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Ephemerides for Visual Binaries of Quasi-Parabolic Orbits

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Abstract. In this paper, two unified algorithms are developed for the ephemerides of visual binaries of quasi-parabolic orbits for either the elliptic or hyperbolic case. The numerical applications proved the efficiency of the developed algorithms.

Introduction

The study of visual binaries is of great importance as one of the most essential sources of our present knowledge of stellar masses. Moreover, the use of these masses leads to the discovery of the mass luminosity relationship which in turn becomes weighty support of many theories of stellar evolution. The determination of visual binary orbits is the problem of computing orbital elements of a binary at a given epoch from a set of observed positions. The inverse problem is the computation of the position (θ°, ρ'') at a given epoch from a set of orbital elements, where θ is the apparent angle in degrees and ρ is the angular separation in arc seconds. What concerns us in the present paper is the computation of the ephemerides. In general this type of computation plays an important role in the orbit determination of visual binaries. Because, when a set of elements is known, (θ, ρ) at the observing times t are recalculated by the ephemerides formulae, and the residuals observed-calculated (O-C) can be found. They should be sufficiently small and mostly randomly distributed for an acceptable orbit. Although most of the known orbits correspond to the elliptic case, in theory, there is no restriction to the fact that the orbit may be parabolic or hyperbolic. The ephemerides formulae for these cases are exactly the same formulae which relate position and time in the corresponding conic section of the two bodies motion, according to the celestial mechanics[1] together with the well known formulae

$$\tan(\theta - \Omega) = \tan(f + \omega)\cos i, \tag{1.1}$$

and

$$\mathrm{r} = r\cos(f+\omega)\sec(\theta+\Omega), \tag{1.2}$$

where Ω, ω, i, f, and r have their usual meaning for orbits. Equations (1.1) and (1.2) convert the values of f and r of the companion in the true orbit into ∂ and ρ. In the orbit determination of visual binaries, provisional quasi-parabolic orbits are used to represent the periastron section of a high eccentricity orbit of long and indeterminate period[2]. In other words, the different types of conic motion not only exist naturally, but can also be used to solve some critical orbital situations of visual binary systems. The serious problem of the quasi-parabolic orbits, for both elliptic and hyperbolic cases is due to the indeterminacy of Kepler's equation as the eccentricity e tends to unity. On the other hand, as the semimajor axis a increases, both the mean anomaly and the eccentric anomaly become vanishingly small but of course of definite values.

Consequently, the conventional series solution for these critical orbits leads to divergent or at best weak convergent series. Motion predictions of these very critical orbits can not therefore be treated by the conventional methods of orbit determination and need special devices. The above mentioned importance of computation of the ephemerides and critical situation of quasi-parabolic orbits are what motivated our work to develop the two unified algorithms for computing visual binaries ephemerides of quasi-parabolic orbits for both elliptic and hyperbolic cases. We present the applications of the algorithms to the binaries ADS 13103 and ADS 11632.

Formulations

In both algorithms (referred to in Section 3), the ephemerides (θ, ρ) of the quasi-parabolic orbits of visual binaries are obtained by two steps: (1) computation of the radial distance r and the true anomaly f at any time t, then, (2) the solution of Equations (1.1) and (1.2) for the ephemerides (θ, ρ) at time t. In what follows, the basic equations of (r, f) are formulated for each algorithm, while their utilization on digital computers for (θ, ρ) will be given in Section 3.

