Analysis of Saltwater Intrusion Beneath an Oceanic Island

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ABSTRACT. The position of saltwater-freshwater interface beneath an oceanic island was analyzed. Governing partial differential equations were formulated by combining Darcy's Law and continuity equation by using Ghyben-Herzberg Principle and Dupuit assumptions. For the solution of the governing equations, Finite Difference Method was used. A computer program based on two-dimensional steady-state mathematical model was improved and modified. As sufficient and reliable field data are unavailable for an oceanic island, the program has been verified using two simplified flows for which analytical solutions are available, namely, radial flow in a circular island and one dimensional flow in a strip island. The program was then applied for three other aquifers of square, rectangular, and irregular shape. The effects of island geometries, aquifer characteristics and various imposed conditions on saltwater intrusion were analyzed.

Introduction

An oceanic island is physically a land mass surrounded by sea from all sides. In the aquifers of such an island, the lighter freshwater is underlain by heavier saltwater. Under normal conditions, there exists an equilibrium between these two liquids which differ in their densities. The movement of the freshwater towards the sea acts as a barrier to the intrusion of the saline water inland and into the aquifer (Henry 1959).

For these oceanic islands, the aquifers are generally the only sources of water supply. With increasing urbanization, agricultural and other activities, the need to develop the groundwater aquifer increases. However, inadequate data and often hasty decisions in the development process of the aquifer results in surpassing its safe yield. This results in lowering of the water table, which causes an imbalance in the dynamic equilibrium attained under normal conditions between the fresh and saline waters. As a consequence, the normally seaward gradient of the water table is altered, allowing the saline water to move towards the freshwater aquifer.

It has been known that upconing of the saltwater and rate at which freshwater turns saline depends on rate of discharge, duration of pumping and hydrologic conditions of the aquifer (Reilly and Goodman 1985). Hence, the ability to locate the saltwater interface and to predict its behavior would assist in placing pumping or recharge wells at appropriate locations.

In the light of the foregone discussion, it is apparent that a quantitative understanding of the nature of both movement and mixing between fresh and saltwater and other factors influencing these processes is invaluable to the protection and proper management of these aquifers in the future (Reilly and Goodman 1985).

Theoretical Background

The earliest work related to sea water intrusion problems was undertaken by Badon-Ghyben (1888) and Herzberg (1901). Their works have been known as Ghyben-Herzberg Principle, and it marks the beginning of quantitative analysis to locate the saltwaterfreshwater interface in coastal aquifers. Since then, numerous investigations have been performed and, therefore, a considerable number of publications are available in the literature.

The problem considered in this paper is confined to saline water intrusion in islands, consequently only the studies related to saltwater intrusion in oceanic islands will be mentioned. For a comprehensive literature review on quantitative analysis of saltwater-freshwater relationships in groundwater systems, the reader is referred to, for example, Reilly and Goodman (1985).

When the freshwater body floats on top of the seawater, the interface has the shape of a lens, hence it is designated as lens model. The following studies dealt with such a model.

Fetter (1972) developed a mathematical model describing a two-dimensional steady-state position of saltwater-freshwater interface in an unconfined aquifer by making use of Dupuit-Forchheimer approximations. He used finite difference technique to numerically solve his model. He also presented an application to show how his numerical model can be used to generate the known position of the saline water interface beneath the South Fork of Long Island, New York.

Anderson (1976) proposed two one-dimensional models based on the Dupuit-Forchheimer assumptions to simulate flow beneath strip oceanic islands. Both models are solved by finite differences. Anderson used his second model to predict the response of water levels to modifications in recharge patterns on the South Fork of Long Island.

Contractor (1981) presented a two-dimensional saltwater intrusion model based on finite element method, and later applied it to the aquifer in Northern Guam (Contractor 1983). He emphasized how the model can be used to make an infinite variety of management and planning studies.

Mathematical Model

A mathematical model offers a description of a physical system and the mechanisms that influence it most, by a set of differential equations. Quite often, the physical systems are too complex to be described easily. Thus, certain assumptions have to be made to simplify the problem. These assumptions should be neither too simple nor too complex. This is because they will determine how far the model will approximate the field conditions, which, in turn, determines the reliability of the model.

In this work, the mathematical model describing the position of the groundwater table and the depth of the saltwater interface, both with respect to the mean sea level, is derived using simultaneously :

- 1. Principle of conservation of mass,
- 2. Darcy's law,
- 3. Ghyben-Herzberg principle.

