

Suboptimal Decentralized Control of Multivariable Systems

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ABSTRACT. A state-space design for multivariable, suboptimal decentralized control is presented. It is based upon the scalar multiplier with a cost function βJ . A large-scale system consisting of a number of subsystems is used. The suboptimal control law for the large system is obtained from the suboptimal laws of the subsystems. Since the control laws of the subsystems are obtained by solving the low-order matrix Riccati equations, a considerable saving in computer storage and time is attained. Sufficient conditions for the optimal cost function are also given. The behaviour of the algorithm is illustrated by a simulated example.

KEYWORDS: Optimal control; Suboptimal control; Decentralized systems.

1. Introduction

When the order of the system is very high, the solution of the matrix Riccati equation is required when designing the suboptimal controls for systems with quadratic performance criteria. The number of nonlinear differential equations to be integrated becomes very large, if the order of the system is high. It follows that feedback controller of state information can be proven to be the major cost of the virtual situations such as: power-system control, process control and complex industrial system controls. To overcome this difficulty, many methods mentioned in Meditch^[1], Aoki^[2], Chidambara and Schanker^[3] as well as Isaken and Payne^[4] have been proposed to obtain suboptimal controls for higher-order systems. Hassan and Singh^[5] developed a procedure for computing under optimal or suboptimal decentralized controller. In Reddy and Rao^[6], the suboptimal control law for the large-scale system is derived from the optimal control laws of the subsystems. Since the optimal control laws of the subsystems are obtained by solving the lower-order matrix Riccati equation; Skelton and Xu^[7], Yaz^[8], Ismail^[9], Ismail^[10], Ismail^[11] and Toivonen and Makila^[12] have developed an entirely new approach to the solution of the decentralized suboptimal control problem. In this research paper, a new design procedure, which is motivated by its simplicity and the fact that the algorithm is based on suboptimality multiplier βJ is developed. It uses the well-known standard Riccati equation when reliable conditions are satisfied to form a decentralized output feedback control gains. Numerical results show that the algorithm has numerous advantages.

2. Control Problem

Research in decentralized control has been motivated by using conventional modern control theory as state space equations. In this research paper, a large-scale system consisting of N subsystem that can be defined for a decentralized system is used. The sub-optimal problem of a large-scale system consisting of N subsystem, with βJ for the cost function, can be written as

$$J = \sum_{K=1}^N \int_0^{\infty} e^{2\alpha} (X_K^T Q_K X_K + U_K^T R_K U_K) dt \quad \alpha \geq 0 \quad (1)$$

Where α is a real scalar, $Q_K \geq 0$ and $R_K > 0$. J in equation (1) is minimized according to the linear, time-invariant, continuous system, consisting of N subsystems as follows:

$$\begin{aligned} \dot{X}_K &= A_K X_K + B_K U_K, \quad X_K(0) = X_0 \\ y_K &= C_K X_K, \quad K = 1, \dots, N \end{aligned} \quad (2)$$

where K is a subsystem,

X_K is a state vector of dimension n_K ,

U_K is a control vector of dimension p_K ,

y_K is an output vector of dimension r_K , and

A_K , B_K and C_K are matrices of ranks n_K , p_K and r_K respectively.

It is required to determine a suboptimal decentralized control law of the form:

$$U_K^* = -F_K y_K, \quad K = 1, \dots, N \quad (3)$$

where F_K is of feedback gain matrix of dimension r_K

This function will minimize the cost function of the equation (1).

3. Solution of the Problem

A suboptimal decentralized control U_K^* and its cost J for the k th subsystem are given by:

$$U_K^* = -F_K C_K X_K = -R_K^{-1} B_K^T \tilde{P}_K X_K \quad (4)$$

$$J = X_K^T(0) \tilde{P}_K X_K(0) \quad (5)$$

For evaluating the suboptimal output feedback system, P_K and A_K^n are defined as:

$$\tilde{P}_K = \beta P_K \quad (6)$$

$$\text{and } A_K^n \triangleq A_K + B_K R_K^{-1} B_K^T \tilde{P}_K - B_K F_K C_K \quad (7)$$

where $P_k = P_k^T$ is the unique positive solution of the algebraic Riccati equation:

$$P_K (A_K + \alpha I) + (A_K + \alpha I)^T P_K + Q_K - P_K B_K R_K^{-1} B_K^T P_K = 0 \quad (8)$$

for a given $\beta \geq 1$, if there exists an F_k which satisfies

$$\tilde{P}_K (A_K^n + \alpha I) + (A_K^n + \alpha I)^T \tilde{P}_K + Q_K - \tilde{P}_K B_K R_K^{-1} B_K^T \tilde{P}_K = 0 \quad (9)$$

$$\text{and } C_K^T F_K^T R_K F_K C_K \leq \tilde{P}_K B_K R_K^{-1} B_K^T \tilde{P}_K \quad (10)$$

In this case, there exists a suboptimal decentralized output feedback control of equation (4), with which the subsystem (2) yields the minimized cost function of equation (1).

