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New Analytical Expressions for the Intrinsic Color Index

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Abstract. In this paper, new analytical formulae for the interinsic color index are established for two photometric systems.

1. Introduction

One of the enigmatic problems of stellar spectrophotometry in the 1920s was the observation that some stars having O-and B-type spectra have red colors characteristic of much later spectral types. This problem posed the puzzling question "How does a star that appears to have a high atmospheric temperature, as judged by the degree of excitation and ionization of the material, radiate a continuous spectrum that is appropriate to a much cooler temperature? "Once the existence of interstellar absorption was recognized, the mystery evaporated and the answer emerged clearly: It doesn't! It radiates the same continuous spectrum as any other star of its type. But the light we receive is reddened by selective absorption processes in the interstellar medium, which dim the light more efficiently at short wavelengths.

Astronomers now know that the obscuring interstellar material is in the form of small soil specks, dust grains whose chemical composition may be silicates (like sand) or carbon-containing compounds (like graphite or silicon carbide). The obscuration of starlight is believed to arise from a combination of true absorption and scattering, and this combination goes by general name of *extinction* of starlight. Most of the dust grains are slightly smaller than the wave length of the visual light, since, apart from a general extinction of starlight, interstellar dust also produce a *reddening* of starlight. Fig. 1 helps to explain the physical basis of this phenomenon.

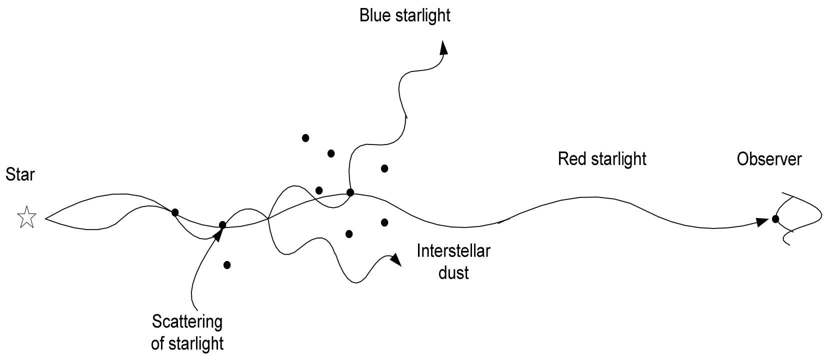


Fig. 1. The mechanism of the interstellar reddening.

Interstellar reddening occurs because blue light is scattered out of the beam of starlight directed toward us more than red light is. Interstellar reddening effects, measured by color excesses, can be determined directly from observation.

The colour index is a simple numerical expression that determines the temperature of a star. To measure the index, one observes the magnitude of an object successively through two different filters, such as U and B, or B and V, where U is sensitive to ultraviolet rays, B is sensitive to blue light, and V is sensitive to visible (green-yellow) light. The difference in magnitudes found with these filters is called the U-B or B-V colour index, respectively. The smaller the colour index, the more blue (or hotter) the object is.

Colour indices of distant objects are usually affected by interstellar extinction *i.e.* they are redder than those of closer stars. The amount of reddening is characterized by colour excess defined as the difference between the observed colour index and the normal colour index (or intrinsic colour index). For example, in the UBV photometric system we can write it for the B-V color :

$$E(B-V) = (B-V) - (B-V)_0$$

where, $(B-V)_0$ is the intrinsic color index and E(B-V) is colour excess. The relation between the color excess and interstellar reddening A is:

$$A = 3E(B-V) .$$

Since (B-V) can be determined directly from observations, it remains to compute the interstellar reddening A, the intrinsic color index $(B-V)_0$. Due to its role, the present paper is devoted to establish new analytical formulae for the interinsic color index for two photometric systems.