Assumptions

In oceanic islands, where the freshwater lens is usually thin, compared with the lateral extent, and the slope of water table is small, the groundwater flow in the aquifer is generally assumed to be horizontal under normal conditions. In such a case, the equipotential surfaces are vertical and the velocity is uniform over the depth of the flow. Groundwater in such aquifers are almost always tackled using these approximations (Fetter 1980). They are referred to as Dupuit-Forchheimer assumptions.

In the mathematical formulation, in addition to the above, the following simplifying assumptions are used.

1. The medium of the aquifer is both homogeneous and isotropic.

2. Both fresh and saltwater are immiscible. A sharp interface exists between them, and there is no flow across this interface.

3. Fresh and saltwater are slightly compressible, and their viscosities are constant.

4. Seawater is stagnant and tidal action is negligible.

5. Vertical flow is only considered for accretion.

Governing Equations

Typical cross-section of a coastal unconfined aquifer is depicted in Fig. 1. The governing partial differential equation which describes the unsteady position of the water table, h, for this flow system is :

$$K(1 + \delta) \left[\frac{\partial}{\partial x} \left(h \frac{\partial h}{\partial x} \right) \right] + K(1 + \delta)$$
$$\left[\frac{\partial}{\partial y} \left(h \frac{\partial h}{\partial y} \right) \right] + W = S(1 + \delta) \frac{\partial h}{\partial t} \quad (1)$$

with $\delta = \gamma_f / (\gamma_s - \gamma_f) = \rho_f / (\rho_s - \rho_f)$



FIG 1. Typical cross-section of a coastal unconfined aquifer.

The depth of interface, with respect to mean sea level, is then given by Ghyben-Herzberg relationship.

$$B = \delta h \tag{2}$$

The symbol used are as follows :

- K hydraulic conductivity [L/T]
- h water table elevation [L]
- W recharge rate [L/T]
- S specific yield, dimensionless
- x space coordinate[L]
- y space coordinate[L]
- B depth of interface with respect to mean sealevel [L]
- t time[T]
- γ specific weight of water $[M/T^2L^2]$
- ρ density of water $[M/L^3]$

In the above equations, f and s refer to fresh and saltwater, respectively.

Details of the above equations may be found elsewhere (Onder 1988). Slightly different derivation is given by Bear (1979).

When the flow is steady, the variation of parameters with respect to time is zero, *i.e.* the right hand side of Eq. 1 becomes null, and it takes the following form :

$$K(1 + \delta) \left[\frac{\partial}{\partial x} \left(h \frac{\partial h}{\partial x} \right) \right] + K(1 + \delta)$$
$$\left[\frac{\partial}{\partial y} \left(h \frac{\partial h}{\partial y} \right) \right] + W = 0$$
(3)

When it is further manipulated it becomes

$$\frac{\partial^2 h^2}{\partial x^2} + \frac{\partial^2 h^2}{\partial y^2} = \frac{-2W}{K(1+\delta)}$$
(4)

When h^2 in Eq. 4 is replaced by v it is easily seen that it presents the well-known Poisson Equation.

Boundary Conditions

Equation 1 and its special forms Eq. 3 and 4 are all partial differential equations. For a complete mathematical description to these equations, a set of appropriate boundary and initial conditions should be added.

In general, the following boundary conditions are encountered in the field (Bear 1979) :

1. Known head boundary condition (Drichlet Condition)

2. Known flow rate boundary condition (Neumann condition)

3. Mixed boundary conditions (Combination of 1 and 2).

When the flow is unsteady, in addition to boundary conditions, the initial conditions must also be specified. The initial conditions are meaningful set of head values at the initial time. Specification of the boundary and initial conditions complete the mathematical formulation of the physical problem. The solution of the resulting equations by using an appropriate technique gives the elevation of the water table with respect to mean sea level. The depth of the interface is then easily obtained using Ghyben-Herzberg principle.

Method of Solution

Numerical Model

In this work, for the solution of mathematical model, numerical solution technique is adopted. Usually analytical solutions are confined to simplified and special cases. This is because of the fact that it is extremely difficult, if not impossible, to handle irregular boundaries and, in the case of unsteady flow, to obtain the general solution for two dimensional flow. However, analytical solutions have been used for verification purposes and to understand the mechanism of flow behavior.