Unified Gauss Method

The first algorithm uses unified Gauss method for quasi-parabolic orbits recently developed by Sharaf *et al.*[3] (hereafter referred to as Paper I0. What concerns us in the present paper among the formulations of Paper I are the following

$$\tan^2 \frac{1}{2} f = \left\{ \frac{5+5e}{1+9e} \times \frac{W^2}{1 \mp \frac{4}{5} A + C} \right\} , \tag{2.1}$$

$$r = q \frac{1 \mp 0.8 \quad A + C}{1 \pm 0.2 \quad A + C} \left(1 + \tan^2 \frac{1}{2} f \right) , \tag{2.2}$$

where the pericenter distance q is

$$q = \pm a(1-e) \ , \tag{2.3}$$

$$A = \pm \frac{5(1-e)}{1+9e} W^2 \ . \tag{2.4}$$

W is the solution of the cubic equation

$$W^3 + 3W = \frac{3}{B} \left(\frac{\mu}{q^3} \times \frac{1+9e}{20} \right)^{1/2} (t - \tau) \ , \tag{2.5}$$

τ, μ are respectively, the time of periastron passage and the gravitational constant. With the units usually used in visual binaries computation, μ is given as

$$\mu = 4\pi^2 \, (m_1 + m_2) \, \pi''^3 \quad , \tag{2.6}$$

where π'' is the parallax in arcseconds and $m_{1,2}$ the masses of the components of the visual binary system in solar mass units. The constants B and C are given in Paper I as power series in A developed up to the twentieth power. In the present paper, these constants are given up to the fourth power, which is very sufficient for ephemerides calculations. These are

$$B = 1 + \frac{3}{175} A^2 \ \frac{2}{525} A^3 + \frac{471}{336875} A^4 \ , \tag{2.7}$$

$$C = \frac{8}{175} A^2 \pm \frac{8}{525} A^3 \ \frac{1896}{336875} A^4 \quad . \tag{2.8}$$

Finally, the upper sign is used for elliptic orbits, while the lower sign for hyperbolic orbits.

Method of Successive Approximations

The second algorithm uses a successive method for quasi-parabolic orbits developed as follows.

Let

$$\lambda = \frac{1-e}{1+3} \quad . \tag{2.9}$$

The relation between the radial distance r and the orbital parameter p is given for both types of orbits as

$$r = \frac{p}{1+e\ \cos\ f} \ , \tag{2.10}$$

which could be written as

$$r = q\frac{1+\psi^2}{1+\lambda\psi^2} \ , \tag{2.11}$$

where

$$\psi = \tan\frac{1}{2}\ \ f \ . \tag{2.12}$$

We seek a solution of Kepler's equation as a power series in λ,

$$\psi = \sum_{j=1}^{\infty} a_j\lambda^j \quad . \tag{2.13}$$

Now, according to the law of areas, we have

$$\frac{\sqrt{\mu\ \ p}}{2q^2}dt = \frac{1+\psi^2}{(1+\lambda\psi^2)^2}d\omega \ . \tag{2.14}$$

Expanding the right-hand side by polynomial division to produce a power series in λ and integrating term by term, yields

$$\frac{\sqrt{\mu\ \ p}}{2q^2}(t-\tau) = \sum_{j=0}^{\infty}(-1)^j(j+1)\left\{\frac{\psi^{2j+1}}{2j+1}+\frac{\psi^{2j+3}}{2j+3}\right\}\lambda^j \quad . \tag{2.15}$$

Finally, we substitute for ψ from Equation (2.13) and equate coefficients of corresponding powers of λ. The *zero*th-order term a_o is the one and only real root of

$$a_o^3 + 3a_o = \frac{3\sqrt{\mu\ \ \rho}}{2q^2}(t-\tau) \tag{2.16}$$

The higher-order terms can be obtained up to any power; again for ephemerides calculations it seems that the following coefficients is sufficient:

$$
\begin{aligned}
a_1 &= (10a_o^3 + 6a_o^5)/15(1+a_o^2) \\
a_2 &= -(21a_o^5 + 15a_o^7 - 70a_o^2 a_1 - 70a_o^4 a_1 + 35a_o a_1^2)/35(1+a_o^2) \\
a_3 &= (36a_o^7 + 28a_o^9 - 189a_o^4 a_1 - 189a_o^6 a_1 + 126a_o a_1^2 + 252a_o^3 a_1^2 - 21a_1^3 \\
&\quad +126a_o^2 a_2 + 126a_o^4 a_2 - 126a_o a_1 a_2)/63(1+a_o^2)
\end{aligned} \tag{2.17}
$$

It should be noted that Equations (2.9) to (2.17) could be applied to both elliptic and hyperbolic orbits and with λ positive in the former case and negative in the latter, so that the second algorithm is also unified.