4. Summary of the Algorithm

The proposed algorithm was tested on one system, and the minimum a priori knowledge required by the suboptimal decentralized control is identified. The constraints and the procedures of the algorithms are as follows:

1 - $\alpha \geq 0$ and $\beta \geq 1$ are real scalars,

2 - $F_K \in \theta_K$ is chosen,

where $\theta_K = F_K \in R^{p \times r} : A_K + B_K F_K C_K$ is asymptotically stable, and $j = 1$, where j = number of iterations,

3 - $F_K^J \in \theta_K$ is found, so that:

$$\tilde{P}_K^J (A_K^n + \alpha I) + (A_K^n + \alpha I)^T \tilde{P}_K^J + Q_K - \tilde{P}_K^J B_K R_K^{-1} B_K^T \tilde{P}_K^J = 0,$$

$$\text{and } C_K^T F_K^T R_K F_K C_K \leq \tilde{P}_K^J B_K R_K^{-1} B_K^T \tilde{P}_K^J.$$

Consequently, equations (9) and (10) are satisfied. They represent the sufficient conditions for the suboptimal decentralized control.

4 - $j := J + 1$ is updated

5 - Then, go to No. 2.

5. A Numerical Example

A control system consisting of three subsystems is considered. It is defined by the following matrices (Yu and Siggers^[13], Yu and Moussa^[14]:

$$A_{11} = \begin{bmatrix} -0.922 & 1.00 & -2.06 & -0.099 \\ -2.750 & -2.78 & -1.36 & -0.037 \\ 0.000 & 0.00 & 0.00 & 0.000 \\ -4.950 & 0.00 & -5.55 & -0.039 \end{bmatrix}, A_{12} = \begin{bmatrix} 0.024 & 0.0 & -0.087 & 0.002 \\ -0.158 & 0.0 & 1.110 & -0.001 \\ 0.000 & 0.0 & 0.000 & 0.000 \\ 0.222 & 0.0 & 8.170 & 0.004 \end{bmatrix}$$

$$\begin{aligned}
A_{13} &= \begin{bmatrix} 0.072 & 0.0 & -0.25 & 0.003 \\ 0.460 & 0.0 & 2.80 & -0.020 \\ 0.000 & 0.0 & 0.00 & 0.000 \\ 0.924 & 0.0 & 1.75 & 0.020 \end{bmatrix}, B_1 = \begin{bmatrix} 0.0 & 0.4 \\ 0.0 & 0.9 \\ 0.8 & 0.4 \\ 0.0 & 0.9 \end{bmatrix}, C_1 = I \\
A_{21} &= \begin{bmatrix} 0.21 & 0.0 & 0.121 & 0.003 \\ -1.10 & 0.0 & -0.620 & -0.015 \\ 0.00 & 0.0 & 0.000 & 0.000 \\ 0.92 & 0.0 & 1.750 & 0.020 \end{bmatrix}, A_{22} = \begin{bmatrix} 0.21 & 1.0 & 1.60 & -0.005 \\ -1.90 & -1.8 & 9.30 & -0.120 \\ 0.00 & 0.0 & 0.05 & 1.000 \\ -3.10 & 0.0 & -5.55 & -0.032 \end{bmatrix} \\
A_{23} &= \begin{bmatrix} 0.60 & 0.0 & 0.46 & 0.002 \\ -1.00 & 0.0 & 1.49 & -0.040 \\ 0.00 & 0.00 & 0.00 & 0.000 \\ 0.12 & 0.0 & 2.98 & -0.280 \end{bmatrix}, B_2 = \begin{bmatrix} 0.9 & 0.9 \\ 0.0 & 0.0 \\ 0.8 & 0.4 \\ 0.0 & 0.9 \end{bmatrix}, C_2 = I \\
A_{31} &= \begin{bmatrix} -0.002 & 0.0 & 0.089 & 0.00 \\ -6.780 & 0.0 & -1.100 & -0.09 \\ 0.000 & 0.0 & 0.000 & 0.00 \\ -1.240 & 0.0 & 2.980 & -0.28 \end{bmatrix}, A_{32} = \begin{bmatrix} 0.01 & 0.0 & 0.22 & 0.000 \\ -2.10 & 0.0 & 1.70 & 0.123 \\ 0.00 & 0.0 & 0.00 & 0.000 \\ 0.07 & 0.0 & 0.37 & 0.011 \end{bmatrix} \\
A_{33} &= \begin{bmatrix} -0.197 & 1.0 & -1.20 & -0.003 \\ -5.44 & 2.0 & 7.00 & -2.370 \\ 0.0 & 0.0 & 0.05 & 1.000 \\ -3.4 & 0.0 & -2.10 & -0.017 \end{bmatrix}, B_3 = \begin{bmatrix} 0.8 & 0.4 \\ 0.0 & 0.9 \\ 0.8 & 0.4 \\ 0.0 & 0.9 \end{bmatrix}, C_3 = I
\end{aligned}$$