2. Linear Model Analysis of Observational Data in the Sense of Least –Squares Criterion

Let y be represented by the general linear model of the form $\sum_{i=1}^{n} c_i\ \varphi_i(x)$ where φ's are linearly independent functions of x. Let **c** be the vector of the exact values of the c's coefficients and $\hat{\mathbf{c}}$ the least-squares estimators of **c** obtained from the solution of the normal equations of the form $\mathbf{G}\hat{\mathbf{c}} = \mathbf{b}$. The coefficient matrix $\mathbf{G}(n \times n)$ is symmetric positive definite, that is, all its eigenvalues λ_i ; $i = 1,2,\cdots,n$ are positive. Let $E(z)$ denotes the expectation of z and σ^2 the variance of the fit, defined as :

$$\sigma^2 = \frac{q_n}{N-n}, \tag{1}$$

where

$$q_n = \left(\mathbf{y} - \Phi^T\hat{\mathbf{c}}\right)^T\left(\mathbf{y} - \Phi^T\hat{\mathbf{c}}\right), \tag{2}$$

N is the number of observations, **y** is the vector with elements y_k and $\Phi(n \times N)$ has elements $\Phi_{ik} = \Phi_i(x_k)$. The transpose of a vector or a matrix is indicated by the superscript "T". According to the least-squares criterion, it could be shown that[1].

1- The estimators $\hat{\mathbf{c}}$ by the method of least-squares gives the minimum of q_n

2- The estimators $\hat{\mathbf{c}}$ of the parameters **c**, obtained by the method of least-squares are unbiased; *i.e.* $E(\hat{\mathbf{c}}) = \mathbf{c}$.

3- The variance –covariance matrix $\mathrm{Var}(\hat{\mathbf{c}})$ of the unbiased estimators $\hat{\mathbf{c}}$ is given by:

$$\mathrm{Var}(\hat{\mathbf{c}}) = \sigma^2\mathbf{G}^{-1}. \tag{3}$$

4- The average squared distance between $\hat{\mathbf{c}}$ and **c** is :

$$E(L^2) = \sigma^2\sum_{i=1}^{n}\frac{1}{\lambda_i}. \tag{4}$$

5- The average squared Euclidean norm of $\hat{\mathbf{c}}$ is:

$$E(\hat{\mathbf{c}}^T\mathbf{c}) = \mathbf{c}^T\mathbf{c} + \sigma^2\sum_{i=1}^{n}\frac{1}{\lambda_i}. \tag{5}$$

Finally it should be noted that if the precision is measured by the probable error e, then

$$e = 0.6745\,\sigma \tag{6}$$

3. Analytical Relation Between M_V Versus $(B-V)_0$

3.1 *Empirical Relation*

An empirical relation between the absolute visual magnitude M_V versus $(B-V)_0$ is given in reference[2] and is listed in Table 1.

Table 1. Empirical relation between M_V versus $(B-V)_0$

$(B-V)_0$	M_V	$(B-V)_0$	M_V
-0.30	-3.50	0.30	2.80
-0.25	-2.30	0.40	3.35
-0.20	-1.30	0.50	4.05
-0.15	-0.50	0.60	4.60
-0.10	-0.30	0.70	5.20
-0.05	0.9	0.80	5.70
0.00	1.30	0.90	6.10
0.05	1.55	1.00	6.60
0.10	1.80	1.10	7.00
0.20	0.25	1.20	7.45

3.2 *Analytical Representation*

We established using linear model analysis of Section 3.1, an analytical relation between $(B-V)_0$ versus M_V as:

$$(B-V)_0 = -0.0974 + 0.0989M_V + 0.01M_V^2 . \tag{7}$$

This relation is extremely accurate as indicated in the error analysis report 1 of Appendix A

4. The Q Values

It is possible to define photometric parameter that depends only on the spectral type of a star independent of the reddening. In the UBV, this parameter[2] is:

$$Q = (U-B) - 0.72\ (B-V) . \tag{8}$$

The Q values for stars of spectral types O through B9 are listed in Table 2. The importance of this parameter is that:1-For early- type stars, it uniquely determines a star's intrinsic color from photometric data only without the need for spectra. 2- Furthermore Q can be measured easily for stars that are much too faint for spectral classification.

Table 2. Q versus spectral class for early-type stars.