Computational Developments

Solution of Cubic Equation

The cubic equation encountered in both algorithms (Equations (2.5), (2.6)) is of the form

$$\mathrm{X}^3 + 3\mathrm{X} = 2S, \tag{3.1}$$

where S is non-negative, so according to Descartes' rule of changing signs, there is only one real solution of Equation (3.1). This solution is easily obtained analytically as

$$\mathrm{X} = [S + (S^2+1)^{1/2}]^{1/3} - [S + (S^2+1)^{1/2}]^{-1/3} \quad . \tag{3.2}$$

In what follows, the implementations of the formulations of Section 2 for digital computers will be given through the following two computational algorithms, each described by its purpose, input and its computational sequence.

Computational Algorithm 1

● *Purpose:* To compute (θ, ρ) or a visual binary system of quasi-parabolic orbit for both elliptic and hyperbolic cases at time t by using the unified Gauss method of Subsection 2.1.

● *Input:* q'', i, Ω, ω, μ, t, τ, e, Tol (specified tolerance).

● *Computational Sequence:*

(1) Set B = 1
(2) Solve the cubic Equation (2.5) for W by the method given in Subsection 3.1.

(3) Compute A from Equation (2.4).

(4) Calculate new value of B from power series (2.7).

(5) Repeat step 2 to 4 until A ceases to change within the specified Tolerance Tol.

(6) With this value of A calculate C from power series (2.8).

(7) Calculate f from Equation (2.1).

(8) Calculate r from Equation (2.2).

(9) Calculate Q from

$$Q = \tan^{-1}\left\{\frac{\sin(f+\omega)\ \cos i}{\cos(f+\omega)}\right\}$$

(10) Calculate ∂ from

$$\theta = (\Omega + Q) \times 180^{\circ} / \pi$$

(11) Calculate ρ from

$$\rho = \frac{r\ \cos(f+\omega)}{\cos Q}$$

(12) The algorithm is completed.

Computational Algorithm 2

● *Purpose:* To compute (θ, ρ) for a visual binary system of quasi-parabolic orbit for both elliptic and hyperbolic cases at time t by using the method of successive approximations of Subsection 2.2.

● *Input:* q'', i, Ω, ω, μ, t, τ, e.

● *Computational Sequence:*

(1) Calculate λ from Equation (2.9).

(2) Compute p from $p = q\,(1 + e)$.

(3) Solve the cubic Equation (2.16) for a_o, y the method given in Subsection 3.1.

(4) Calculate a_1, a_2 and a_3 from Equations (2.17).

(5) Compute ψ from $\psi = a_o + a_1\lambda + a_2\lambda^2 + a_3\lambda^3$.

(6) Compute r from Equation (2.11).

(7) Compute f from Equation (2.12).

(8) Compute Q from

$$Q = \tan^{-1}\left\{\frac{\sin(f+\omega)\ \cos i}{\cos(f+\omega)}\right\}.$$

(9) Compute θ from

$$\theta = (\Omega + Q) \times 180^{\circ} / \rho.$$

(10) Calculate ρ from

$$\rho = \frac{r \cos(f + \omega)}{\cos Q} \quad .$$

(11) The algorithm is completed.

