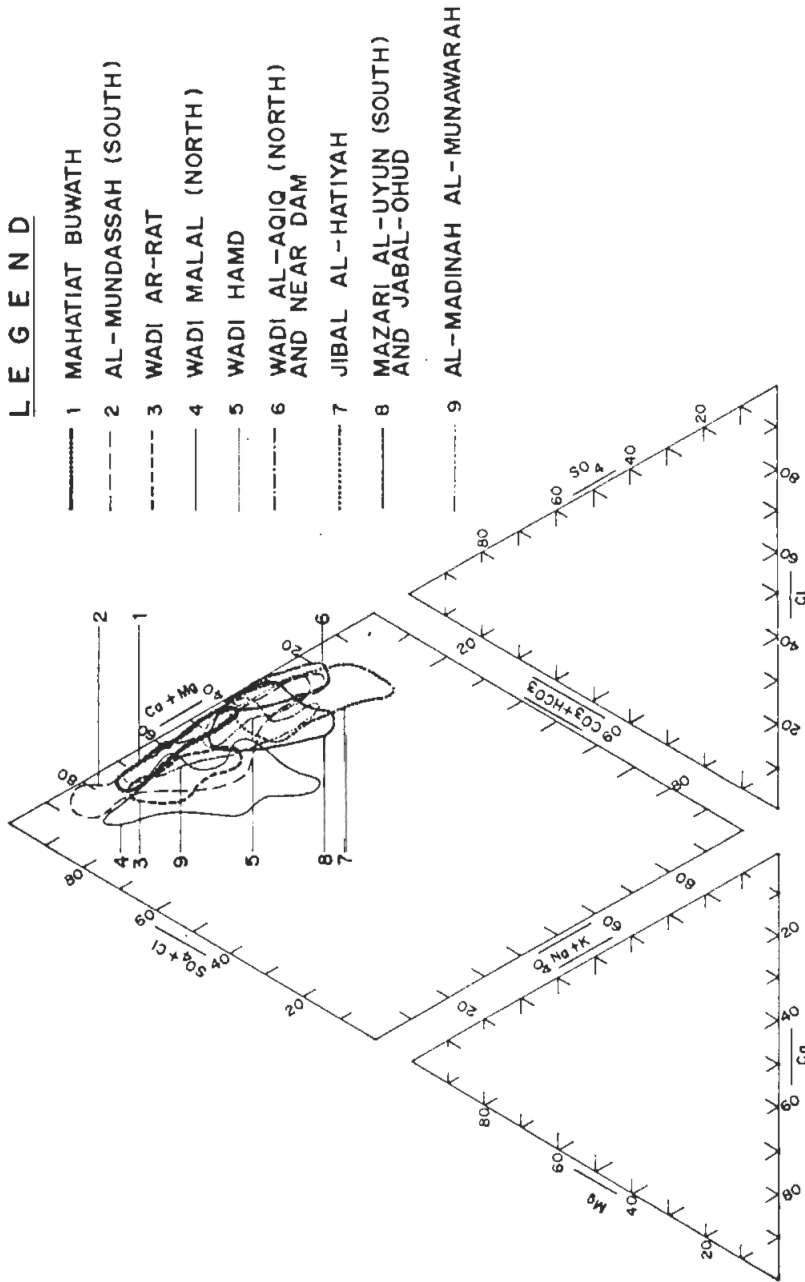


FIG. 5. Trilinear diagram showing the hydrochemical facies the groundwater of Al-Madinah Al-Munawarah basin.



### Conclusion

The conclusions of this study can be summarized as follows :

1. The value of electrical conductivity increases with distance from upstream, indicating poorer quality.
2. The ratio of  $\text{SO}_4^{--}/\text{Cl}^-$  decreases with the distance from upstream.
3. The relationship of electrical conductivity with hydraulic gradient and the number of joining tributaries is poor.
4. The quality of the water in the sampled wells in the Al-Madinah Al-Munawarah region, at present, is of good quality and mainly chloride and sulfates of  $\text{Ca}^{++}$  and  $\text{Mg}^{++}$ .
5. Various relationships developed can be utilized in determining the groundwater quality (upstream or downstream), if the quality at a certain well is known.

