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Utilization of GERT in Modeling Higher Education System

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ABSTRACT The graphical evaluation and review technique (GERT) has been applied to many industrial and managerial problems. However, its application in the educational field is very limited. In this paper we have demonstrated that GERT can be applied to higher educational system in Saudi Arabia to determine the mean time spent by a student before graduation with a B.Sc. degree, as well as the expected probability of his graduation. The 99 percent confidence interval around the mean number of graduates per year is also derived. A computer program on PC has been developed for this purpose. Data collected from the Faculty of Engineering at King Abdulaziz University are used to illustrate the application of this technique and the developed PC package.

1. Introduction

Many project management techniques are available in the literature. These techniques can be divided into two groups. CPM (critical path method) and PERT (program evaluation and review technique) fall into a group known as deterministic one. The second group known as stochastic group contains GAN (generalised activity network) and GERT (graphical evaluation and review technique).

CPM and PERT methods cannot deal with problems of repetition of activities (such as a student repeating a year, or semester or a course). Elmaghraby[1,2] introduced the algebra of networks consisting of multiparameter branch and decision nodes. This is what he called GAN. His algebra deals with constant time parameter. Later on, Pritsker[3] introduced a new graphical technique, called GERT, to study stochastic problems.

Since the introduction of GERT, many papers, such as Pritsker & Happ[4], Pritsker & Whitehouse[5], Whitehouse and Pritsker[6], and Whitehouse[7], have come out on application of GERT in Industrial Engineering, management, research, development, ...*etc*.

Whitehouse[7], in his book, has discussed many models of educational institutions to estimate the percentage of students graduating and the average time spent by a student in the institution.

Ahmad[8] in a recent paper, used GERT to model the system of a college of engineering, where a student can repeat a whole semester. His model estimates the mean time spent by a student in the college before he graduates, the probability of his graduation is also estimated.

In this paper we show the application of GERT to the present academic system of the Faculty of Engineering at KAU. GERT is used in evaluating the mean time spent by a student in the college before he graduates; the probability of graduation is also calculated. A computer program in BASIC has also been developed to derive the two characteristics as mentioned above. A brief description of basic network algebra, for EXCLUSIVE-OR node, is given by Ahmad[8] to facilitate the understanding of the derivation.

2. Basic Networks (EXCLUSIVE-OR nodes) Algebra

2.1 Series System

Figure 1 represents a network showing two activities a and b in series. $W_i(s)$ is the transmittance function representing p_i (the probability that activity i will be realized), where s is the parameter of transmittance, and t_i, the time activity i will take. Then $W_e(s)$, the equivalent transmittance of the series system, is given by $W_a(s)\ W_b(s)$. The equivalent probability, that the system will be realized, is given by $W_e(0)$, and the equivalent time of realization of the system is

$$t_e = \frac{W'_e(s)\mid_{s=0}}{W_e(0)} \tag{1}$$

where $W'_e(s)$ is the first derivative of $W_e(s)$.

Thus, if $W_a(s) = p_a \exp(s.t_a)$ and $W_b(s) = p_b \exp(s.t_b)$ the equivalent probability of the system is $p_a p_b$, and the expected time $t = t_a + t_b$.

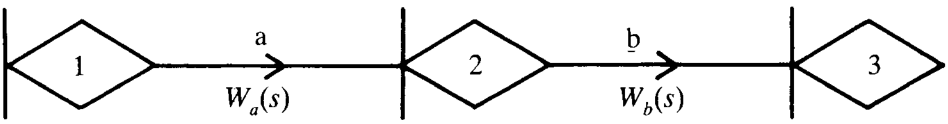

FIG. 1. Series system.

2.2 Parallel System

Figure 2 represents a network in which activities a and b are in parallel. In this system $W_e(s) = W_a(s) + W_b(s)$, $W_e(0) = p_a + p_b$, and $t = (p_a t_a + p_b t_b) / (p_a + p_b)$.

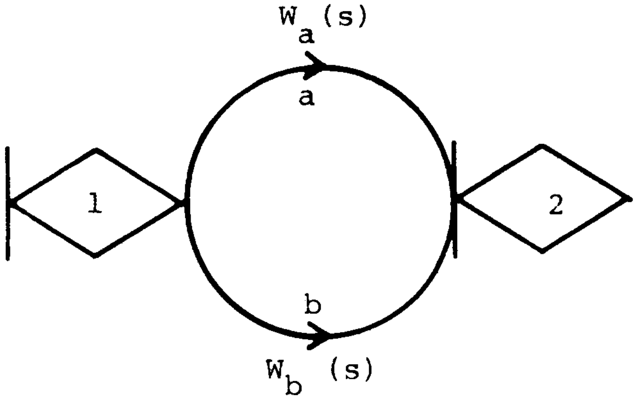

FIG. 2. Parallel system.

