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On the Solar Motion from Radial Velocities

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ABSTRACT. In this paper, the error controlled method to determine the solar motion from radial velocities of a previous study is summarized. An application of the method for the stars of the fifth fundamental catalog with certain selection and solution criteria gives very accurate results using about 6% only of the total number of stars. A result which confirms that, by using controlling criteria one can construct an optimum economical computational algorithm that minimize both the storage and the execution time with a unique and accurate solar motion elements as output.

Introduction

The determination of the solar motion with respect to stellar groups plays an essential role in the stellar kinematics. Also, the determination of the solar motion provides important parameters in the correlation studies with the physical properties of the stellar group which it refers to[1]. The solar motion has been determined by numerous authors with respect to the great range of stellar and interstellar constituents of the Galaxy, and there are extensive literature on the subject, *e.g.*[2-7]. The values vary however considerably from one author to another.

Today observational astronomy has been characterized by an enormous growth in data acquisition stimulated by the advent of new technologies in telescopes and detectors. The more volume of astronomical data collection needed for the solar motion determination is in fact impressive. It is now possible to compile catalogs of 10^5 or even more stars. To design an optimum economical algorithm of the solar motion it should be directed to minimize the two major computational problems which are, the storage and execution time. For such algorithm, selection and solution criteria are to be formulated so as to reduce the database to small number of stars which at the same time gives a unique and accurate solar motion elements. Recently[8] established an analytical error controlled method to determine the solar motion from radial velocities.

By this method one can define an acceptable solution set which in turn provides a unique accurate set of the solar motion elements (with respect to a given group of objects). In the present paper, the above method is summarized in the next section, and applied to the stars of the fifth fundamental catalog[9]. The adopted selection and solution criteria [Equations (3.1) and (3.2)] permits us to use about 6% only of the total number of stars and obtain very accurate solar motion elements with respect to these stars. A result which confirms with the other experimentations of [1], that the quality of the data controlled by the selection and solution criteria is the fundamental tool (not the very large data set) to secure *an optimum economical computational algorithm that minimize both the storage and the execution time with a unique and accurate solar motion elements as output.*

Basic Equations

Elements of the Solar Motion

Let X, Y, and Z to be components of the star's space velocity V along the x, y, z axes of a coordinate system whose center is the Sun and such that the x-axis points towards the point ($\alpha = 0^h$, $\delta = 0^o$), the y-axis is oriented towards the point ($\alpha = 6^h$, $\delta = 0^o$), and the z-axis towards the north celestial pole at a definite epoch. Let α, δ, and ρ be respectively the star's right ascension, declination and radial velocity, then we have[10]

$$\rho = X \cos\delta \cos\alpha + Y \cos\delta \sin\alpha + Z \sin\delta, \qquad (1)$$

where all the velocity components are in km/sec.

For a group of N stars, Equation (1) can be considered as the condition equation for the least-squares solution, X, Y, and Z. Having obtained (X, Y, Z) then the components of the Sun's velocity with respect to the same group and referred to the same axes as X, Y, and Z are given as

$$X_s = -X; \; Y_s = -Y; \; Z_s = -Z \qquad (2)$$

The elements of the solar motion with respect to the group are (α_A, δ_A, S), where α_A and δ_A are the right ascension and declination of the solar apex. Finally, S is the absolute value of the Sun's velocity relative to the group.

The relations between the components of the solar velocity and the elements are given as

$$\alpha_A = \tan^{-1}\left\{\frac{Y_s}{X_s}\right\} , \tag{3}$$

$$\alpha_A = \tan^{-1}\left(\frac{Z_s}{(X_s^2 + Y_s^2)^{1/2}}\right) , \tag{4}$$

$$S = \left(X_s^2 + Y_s^2 + Z_s^2\right)^{1/2} . \tag{5}$$

Determination of the Solar Motion and Its Error Estimates

According to[1], the determination of the solar motion and its error estimates are summarized as follows. Written Equation (1) as