### References

- Alkein, O.A. (1984) *Obshchaya Gidrokimiya* – Leningrad.
- Altoviski, M.E. (1962) *Disctionary for Hydrologist*, Gosgeolitchizdat, Moseow, USSR, 616 pp., (In Russian).
- Back, W. (1961) Techniques for Mapping of Hydrochemical Facies, *U.S. Geol. Sur. Prof. Paper. 424D*: 380-382.
- Back, W. (1966) Hydrochemical Facies and Groundwater Flow Patterns in northern part of Atlantic Coastal Plain, *U.S. Geol. Surv. Prof. Paper. 498A*: A1-A42.
- Back, W. and Hanshaw, B. (1965) Chemical Geohydrology, in: Chow, V.T. (ed.) *Advances in Hydroscience*, Vol. 2: Academic Press, Inc., New York, 49-109.
- Blaney, H.F. (1952) Consumptive Use of Water, *Trans. ASCE 117*: 949-973.
- Bokhari, Y.A. and Khan, M.Z.A. (1990) Systems Approach in Assessing Suitability of Groundwater for Irrigation in western region of Saudi Arabia, *Bull. Fac. Earth Sci. K.A.U. 4*: 1-14.
- Chebotaiev, I.I. (1955) Metamorphism of Natural Waters in the Crust of Weathering, *Geochim. Cosmochim. Acta. 8*: 22-48, 137, 170 and 198-212.
- Feth, J.H. and Brown, R.J. (1962) Method of Measuring Upward Leakage from Artesian Aquifers, Using Rate of Salt-Crust Accumulation, *U.S. Geol. Sur. Prof. Paper. 450-B*: 100-101.
- Feth J.H., Barker, D.A., Moore, I.G. and Brown, R.J. (1966) Lake Bonneville: Geology and Hydrology of the Weber Delta district, including Ogden, Utah, *U.S. Geol. Sur. Prof. Paper. 518*.
- Goldschmidt, V.M. (1937) The Principles of the Distribution of the Chemical Elements in Minerals and Rocks, *J. Chem. Soc.*, London, 655-673.
- Gorrell, H.A. (1958) Classification of Formation Waters Based on Sodium Chloride Content, *Bull. Amer. Assoc. Petrol. Geologists. 42*:(2) p. 2513.
- Hem, J.D. (1959) Study and Interpretation of the Chemical Characteristics of Natural Water, *U.S. Geol. Sur. Water Supply Paper 1473*: 269 p.
- Hem, J.D. (1970) Study and Interpretation of the Chemical Characteristics of Natural Water, *Geol-Survey Water Supply Paper 1473*: 363 p.
- Ivanov, V.V., Barabanov, L.N. and Plotnikova, G.N. (1968) The Main Genetic Types of the Earth's Crust Mineral Waters and their Distribution in the U.S.S.R. In: Malkovsky, M. and Kacura (Eds.) *Report of the 23rd Session IGC Genesis of Mineral and Thermal Waters*, Prague, 33-39.
- Jetel, J.M. and Paces, T.A. (1978) Simple Classification of Water Chemistry Based on Molar Concentration, *Vestnik Ustr., Ust. Geol. 53*, Prague.
- Kazmann, R.G. (1972) *Modern Hydrology*, 2nd ed. Harper and Row, New York, 365 p.
- Linsley, R.K., Kohler, M.A. and Paulhus, J.L.H. (1975) *Hydrology for Engineers*, 2nd ed. New York, McGraw-Hill, 482 p.