2.3 Self-loop

In Fig. 3, activity b represents a self-loop. In this system, $W_e(s) = W_a(s) / (1-W_b(s))$, $W_e(0) = p_a/(1-p_b)$, and $t_e = t_a + [t_b \cdot P_b / (1-P_b)]$.

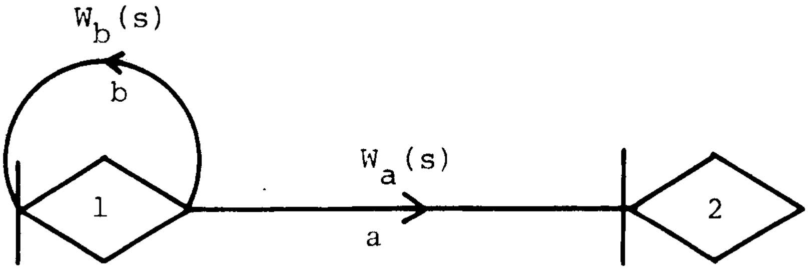

FIG. 3. Self-loop

2.4 General Network

This consists of series, parallel and self-loop systems. Such networks can be solved by using the topological equation of a closed network.

$$H = 1 - L_1 + L_2 - \dots + (-1)^i L_i + \dots = 0 \quad (2)$$

where L_i is the summation of values of all ith-order loops. The reader interested in details of derivaitons of the above expressions may refer to the references[1-5].

3. College of Engineering Education System

In College of Engineering, King Abdulaziz University, Jeddah, course study period is one semester. Admission of student in the college is allowed in any semester. A minimum of 145 credit hours is required for graduation. Some of the credit hours of work done at other university can be transferred with the residence condition that a student must complete at least 45 credit hours in the college to obtain a bachelor's degree from the university. The total minimal requirements for credit registration per semester is 12 hours and the maximal is 21 hours. This means a student who has to clear only the residence condition must study a minimum of 3 semesters. However, minimum number of semesters spent by a student to graduate is found to be five semesters[9]. This was actually a transfer student from other university. A student who joins the college directly after graduation from high school may take a minimum of seven semesters to graduate. Since students fail in some courses or repeat some courses because of poor grades, the maximum number of semesters to graduate was found to be 17.

4. GERT Formation

Figure 4 represents the system of study in the college. Node 0 represents the state from where a student chooses to enter the system. A branch (link) between node 0 and node n represents the admission of a student at the beginning of the nth semester. Thus P_n is the probability of this activity. A branch between nodes n and $(n+1)$ represents the continuation of study in the nth semester. Thus $P_{n,n+1}$ is its probability. A brach between nodes n and the last node G represents the last semester of study before graduation. $P_{n,G}$ is thus the probability of graduation after n semesters. A branch leading to a complemented node $\bar{n}$ represents a dropout, expulsion, or transfer to some other faculty at the beginning of or during the nth semester. Thus $P_{n,\bar{n}}$ is the probability of dropout after $(n–1)$ semesters.

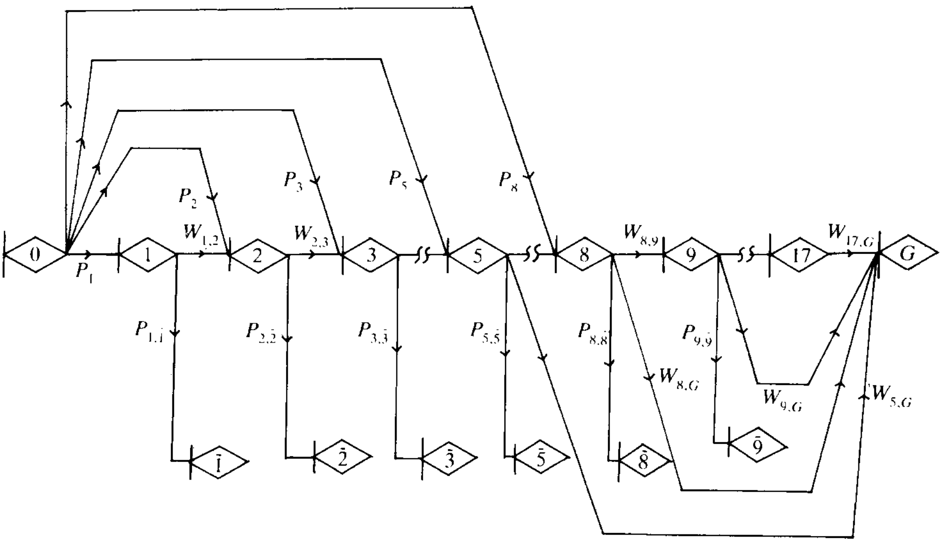

FIG. 4. GERT Network of College of Engineering Education System.