The most commonly used numerical techniques are finite difference and finite element methods. In this research work, finite difference method is selected. The basic idea in finite difference method is that the partial differential equation governing the behavior of the fresh and saltwater interface is approximated by finite differences. This leads to a set of algebraic equations, the solution of which is much simpler. Finite difference approximations of the governing partial differential equation is given in the following paragraphs.

Using central difference approximation (Fig. 2), Eq. 4, at any nodal point (i, j) may be approximated as



FIG. 2. Grid system for central difference scheme.



where $v_{i,j} = h_{i,j}^2$

W – net recharge rate

δ – constant

The increment in y direction is taken to be equal to Δx .

Applying Eq. (5) to all nodal points will give a system of approximating algebraic equations. A typical set of n equations in n unknowns would have the form

$$a_{1,1} v_1 + a_{1,2} v_2 + \dots a_{1,n} v_n = b_1$$

$$a_{2,1} v_1 + a_{2,2} v_2 + \dots a_{2,n} v_n = b_2$$
(6)
$$\dots = \dots$$

$$a_{n,1} v_1 + a_{n,2} v_2 + \dots a_{n,n} v_n = b_n$$

This set of equation can be written in matrix form as :

 $\begin{bmatrix} a_{1,1} & a_{1,2} & \dots & a_{1,n} \\ a_{2,1} & a_{2,2} & \dots & a_{2,n} \\ \dots & \dots & \dots & \dots \\ a_{n,1} & a_{n,2} & a_{n,n} \end{bmatrix} \cdot \begin{cases} v_1 \\ v_1 \\ \dots \\ v_1 \end{cases} = \begin{cases} b_1 \\ b_1 \\ \dots \\ b_1 \end{cases}$ (7)

where

 $v_{i,j}$, *i* and $j = 1, 2, \dots, n$ are unknown parameters b_j , $i = 1, 2, \dots, n$ are known values, a_i , $i = 1, 2, \dots, n$ are known coefficients.

Equation 7 can be rewritten using capital letters to denote each array, brackets to denote a square matrix, and braces to denote a column matrix as :

$$[A] \{V\} = \{B\}$$
(8)

Solution of Linear Algebraic Equations

In general, linear algebraic equations are solved by iteration or direct methods. In this work, the set of linear algebraic equations given by Eq. 8 is solved by Gauss Elimination Method, which is a feasible direct method, when matrix equation contains less than 1000 unknowns.

This numerical method has been used successfully by Onder and Kalyoncu (1987), Onder (1988) and Hassan (1988).

The listing of computer program used in the solution of Eq. 8 and the details of it may be found in Onder (1988) and Hassan (1988).

Verification of Numerical Model

Often, due to lack of field data, comparison of numerical results with actual field observations is not possible. An alternative way of verification is to test the numerical model against known solutions to the partial differential equation. This is the case in the present work.

In this context, to demonstrate the accuracy of the solution technique used in the numerical model, two analytical solutions are used, namely, for one-dimensional steady flow in an infinitely long strip island, and for axisymmetric radial flow in a circular island.

Analytical Solutions

Strip Island

Cross-sectional view of the flow system in a strip aquifer along the width of the island is depicted in Fig. 3. The width of the island in x-direction is 2L, and the island has an infinite length in y-direction. The flow is practically one-dimensional in x-direction.



FIG. 3. One-dimensional flow beneath an oceanic strip island.

i) Governing Equations. The partial differential equation that describes the flow is one-dimensional version of Eq. 4.

$$\frac{d}{dx}\left(\frac{dh^2}{dx}\right) + \frac{2W}{K(1+\delta)} = 0 \tag{9}$$

The meaning of the symbols are as defined previously.

ii) Boundary conditions. Let the origin x = 0 be located on the centerline of the island. The x-axis is normal to the longitudinal axis. Referring to Fig. 3, the boundary conditions in terms of the water table may be written as follows :

$$x = 0, \qquad h \frac{dh}{dx} = 0 \qquad (10)$$

$$x = L, \qquad h = 0 \qquad (11)$$

Where L is the half width of the strip aquifer.

iii) Solution. Integrating Eq. 9 and forcing it to satisfy the conditions given by Eq. 10 and 11 gives :

$$h^{2} + \frac{W}{K(1+\delta)} (L^{2} - x^{2}) = 0$$

or

$$h = \sqrt{\frac{W}{K(1+\delta)} (x^2 - L^2)}$$
 (12)

is obtained. This is the solution for the water table elevation, h with respect to mean sea level. The depth of the interface with respect to mean sea level is then obtained using Eq. 2.