- Maxey, G.B. and Mifflin, M.D.** (1966) Occurrence and Movement of Groundwater in Carbonate Rocks of Nevada, *Natl. Speleol. Soc. Bull.* **28**(3): 141-157.
- Mifflin, M.D.** (1968) Delineation of Groundwater Flow Systems in Nevada, *Desert Res. Inst., Tech. Rept. Ser. H-W*, (4) Reno, Nev.
- Penman, H.L.** (1956) Estimating Evaporation, *Trans. Am. Geophys. Union* **37**: 43-50.
- Piper, A.M.** (1944) A Graphical Procedure in the Geochemical Interpretation of Water Analyses, *Trans. Amer. Geophys. Union* **25**: 914-923.
- Piper, A.M., Garret, A.A. and Others** (1953) Native and Contaminated Waters in the Long Beach-Santa Ana area, California, *U.S. Geol. Survey Water Supply Paper* **1136**: 320 p.
- Rainwater, F.H. and Thatcher, L.L.** (1960) Methods for Collection and Analysis of Water Samples, *Geol. Survey Water Supply Paper* **1454**: 301 p.
- Schoeller, H.** (1959) Arid Zone Hydrology, *Recent Developments*, UNESCO Rev., Reicardi, 12.
- Schoeller, H.** (1962) *Les Eaux Souterraines*, Masson et Cie, Paris.
- Standard Methods for Water and Wastewater Examination** (1989) AWWA, APHA, WPCF.
- Thorntwaite, C.W.** (1948) An Approach toward a Rational Classification of Climate, *Geogr. Rev.*, **38**: 55-94.
- Tóth, J.** (1962) A Theory of Groundwater Motion in Small Drainage Basins in Central Alberta, Canada, *J. Geophys. Res.* **67**(11): 4375-4387.
- Tóth, J.** (1963) A Theoretical Analysis of Groundwater Flow in Small Drainage Basins, *J. Geophys. Res.* **68**(16): 4795-4812.
- Tóth, J.** (1966a) Mapping and Interpretation of Field Phenomena for Groundwater Reconnaissance in a Prairie Environment, Alberta, Canada, *Internat. Assoc. Sci. Hydrology Bull.* **11**(2): 20-68.
- Tóth, J.** (1966b) Groundwater Geology, Movement Chemistry, and Resources near Olds, Alberta, Res. Council Alberta (Can.) *Geol. Div. Bull.* **17**: 126 p.
- Tóth, J.** (1968) A hydrogeological Study of the Three Hills area, Alberta, Res. Council Alberta (Can.) *Geol. Div. Bull.* **24**: 117 p.
- White, D.E.** (1957a) Thermal Waters of Volcanic Origin, *Bull. Geol. Soc. Amer.* **68**(12): 1637-1657.
- White, D.E.** (1957b) Magmatic, Connate, and Metamorphic Waters, *Bull. Geol. Soc. Amer.* **68**(12): 1659-1682).
- White, D.E., Hem, J.D. and Waring, G.A.** (1963) Chemical Composition of Subsurface Waters, Chapter F, Data of Geochemistry, *U.S. Geol. Sur. Prof. Paper.* **440-F**: 76 p.

## النمذجة المُحدَّدة لنوعية المياه الجوفية في المدينة المنورة باستخدام مدخل «معامل التجمع»

عاصم يحيى بخاري\* و محمد ذوالفقار علي خان\*\*  
\* كلية علوم الأرض - \*\* قسم الهندسة المدنية ، كلية الهندسة ،  
جامعة الملك عبد العزيز ، جدة ، المملكة العربية السعودية

المستخلص . أجريت تحاليل لعينات من المياه الجوفية لعدد ١٠٥ بئرا في منطقة المدينة المنورة ، بالمملكة العربية السعودية باستخدام مدخل معامل التجمع والتحليل الارتدادي ، ونتج عنها معادلات (قيم  $R^2$  المرتفعة) بين التوصيل الكهربائي (ت) والأيونات الأخرى ، ص<sup>+</sup> + بؤ<sup>-</sup> والأيونات الأخرى ، م ك والمسافة (كم) من المنبع وكذا نسبة ك ب أ<sup>٤</sup> / كل والمسافة من المنبع . ومن السهل استخدام المعادلات الناتجة لتحديد نوعية المياه الجوفية في مياه الآبار عند المنبع وعند المصب من قيمة ت ك معروفة في بئر محددة . وأغلب المياه الجوفية الحالية من نوع كبريتات وكلوريدات الكالسيوم والمغنسيوم ، وهي صالحة للاستخدام المنزلي .

وتبين التحليل أن نسبة ك ب أ<sup>٤</sup> / كل تتناقص مع المسافة من المنبع ، في حين يزداد ت ك مع المسافة . والعلاقة بين عدد الفروع والميل الهيدرولي مع المسافة من المنبع ضعيفة .  
الكلمات المفتاحية .

نمذجة مُحدَّدة ، تحليل ارتدادي ، نوعية المياه الجوفية في منطقة المدينة المنورة ، التوصيل الكهربائي ، تأثير الفروع والميل الهيدرولي .