where $Q = I$, $R = I$, $P = I$, and I is a unity matrix.

The suboptimal decentralized feedback of this example is carried out through the use of computer program devised for the present algorithm which depends on the minimization of the cost function βJ^* . The results of the computations are tabulated hereafter for the three subsystems stated in Table (1). It is observed from Table (1), with the suboptimal controls laws of the subsystems used in the example, that the algorithm can be applied to a large-scale linear systems. Besides, the step response of the suboptimal closed-loop was determined as shown in Fig. (1). From the four curves, it is clear that the states of the variables (VAR_1 , VAR_2 , VAR_3 , VAR_4) for subsystem₁, subsystem₂ and subsystem₃ reached the steady state in a short time.

TABLE 1. Results of the algorithm applied to the example.

	Subsystem ₁	Subsystem ₂	Subsystem ₃
m_A	$\begin{bmatrix} 1.80 & -3.8 & -2.3 & 0.0 \\ 0.00 & -2.9 & 0.0 & 0.0 \\ 1.21 & 0.0 & -2.3 & 0.0 \\ -1.36 & 0.0 & 0.0 & 0.0 \end{bmatrix}$	$\begin{bmatrix} -7.1 & -2.6 & 7.4 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 \\ -19.2 & 0.0 & 6.6 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 \end{bmatrix}$	$\begin{bmatrix} -1.0 & -4.5 & -1.0 & 0.0 \\ 0.0 & -2.7 & 0.0 & 0.0 \\ 0.9 & 0.0 & -1.1 & 0.0 \\ 2.3 & 0.0 & 0.0 & 0.0 \end{bmatrix}$
$\bar{P}$	$\begin{bmatrix} 10.5 & 8.4 & -4.6 & 0.0 \\ 1.0 & 1.0 & -1.0 & 0.0 \\ -7.9 & 0.0 & -4.9 & 0.0 \\ -2.7 & 0.0 & -1.4 & 1.0 \end{bmatrix}$	$\begin{bmatrix} 13.4 & 12.0 & 1.2 & 0.0 \\ 0.0 & 1.0 & -0.1 & 0.0 \\ 0.6 & 0.0 & 3.9 & 0.0 \\ -0.9 & 0.0 & 1.5 & 1.0 \end{bmatrix}$	$\begin{bmatrix} 12.4 & 11.2 & -1.7 & 0.0 \\ 0.0 & 1.0 & -0.10 & 0.0 \\ -1.5 & 0.0 & 3.60 & 0.0 \\ -6.7 & 0.0 & -1.10 & 0.0 \end{bmatrix}$
F	$\begin{bmatrix} 2.1 & 6.7 & 0.0 & -7.6 \\ -0.5 & 4.3 & 0.0 & -5.1 \end{bmatrix}$	$\begin{bmatrix} 12.5 & 9.6 & 0.0 & 4.3 \\ 11.4 & 5.7 & 0.0 & 4.1 \end{bmatrix}$	$\begin{bmatrix} 8.7 & 8.9 & 0.0 & 1.6 \\ -1.8 & 5.4 & 0.0 & 0.3 \end{bmatrix}$
${}^m_{A+BFC}$	$\begin{bmatrix} -5.1 & -1.0 & 2.6 & 0.1 \\ 2.7 & -3.2 & 1.4 & 0.0 \\ 0.0 & 0.0 & -6.0 & 0.0 \\ 4.9 & 0.0 & 5.5 & -5.9 \end{bmatrix}$	$\begin{bmatrix} -6.60 & 0.0 & -0.5 & -0.1 \\ 1.00 & -6.0 & -1.5 & 0.1 \\ 0.00 & 0.0 & -6.0 & 0.0 \\ -0.12 & 0.0 & -2.9 & -5.7 \end{bmatrix}$	$\begin{bmatrix} -5.9 & 0.0 & -0.1 & 0.0 \\ 6.8 & -6.0 & 1.1 & 0.1 \\ 0.0 & 0.0 & -6.0 & 0.0 \\ 1.2 & 0.0 & -3.0 & -5.7 \end{bmatrix}$
α	1.000	1.000	1.000
β	1.500	1.500	1.500
* J	34.519	32.632	17.022

Conclusions

An output feedback controller of suboptimality degree β is applied to a large-scale linear system to solve the suboptimal decentralized control problem. The algorithm is based on a standard Riccati equation, and is applicable when certain conditions are satisfied to form a suboptimal decentralized output feedback control gain. The developed suboptimal decentralized controller is easy to implement, simple to iterate (low number of iterations), requires insignificant computation time and does not require large memory. This has been illustrated by an example.