Numerical Applications

The above computational algorithms are applied to obtain the ephemerides for visual binaries Σ 2597 = ADS 13104, Heints[4] and Σ 2398 = ADS 11632, Knudsen[2] of elliptic and hyperbolic orbits respectively. Ten epochs are considered for each binary and are selected between the years 1994.0 to 2006.0 for the elliptic orbit, and between the years 1945.0 to 1990.0 for the hyperbolic orbit. The input data for the binaries taken from their respective references are listed in Table 1. The ephemerides of each binary are listed in Tables 2 and 3 for the two algorithms. The adapted constants are taken as Tol. = 10^{-10} and μs computed for each binary from Equation (2.6). The accuracy of the computations are checked for both orbits by the two bodies condition,

$$r - q(1 + e)\,(1 + e \cos f)^{-1} = 0$$

TABLE 1. Elements.

ADS	Name	$\tau(y)$	q''	e	i''	ω°	Ω°	π''	$m_1 + m_2$
13104	Σ 2579	1972.50	0.0698	0.936	101.5	142	82.5	0.015	2.68 M
11632	Σ2398	1871.53	16.547	1.043	76.74	345.6	145.91	0.286	0.696

TABLE 2. Ephemerides from algorithm 1.

ADS 13104			ADS 11632		
t	θ°	ρ°	t	θ°	ρ°
1994.0	107.127	0.334	1945.0	158.554	16.074
1995.0	106.451	0.351	1950.0	159.758	15.830
1996.0	105.839	0.365	1955.0	161.000	15.571
1997.0	105.280	0.354	1960.0	162.285	15.299
1998.0	104.768	0.402	1965.0	163.618	15.017
1999.0	104.296	0.418	1970.0	165.002	14.727
2000.0	103.860	0.435	1975.0	166.442	14.431
2002.0	103.077	0.467	1980.0	167.944	14.132
2004.0	102.393	0.498	1985.0	169.510	13.830
2006.0	101.789	0.528	1990.0	171.146	13.528

TABLE 3. Ephemerides from algorithm 2.

ADS 13104			ADS 11632		
t	θ^o	ρ^o	t	θ^o	ρ^o
1994.0	107.171	0.333	1945.0	158.550	16.075
1995.0	106.499	0.350	1950.0	159.753	15.831
1996.0	105.890	0.367	1955.0	160.996	15.571
1997.0	105.335	0.388	1960.0	162.281	15.300
1998.0	104.826	0.400	1965.0	163.615	15.017
1999.0	104.358	0.416	1970.0	165.000	14.727
2000.0	103.26	0.432	1975.0	166.440	14.432
2002.0	103.151	0.463	1980.0	167.942	14.132
2004.0	102.476	0.494	1985.0	169.510	13.830
2006.0	101.881	0.523	1990.0	171.147	13.528

and was found to be at least of order 10^{-7} for each orbit.

Conclusion

In concluding this paper, two unified algorithms are developed for ephemerides of visual binaries of quasi-parabolic orbits for either elliptic or hyperbolic cases. The first algorithm uses unified Gauss method, while the second uses successive approximations method. The numerical applications proved the efficiency of the developed algorithms.

References

[1] **Danby, J.M.A.,** *Fundamentals of Celestial Mechanics,* 2nd ed, William-Bell, Inc., Richmond, Virginia, USA (1988).
[2] **Knudsen, N.W.,** *Pub. Lund. Obs.* (12) (1953).
[3] **Sharaf, M.A., Saad, A.S.** and **Sharaf, A.A.,** *Celestial Mechanics and Dynamical Astronomy,* **70:** 201 (1998).
[4] **Heints, W.D.,** *Astron. J.,* **111**(1): 412 (1996).

حساب تقاويم المزدوجات البصرية ذات المدارات شبه المكافئة

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المستخلص . تم في هذا البحث تكوين خوارزمتين موحدتين ، وذلك لحساب تقاويم المزدوجات البصرية ذات المدارات شبه المكافئة بحيث يصلح كل منهما في الحالات الإهليلجية والزائدية . وقد أثبتت التطبيقات الحسابية فاعلية هاتين الخوارزمتين .