$$y = C_1 \; \Phi_1 + C_2 \; \Phi_2 + C_3 \; \Phi_3 , \tag{6}$$

where

$$y = \rho, \quad C_1 X = -X_s, \quad C_2 = Y = -Y_s, \quad C_3 = Z = -Z_s$$

and

$$\Phi_1 \equiv \Phi_1 \; (\alpha, \delta) = \cos\delta \; \cos\alpha \; ,$$

$$\Phi_2 \equiv \Phi_2 \; (\alpha, \delta) = \cos\delta \; \cos\alpha \; ,$$

$$\Phi_3 \equiv \Phi_3 \; (\alpha, \delta) = \cos\delta \; \cos\delta \; .$$

For N observational data, let $(\alpha_n , \delta_n , y_n)$; $n = 1,2,3 \ldots , N$ be known, then the exact solutions of Equation (6) in the sense of the least-squares criterion are

$$C_1 = (AW_7 + HW_8 + GW_9) / \Delta , \tag{7}$$

$$C_2 = (HW_7 + BW_8 + FW_9) / \Delta , \tag{8}$$

$$C_3 = (GW_7 + FW_8 + DW_9) / \Delta , \tag{9}$$

where

$$A = W_3 \; W_6 - W_5^2; \quad B = W_1 W_6 - W_4^2 , \tag{10}$$

$$D = W_1 \; W_3 - W_2^2; \quad H = W_4 W_5 - W_6 W_2 , \tag{11}$$

$$G = W_2 \; W_5 - W_3 W_4 ; \; F = W_2 W_4 - W_1 W_5 , \tag{12}$$

$$\Delta = AW_1 + HW_2 + G W_4 , \tag{13}$$

$$
\begin{aligned}
W_1 &= \sum_{j=1}^{N} \Phi_{1j}^2; & W_2 &= \sum_{j=1}^{N} \Phi_{1j}\Phi_{2j}; & W_3 &= \sum_{j=1}^{N} \Phi_{2j}^2, \\
W_4 &= \sum_{j=1}^{N} \Phi_{1j}\Phi_{3j}; & W_5 &= \sum_{j=1}^{N} \Phi_{2j}\Phi_{3j}; & W_6 &= \sum_{j=1}^{N} \Phi_{3j}^2, \\
W_7 &= \sum_{j=1}^{N} y_j\Phi_{1j}; & W_8 &= \sum_{j=1}^{N} y_j\Phi_{2j}; & W_9 &= \sum_{j=1}^{N} y_j\Phi_{3j}, \\
W_{10} &= \sum_{j=1}^{N} y_j^2
\end{aligned}
\tag{14}
$$

and $\Phi_{ky} = \Phi_k\ (\alpha_j, \delta_j)$.

While the error estimates of the method are given as:

$$
\begin{aligned}
\sigma = [\frac{-1}{N-3}\{ & W_{10} - C_1^2 W_1 - C_2^2 W_3 - C_3^2 W_6 - 2C_1C_2W_2 - \\
& 2C_1C_3W_4 - 2C_2C_3W_5\}]^{1/2} \quad ; \quad e = 0.6745\ \sigma,
\end{aligned}
\tag{15}
$$

$$\sigma_{c_1} = s\sqrt{A/\Delta} \quad ; \quad e_{c_1} = 0.6745\sigma c_1 \ , \tag{16}$$

$$\sigma_{c_2} = s\sqrt{B/\Delta} \quad ; \quad e_{c_2} = 0.6745\sigma c_2 \ , \tag{17}$$

$$\sigma_{c_3} = s\sqrt{D/\Delta} \quad ; \quad e_{c_3} = 0.6745\sigma c_3 \ , \tag{18}$$

$$Q = \sigma^2 \left(\frac{W_3W_6 + W_1W_6 + W_1W_3 - W_5^2 - W_2^2 - W_4^2}{W_1W_3W_6 + 2W_2W_4W_5 - W_5^2W_1 - W_4^2W_3 - W_2^2W_6} \right) . \tag{19}$$

$$
\begin{aligned}
\sigma_{\alpha_A} &= \frac{\sigma}{C_1^2 + C_2^2} [\frac{1}{\Delta} \{AC_2^2 + BC_1^2 - 2HC_1C_2\}]^{1/2} \ , \\
e_{\alpha_A} &= 0.6745\sigma_{\alpha_A}
\end{aligned}
\tag{20}
$$