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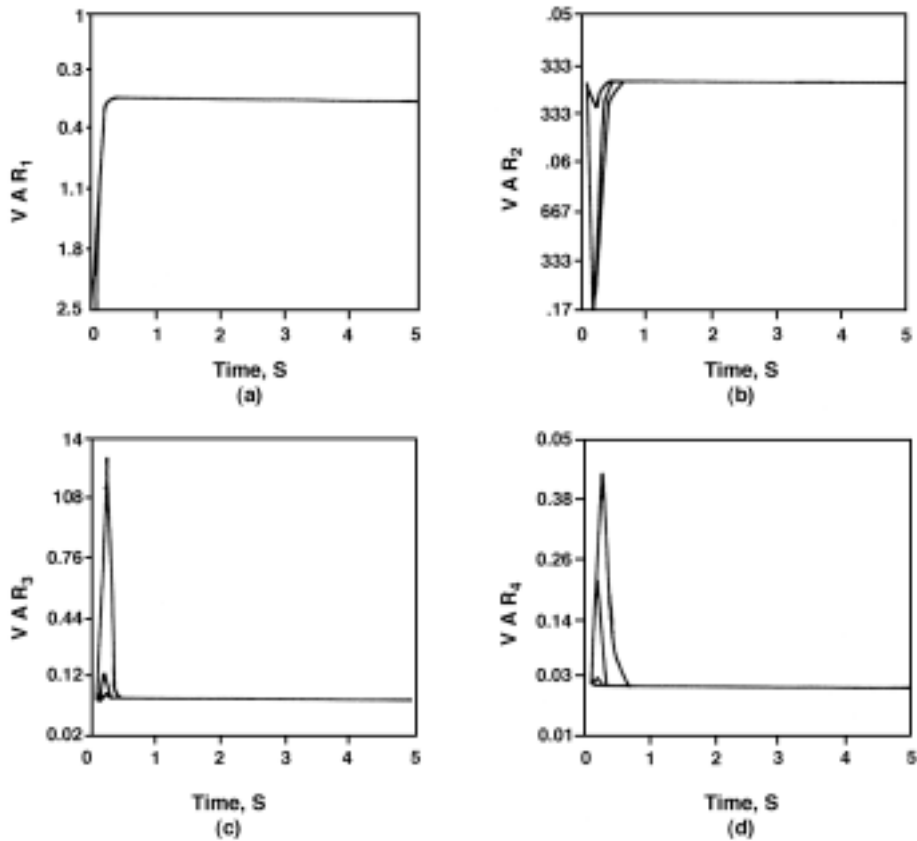


FIG. 1. Suboptimal responses of (a) VAR_1 , (b) VAR_2 , (c) VAR_3 and (d) VAR_4 for the three subsystems.

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حاكم لامركزي شبه مثالي للأنظمة الخطية متعددة المتغيرات

اسماعيل الشحات

هيئة المواد النووية - المعادي - القاهرة - جمهورية مصر العربية

المستخلص . يبرز هذا البحث تصميمًا لحاكم لامركزي شبه مثالي في مجال معين للأنظمة الخطية متعددة المتغيرات . ينبنى هذا الحاكم على أساس مضاعف عددي ذي دالة تكلفة (BJ^*) بهدف الحصول على أنسب تكلفة مثلي للنظام . تم استخدام نظام ذي نطاق واسع يتألف من عدد من الأنظمة الفرعية . تم احراز قاعدة حاكمة شبه مثالية للنظام الواسع ككل من القواعد شبه المثالية للأنظمة الفرعية . حيث إنه يمكن صياغة قواعد ضابطة للأنظمة شبه المثالية عن طريق حل معادلات ريكاتي ذات المصفوفات من الرتب الدنيا ، فإنه يمكن الحصول على وفر كبير في السعة ووقت الحاسب الآلي . تم كذلك توفير ظروف كافية لتكلفة شبه مثالية ، كما تم تبين سلوك الخوارزمية وفقًا لمثال محاكي .