Circular Island

Cross-sectional view of the aquifer beneath a circular island along the diameter is depicted in Fig. 4.



FIG. 4. Cross-sectional view of an aquifer beneath a circular island.

i) Governing Equation. Eq. 9 applies also for this flow. Since the flow is radially axisymmetric, it is convenient to write it in polar coordinates. Then, it takes the following form :

$$\frac{1}{r} \frac{d}{dr} \left(r \frac{dh^2}{dr} \right) + \frac{2W}{K(1+\delta)} = 0$$
(13)

The meaning of the symbols are as previously defined.

ii) Boundary Conditions. Let the origin r = 0 be located on the center of the island (Fig. 4). Eq. 13 is subjected to the following boundary conditions.

$$r = 0, \qquad h \frac{dh}{dr} = 0 \tag{14}$$

(15)

$$r = R, \qquad h = 0$$

Where R is the radius of the aquifer.

iii) Solution. Integrating Eq. 13 leads :

$$h^2 + \frac{W}{2K(1+\delta)} (r^2 - R^2) = 0$$

or

$$h = \sqrt{\frac{W}{2K(1+\delta)}(R^2 - r^2)}$$
(16)

Equation 16 gives the variation of the water table elevation, h, with respect to mean sea level and as a function of the radial distance, r.

Comparison of Analytical and Numerical Solutions

Strip Island

The vertical cross sectional view of the aquifer along the width of the island is same as the one shown on Fig. 3. The plan view and finite difference grid system is given on Fig. 5. The following data are considered :

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	1	2	3	4	5	6	7
	8	9	10	11	12	13	14
	15	16	17	18	19	20	21
	22	23	24	25	26	27	28
	29	30	31	32	33	34	35
•	36	37	38	39	40	41	42
	43	44	45	46	47	48	49
	50	51	52	53	54	55	56
	57	58	59	60	61	62	63
	64	65	66	67	-68	69	70
	71	72	73	74	75	76	77
	78	79	80	81	- 82	83	84
	85	86	87	88	89	90	91
	92	93	94	95	96	97	98
	99	100	101	102	103	104	105
	106	107	108	109	110	111	112
	113	114	115	116	117	118	119

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Dimensions of the aquifer800 m by 3,600 mHydraulic conductivity20 m/dayNet recharge rate0.00055 m/dayDensity of the freshwater $1,000 \text{ kg/m}^3$ Density of the saltwater $1,025 \text{ kg/m}^3$ Mash size100 m by 100 mGrid size 9×37

The computed values by numerical method of the water table elevations and the depth of interface at nodal points lying on a line across the width of the island are shown in Table 1, in which corresponding analytical results are also given. The results are also plotted in Fig. 6. It is seen that the agreement between numerical and analytical results are almost perfect.

TABLE 1. Results of numerical solution for strip island and comparison with analytical solution.

	Analytical	solution	Numerical solution		
Distance (m)	Water table elevation (m)	Interface depth (m)	Water table elevation (m)	Interface depth (m)	
-400	0.000	0.000	0.000	0.000	
-300	0.217	-8.667	0.219	-8.765	
200	0.284	-11.348	0.287	-11.476	
-100	0.317	-12.688	0.321	-12.829	
0	0.328	-13.104	0.331	-13.250	
100	0.317	-12.688	0.321	-12.829	
200	0.284	-11.348	0.287	11.476	
300	0.217	-8.667	0.219	-8.765	
400	0.000	0.000	0.000	0.000	



FIG. 6. Comparison of analytical and numerical solutions for a strip aquifer.

Circular Island

The vertical cross-sectional view of the aquifer along the diameter of the island is same as the one shown on Fig. 4. The plan view and finite difference grid system is given in Fig. 7. The following data are considered.



FIG. 7. Plan view and finite difference grid for a circular aquifer.