Spectral type	Q	Spectral type	Q
O5	-0.93	B0.5	-0.85
O6	-0.93	B1	-0.78
O8	-0.93	B2	-0.70
O9	-0.90	B3	-0.57
B0	-0.90	B5	-0.44
B6	-0.37	B7	-0.32
B8	-0.27	B9	-0.13
A0	0.00		

It could be shown[2] that:

$$(B-V)_0 = 0.332Q \tag{9}$$

For spectral types later then A0, Q is not unique function of spectral class and hence it is no longer useful in estimating the amount of reddening present.

4.1 *Analytical Representation*

In order to find analytical representation of Q versus spectral class for early-type stars, we propose the following two steps:

1- Construct sequence x (say) of positive numbers for the available spectral types, such that: $x = 1$ for stars of spectral type O5, $x = 2$ for stars of spectral type O6, $x = 7.5$ for stars of spectral type B1. 5, and so on. In this respect the sequence x corresponding to the spectral types of Table 1 is:
$x = \{1,2,4,5,6,6.5,7,8,9,11,12,13,14,15,16\}$.

2- With the sequence x as the independent variables and the Q values as the dependent value we established using linear model analysis of Section 1 analytical representation between Q and x, the relation:

$$Q = -0.88762 - 0.05280\,x + 0.0119\,2\,x^2 - 0.00033\,x^3 \tag{10}$$

the error analysis of Equation (10) will be given in report 2 of Appendix A

In concluding the present paper, new accurate analytical formulae for the intrinsic color index are established for two photometric systems. The importance of these formulae is that, they can be used for any input arguments.

References

1- **Kopal, Z.** and **Sharaf, M.A.** (1980) Linear Analysis of the Light Curves of Eclipsing Variables. *Astrophysics and Space Science*, **70**: 77-101.

2- **Mahlas, D.** and **Banney, J.** (1981) *Galactic Astronomy*, Freeman.

Appendix (A)

Report 1: Error Analysis of Representation (7)

1-The solutions and their probable errors

$$C_1 = -0.0974322 \pm 0.00592847$$

$$C_2 = 0.0989122 \pm 0.00246673$$

$$C_3 = 0.0103065 \pm 0.00040501$$

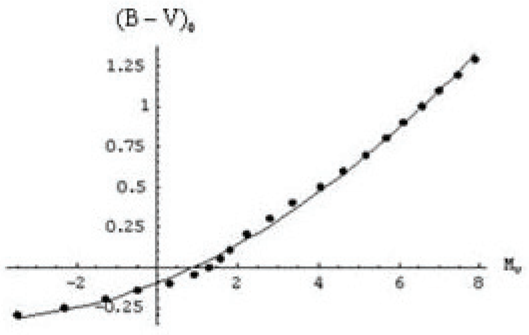
2- The probable error of the fit

e=0.019427

3- The average squared distance between $\hat{c}$ and c

$$E(L^2) = 0.000090982$$

4- Graph of the raw and fitted data



Report 2: Error Analysis of Representation (10)

1-The solutions and their probable errors

$$C_1 = -0.887623 \pm 0.0345456$$

$$C_2 = -0.0527975 \pm 0.0166932$$

$$C_3 = 0.0119198 \pm 0.0022939$$

$$C_4 = -0.000332785 \pm 0.000090497$$

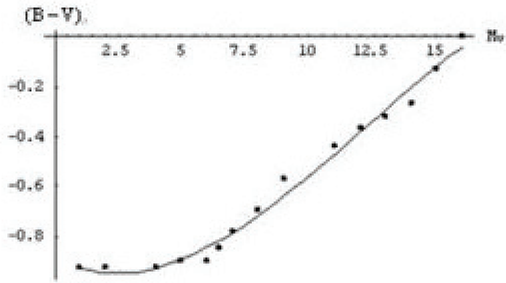
2- The probable error of the fit

e=0.026979

3- The average squared distance between $\hat{c}$ and c

$E(L^2) = 0.00324724$ $Q = 0.00324724$

4- Graph of the raw and fitted data



صيغ تحليلية جديدة لدليل الفائض اللوني

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المستخلص. تم في هذا البحث تشييد صيغ تحليليــة جديــدة لــدليل الفائض اللوني وذلك لنظامين من النظم الفوتومترية.