$$
\begin{aligned}
\sigma_{\delta_A} = \frac{\sigma}{S^2} \{ (& \frac{C_3^2}{(C_1^2 + C_2^2)\Delta} [C_1^2 A + C_2^2 B] + \frac{D}{\Delta}(C_1^2 + C_2^2) - \\
& \frac{2}{\Delta} [FC_2C_3 + GC_1C_3 - HC_1C_2C_3^2 / (C_1^2 + C_2^2)] \}^{1/2} \ ,
\end{aligned}
$$

$$e_{\delta_A} = 0.6745\sigma_{\delta_A} \tag{21}$$

$$\sigma_s = \frac{\sigma}{S}\{\frac{1}{\Delta}[C_1^2 A + C_2^2 B + C_3^2 D + 2FC_2C_3 + 2GC_3C_1 + 2HC_1C_2]\}^{1/2}$$

$$e_{\delta_s} = 0.6745\sigma_{\delta_s} \ , \tag{22}$$

where, σ the standard error of the fit, σ_{C_1}, σ_{C_2}, σ_{C_3}, the standard errors for C's coefficients, while the σ_{α_A}, σ_{δ_A}, σ_s, are the corresponding errors for α_A, δ_A, and S respectively. The error criterion Q is the average squared distance between the least-squares estimators and their true values[11]. Finally, e stands for the probable error.

Numerical Application

The sample was collected from the fifth fundamental catalog[9]. The total number of stars is 3117, while the total number of stars with complete data (α, δ, ρ_o) are 1396. The equatorial coordinates referred to the standard equinox of J2000.0.

The selection criterion is taken as

$$S.C. = |\rho_o - \rho| \leq 1 \ , \tag{23}$$

where ρ_o and ρ are respectively the observed and calculated radial velocities, while the solution criteria are taken as

$$\begin{gathered}
\left|\frac{\delta X_S}{X_S}\right| < 0.15 \ , \quad \left|\frac{\delta Y_S}{Y_S}\right| < 0.15 \ , \quad \left|\frac{\delta Z_S}{Z_S}\right| < 0.15 \ , \\
\left|\frac{\delta\alpha_A}{\alpha_A}\right| < 0.007 \ , \quad \left|\frac{\delta\alpha_A}{\delta_A}\right| < 0.007 \ , \quad \left|\frac{\delta S_A}{S_A}\right| < 0.2 \ , \\
\sigma < 0.60 \ , \qquad Q < 0.06
\end{gathered} \tag{24}$$

As a result of these criteria, the total number of 1396 stars reduced to the 76 stars listed in Table 1, while the solar motion elements together with the relative error criteria are listed in Table 2. The dependence of the solar motion elements on the criteria are shown in Table 3, also the error bars of Fig. 1 for the speed of the solar motion is given as a typical example of this dependence.

TABLE 1. Data of the selected stars.

α (degrees)	δ (degrees)	ρ_o (km/sec)	ρ (km/sec)
5.018	–63.125	8.7	9.103
10.838	–56.537	10.0	9.696
27.865	60.717	–6.8	–6.611
34.351	–9.665	9.0	9.774
39.704	19.901	6.0	6.277
42.674	21.961	8.0	7.141
47.042	55.896	–1.0	–0.127
53.233	40.956	4.0	3.791
56.871	–8.542	15.4	16.184
62.165	24.105	10.1	10.286
72.210	47.713	3.0	3.946
76.269	75.941	–6.0	–5.094
79.402	41.234	7.4	7.474
79.371	–5.156	20.1	19.537
81.283	–33.105	21.2	21.027
86.739	6.350	18.2	17.671
86.939	13.178	20.0	20.859
96.225	–8.330	20.6	20.356
119.195	–51.018	19.1	18.399
129.411	–41.011	18.7	18.008
149.216	–53.432	14.1	14.128
166.254	7.336	4.7	4.075
167.416	44.499	–3.800	–3.288
177.265	14.572	–0.100	–0.773
187.430	20.896	–5.600	–5.042
196.507	55.960	–9.300	–10.159
200.149	–35.288	0.100	0.874
217.957	30.371	–13.700	–14.575
222.720	–15.958	–10.000	–9.264
248.363	–77.103	5.400	5.605
262.608	52.301	–20.000	–19.317
264.866	46.006	–20.000	–20.269
273.912	42.159	–20.500	–20.809
276.043	–33.615	–11.000	–10.576
279.055	65.489	–16.00	–16.815
284.736	32.690	–21.500	–21.092
292.426	51.730	–19.500	–18.688
311.010	–50.079	–1.600	–2.159
317.399	–10.628	–11.800	–11.578
318.941	–52.737	0.000	–0.242
328.482	–36.635	–2.100	–2.795
338.839	0.117	–8.000	–7.725
340.667	–45.115	1.600	1.884
342.420	66.200	–12.400	–11.598
351.733	1.256	–3.200	–3.792
359.668	–2.444	–0.200	–0.519
251.492	82.037	–11.400	–12.163