Diameter of the aquifer	960 m
Hydraulic conductivity	20 m/day
Net recharge rate	0.00055 m/day
Density of the freshwater	$1,000 \text{kg/m}^3$
Density of the saltwater	$1,025 \text{kg/m}^3$
Mash size	80 m by 80 m
Grid size	13×13

The computed values of the water table elevations and the depth of interface at nodal points laying on the diameter are shown in Table 2, in which corresponding analytical results are also shown. The results are also plotted in Fig. 8. It is seen that there is a very good agreement between numerical and analytical results.

 TABLE 2. Results of numerical solution for circular island and comparison with analytical solution.

	Analytical	solution	Numerical solution		
Distance (m)	Water table elevation (m)	Interface depth (m)	Water table elevation (m)	Interface depth (m)	
480	0.000	0.000	0.000	0.000	
-400	0.154	6.146	0.158	-6.242	
-320	0.207	-8.288	0.208	-8.331	
-240	0.241	-9.629	0.241	-9.652	
-160	0.262	-10.483	0.263	10.508	
80	0.274	-10.963	0.275	-10.996	
0	0.278	-11.119	0.279	-11.155	
80	0.274	10.963	0.275	-10.996	
160	0.262	÷-10.483	0.263	-10.508	
240	0.241	-9.629	0.241	-9.652	
320	0.207	-8.288	0.208	-8.331	
400	0.154	-6.146	0.158	-6.242	
. 480	0.000	0.000	0.000	0.000	

Death

and

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FIG. 8. Comparison of analytical and numerical solutions for a circular aquifer.

Application

To investigate the effects of island geometry, aquifer characteristics and various imposed boundary conditions on the aquifer behavior, the model is applied to three hypothetical aquifers, of square, rectangular, and irregular shapes.

Square Island

For the square aquifer, the plan view is shown in Fig. 9. The following data is considered.



FIG. 9. Plan view and finite difference grid for a square aquifer.

Dimensions of the aquifer	960 m by 960 m
Hydraulic conductivity	20 m/day
Net recharge rate	0.00055 m/day
Density of the freshwater	$1,000 \text{kg/m}^3$
Density of the saltwater	$1,025 \text{ kg/m}^3$
Mash size	80 m by 80 m
Grid size	13×13

The summary of the runs is given in Table 3. The results of a typical run is plotted in Fig. 10 and 11. They show the computed values of the water table elevations and the depth of interface at nodal points lying on the symmetry axes.

TABLE 3. Summary of runs in square island.

Run No.	Q	Remarks
1	0	No well
2	+20	One discharge well
3 .	+25	One discharge well
4	-20	One recharge well
5	-25	One recharge well

Water Table Elevation & Interface Depth



FIG. 10. Variations of water table elevation and interface depth along symmetry axes.

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Water Table Elevation & Interface Depth

FIG. 11. Variations of water table elevation and interface depth along x-axis.

Rectangular Island

A rectangular aquifer with the data given below is considered. Figure 12 represents the plan view and the discretization of the aquifer by a grid system.



FIG. 12. Plan view and finite difference grid for a rectangular aquifer.

Dimensions of the aquifer	800 m by 1,200 m
Hydraulic conductivity	20 m/day
Net recharge rate	0.00055 m/day
Density of the freshwater	$1,000 \text{kg/m}^3$
Density of the saltwater	$1,025 \text{kg/m}^3$
Mash size	80 m by 80 m
Grid size	12×26

Four wells are situated along the longitudinal symmetry axis and in the central part of the aquifer.

The distribution of the water table elevations and the position of the interface in the aquifer under steady state conditions, but subjected to various imposed conditions, are computed by the model.

Typical results are plotted in Fig. 13 and 14. Figure 13 shows the elevation of water table and the depth of the interface along symmetry axis in x-direction,



FIG. 13. Variations of water table elevation and interface depth along x-axis.

whereas Fig. 14 along symmetry axis in y-direction.

Irregular Island

The aquifer, the geometry of which is given in Fig. 15, with the following conditions is considered.



FIG.14. Variations of water table elevation and interface depth along y-axis.



FIG. 15. Plan view and discretization of an aquifer with irregular shape.

Hydraulic conductivity	20 m/day
Net recharge rate	0.00055 m/day
Density of the freshwater	$1,000 \text{kg/m}^3$
Density of the saltwater	$1,025 \text{kg/m}^3$

Mash size	80 m by 80 m
Grid size	13×17

Three wells are situated along the line A-A.

Figure 16 shows the elevation of water table and the depth of interface along section A-A.



FIG. 16. Variations of water table elevation and interface depth along section A-A.

Discussion

Numerical simulation of groundwater flow systems depends, to a large extent, upon the availability of field data. The data related to the saltwater interface problems in oceanic island is, however, relatively scarce and difficult to obtain. This is one of the major difficulties that the authors have encountered.

Since the Dupuit-Forchheimer assumptions and the sharp interface assumption of Ghyben-Herzberg are used in the present numerical model, errors may have been introduced. On the other hand, the inaccuracies in aquifer's hydraulic properties and boundary conditions, together with the difficulties in the determination of the exact subsurface geometry of the aquifer would introduce additional errors in the simulation results. However, the latter overshadows the former.

The two-dimensional steady numerical model, in spite of its inherent restrictions and the simplifications introduced in its construction, is a useful tool for the prediction of the aquifer behavior, *i.e.* the position of the water table and the depth of interface. It can be used in the management of the groundwater resources, especially when steady-state local conditions are of interest. When the flow is time dependent, and the proposed model can still be used for simulation purposes. An approximate analytical technique to determine the motion of the seawater interface in a coastal aquifer by the method of successive steady state is presented by Bear and *et al.* (1985). Particularly, when the model is suitably linearized, it can be very helpful for incorporation into a management model of an aquifer in which the location of the interface is one of the criteria.

Summary and Conclusion

Partial differential equations governing the position of saltwater-freshwater interface and the height of water table beneath an oceanic island are provided. For the solution of steady, two-dimensional flow, finite difference method is used.

A computer program based on a two-dimensional, steady-state mathematical model is improved and modified to analyze saltwater intrusion beneath oceanic islands.

As sufficient and reliable field data for an oceanic island is not available to the authors, the verification is accomplished by comparing the results for two aquifers, namely one beneath a strip island and the other beneath a circular island.

The program is then applied to three other hypo thetical aquifers; square, rectangular and irregular in shape. The effects of island geometries, aquifer characteristics and various imposed conditions on saltwater intrusion were analyzed for these aquifers.

The depth to saltwater interface is sensitive to hydraulic conductivity, therefore, the determination of field hydraulic conductivity values should be given due attention.

The model takes into account the spatial variability of the recharge and evaporation rates. The rates of evaporation and recharge are controlled by the vegetation cover and surface soil characteristics. This aspect is important when these parameters vary spatially.

Careful placement of the recharge or extraction wells and judicious selection of their rates is essential for the management of groundwater resources.

When the field conditions do not match with the sharp interface assumption, this model can not be used.

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المستخلص . ركزت هذه الدراسة على موضع تداخل المياه الملحة مع المياه العذبة في الطبقات الواقعة تحت جزر المحيطات .

وقد تم استخدام وربط معادلة حركة المياه (دارسي) بمعادلة الاستمرارية بوساطة نظرية اختلاف كثافة السوائل المعروفة « بغايبن هيرزبيرج » وافتراضات « ديوبيوت » وحل هذه المعادلات بطريقة الفروق العددية .

ونظرًا لعدم توافر المعلومات الخاصة لإثبات نتائج النموذج المقترح المبدئية للتنبؤ بحركة المياه ومواقع تداخلها مع المياه العذبة تحت الجزر ، فقد تم إثبات نتائجه بتطبيقه على حالتين مبسطتين :

الأولى : حركة المياه الدائرية في اتجاه مركز الجزيرة .

والأخرى : حركة المياه الأفقية في اتجاه واحد لجزيرة ذات شكل مستطيل .

وبناء على نتائج التطبيق المبدئية ، تم تطبيق النموذج المقترح على ثلاث حالات بالنسبة لحجم الطبقات الحاملة للماء ذات أشكال مربعة ، ومستطيلة ، وغير متجانسة الشكل .

ومن خلال الدراسة ، تم التعرف على التأثير الناجم من جراء تغيرات أشكال الطبقات الحاملة للمياه والواقعة تحت الجزر ، بالإضافة إلى الخواص الهيدرولية وتغيير نتائج الدراسة . تبين أنه يمكن استخدام النموذج الرياضي المقترح للتنبؤ بمواقع مناطق تداخل المياه الملحة مع المياه العذبة ذات التغيرات الزمنية والخواص الهيدرولية الثابتة .