TABLE 1. Contd.

α (degrees)	δ (degrees)	ρ_o (km/sec)	ρ (km/sec)
4.659	31.517	– 5.300	– 4.478
14.459	28.992	– 0.500	– 1.238
19.450	3.614	5.300	4.993
28.592	– 41.503	12.000	12.818
30.575	54.488	– 2.000	– 2.688
45.733	– 45.025	17.000	16.229
52.654	– 4.925	15.000	15.559
60.383	– 0.450	16.000	16.372
79.894	– 12.823	20.200	20.471
105.430	– 12.065	21.500	21.128
130.073	– 51.078	17.100	17.172
154.509	65.108	– 6.000	– 5.068
169.783	38.186	– 3.000	– 2.723
187.094	– 38.959	5.000	4.934
209.955	– 2.450	– 8.200	– 8.454
242.243	36.491	– 18.200	– 19.171
266.890	– 26.169	– 13.500	– 12.870
281.679	52.988	– 20.000	– 19.229
289.917	– 34.579	– 10.000	– 9.560
307.349	30.369	– 18.400	– 18.550
307.335	36.455	– 18.000	– 18.495
309.631	21.201	– 18.400	– 17.820
316.487	– 16.767	– 10.900	– 10. 421
350.222	38.182	– 8.700	– 9.243
355.102	44.334	– 9.000	– 8.589
358.155	10.947	– 3.000	– 3272
359.440	25.141	– 4.200	– 5.068
276.038	83.175	– 11.200	– 11.982

TABLE 2. Optimum values of the solar elements with their relative errors.

$X_s = -0.824$ (km/sec) $\qquad \dfrac{\delta X_s}{X_s} = \pm\ 0.118$

$Y_s = 19.213$ (km/sec) $\qquad \dfrac{\delta Y_s}{Y_s} = \pm\ 0.124$

$Z_s = -9.77$ (km/sec) $\qquad \dfrac{\delta X_s}{X_s} = \pm\ 0.113$

$\alpha_A = 272^{\circ}.46 \qquad \dfrac{\delta\alpha_A}{\alpha_A} = \pm\ 0.0061$

$\delta_A = 26^{\circ}.93 \qquad \dfrac{\delta\delta_A}{\delta_A} = \pm\ 0.0055$

$S = 21.571$ (km/sec) $\qquad \dfrac{\delta S}{S} = \pm\ 0.1184$

$\sigma = 0.596,$

$Q = 0.0421$

TABLE 3. Determination of the solar motion elements and their relative errors using different number of stars.

No.	X_s km/sec	δX_s km/sec	$\delta X_s/X$	Y_s km/sec	δY_s km/sec	$\delta Y_s/Y$	Z_s km/sec	δZ_s km/sec	$\delta Z_s/Z$
77	– 0.82	0.12	0.143	19.21	0.12	0.006	– 9.77	0.11	0.012
153	– 0.65	0.16	0.247	19.33	0.17	0.009	– 9.50	0.16	0.017
232	– 0.49	0.21	0.418	19.00	0.20	0.011	– 9.72	0.19	0.020
318	– 0.39	0.26	0.651	19.05	0.23	0.012	– 9.70	0.22	0.023
451	– 0.77	0.28	0.365	18.99	0.29	0.015	– 9.66	0.27	0.028
652	– 1.13	0.34	0.304	19.28	0.35	0.018	– 9.80	0.33	0.034
836	– 0.93	0.41	0.441	19.15	0.42	0.022	– 9.50	0.39	0.041
1031	– 0.29	0.50	1.710	19.18	0.52	0.027	– 8.72	0.48	0.055
1172	– 0.46	0.59	1.280	18.75	0.59	0.032	– 7.56	0.56	0.074
1268	– 0.40	0.67	1.690	18.08	0.67	0.037	– 7.94	0.64	0.080
1374	– 0.31	0.84	2.670	17.73	0.83	0.047	– 8.64	0.79	0.092
1395	– 0.51	0.94	1.840	19.12	0.92	0.048	– 9.67	0.88	0.091

TABLE 3. Contd.

No.	α degrees	$\delta\alpha$ degrees	$\delta\alpha/\alpha$ ($*10^5$)	δ degrees	δ_δ degrees	δ_δ/δ ($*10^4$)
77	272.458	0.006	2.2	26.931	0.006	2.2
153	271.900	0.008	2.9	26.100	0.008	3.1
232	271.482	0.011	4.1	27.100	0.009	3.3
318	271.181	0.012	4.4	26.971	0.011	4.1
451	272.318	0.015	5.5	26.941	0.013	4.8
852	273.359	0.018	6.6	26.912	0.016	5.9
831	272.788	0.021	7.7	26.343	0.019	7.2
1031	270.870	0.026	9.6	24.450	0.023	9.4
1172	268.606	0.031	11.5	21.958	0.028	12.7
1268	268.742	0.037	13.8	23.686	0.032	13.5
1374	271.017	0.047	17.3	25.979	0.040	15.4
1395	271.526	0.049	18.0	26.827	0.041	15.3

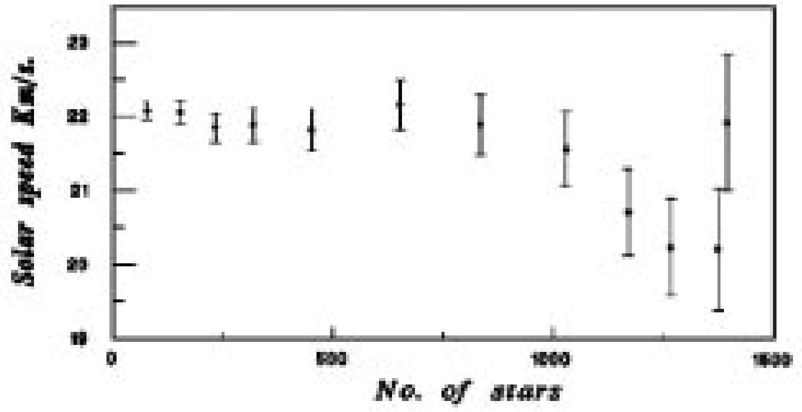

FIG. 1. Error bars for the speed of the solar motion.

From Table 3 and Fig. 1, it is clear that, without using the criteria one may accept some results as elements of the solar motion regardless the number of the stars. But the usage of the criteria has amazing effect on reducing the number of stars that gives accurate elements of the solar motion. A fact which should be considered with the available very large data.

In concluding the present paper, selection and solution criteria enable us to determine very accurately the elements of the solar motion with respect to the 1396 stars of the fifth fundamental catalog using only 76 stars [about 6%]. A result which confirms with the other experimentations of[1], that the usage of these criteria is the fundamental tool that solves the problem of large variation in the values of the solar motion elements with respect of some group. On the other hand, it is also an essential method to secure an optimum economical computational algorithm that minimize both the storage and execution time.

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حركة الشمس من السرعات القطرية للنجوم

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المستخلص . تم في هذا البحث تلخيص طريقة للتحكم في الخطأ لتعيين حركة الشمس من السرعات القطرية للنجوم . طبقت هذه الطريقة على الكتالوج الخامس وفق اختيارات معينة ومعايير محددة للحلول وحصلنا على نتائج دقيقة باستخدام حوالي ٦٪ فقط من العدد الكلي للنجوم . هذه النتيجة تؤكد أنه باستخدام معايير التحكم يمكننا تكوين لوغاريثمات حسابية اقتصادية تقلل من مساحة التخزين وزمن التشغيل مع الحصول على عناصر حركة الشمس بدقة ملائمة .