A break into the graph between nodes n and m means the pattern of graph continues the same way as at node n ($< m$).

5. Data Collection

We collected the data on students enrolment, drop-outs and graduation, starting from the beginning year of the college, *i.e.*, 1975 and ending with the data available upto second semester of 1986. We sorted out all those students who had graduated upto second semester of 1986, and dropped out as well. The total number of students was 3028. We then traced back their semester of enrolment to classify them in three groups which are: a–those who dropped out, b–who graduated, and c–who continued to the next semester. We then calculated the probabilities of all these activities for each semester. The details are given in Table 1. The columns of this table are self-explanatory.

TABLE 1. Activity probabilities.

Semester (n)	No. of continuing student + transfer from outside	Drop-outs	No. of graduates after n semesters	No. of continuing students	Probability of continuation	Probability of dropout	Probability of graduation
1	2537	68	–	2469	0.973	0.027	–
2	2469+195	271	–	2393	0.898	0.102	–
3	2393+156	175	–	2370	0.931	0.069	–
4	2374+100	152	–	2322	0.939	0.061	–
5	2322+23	136	12	2197	0.937	0.058	0.005
6	2197+9	104	26	2076	0.941	0.047	0.012
7	2076+6	84	115	1883	0.904	0.041	0.055
8	1883+2	81	95	1709	0.907	0.043	0.050
9	1709+0	76	212	1421	0.831	0.045	0.124
10	1421+0	87	330	1004	0.707	0.061	0.232
11	1004+0	77	222	710	0.702	0.072	0.221
12	1710+0	77	215	417	0.587	0.110	0.303
13	417+0	76	110	231	0.554	0.182	0.264
14	231+0	38	54	139	0.602	0.164	0.234
15	139+0	17	48	74	0.532	0.123	0.345
16	74+0	24	35	15	0.203	0.324	0.473
17	15	–	15	–	0.000	0.000	1.000

6. Calculation and Analysis

Let $W_{ij}(s)$ represents the transmittance from node i to node j. Also define $W_{ij}(s)$ by

$$W_{ij}(s) = P_{ij} \exp(st) \tag{2}$$

where

P_{ij} = the probability that a student registered in semester i will pass to semester j,
s = parameter of transmittance,
t = time of study.

As time duration in our system is 1 semester, t will always be 1. The equivalent function W_e of the network in Fig. 4 from starting node 0 to end node G (droping s for simplicity) is then obtained in Ahmad, *et al.*[9] as

$$W_e = \sum_{i=1}^{8} P_i E_{i,G} \tag{4}$$

where $E_{i,G}$ represents the equivalent function from node i to node G, and P_i is the probability that a student enters the college at the beginning of ith semester as given in Table 2.

TABLE 2. Probabilities of enrollment in ith semester.

Semester	Number of students enrolled	Probability (P_1)
1	2537	0.8378
2	195	0.0644
3	156	0.0515
4	100	0.0330
5	23	0.0076
6	9	0.0030
7	6	0.0020
8	2	0.0007

In order to show how W_e will be calculated, we shall here derive only $E_{8,G}$.

In Fig. 4, from node 8 to node G one can easily see that there are 10 paths which are enumerated as follows:

8-G, 8-9-G, 8-9-10-G, 8-9-10-11-G, 8-9-10-11-12-G, 8-9-10-11-12-13-G, 8-9-10-11-12-13-14-G, 8-9-10-11-12-13-14-15-G, 8-9-10-11-12-13-14-15-16-G, 8-9-10-11-12-13-14-15-16-17-G.

The transmittances of these paths are then calculated by substituting (3) with proper values for each branch in the path. Consider for example, path 8-9-G.

$$\begin{aligned} \text{Transmittance} &= P_{8,9} \exp(s) \cdot P_{9,G} \exp(s) \\ &= (0.907) \exp(s) \cdot (0.124) \exp(s) \\ &= (0.1125) \exp(2s) \end{aligned} \tag{5}$$

Since all these paths are parallel we add the transmittance of these paths to get our $E_{8,G}$. Thus

$$E_{8,G} = (0.05) \exp(s) + \ldots + (0.008) \exp(10s) \tag{6}$$

Substituting the values of each $E_{i,j}$ for each path of the ten paths, as derived in equation (6), in equation (4) we get W_e of the part of the network from node 8 to node G.

To calculate the probability of going from node 0 to node G we must determine the equivalent transmittances of all the paths remaining starting from node 1 to node 7 *following the same procedure* as used for $E_{8,G}$. The expected time needed to graduate from the college starting at any point can be determined using the following equation.

$$E(t) = W'_e(0) / W_e(0) \tag{7}$$

where $W'_e(0)$ is the value of the first derivative of equation (3) with respect to s at $s = 0$.

The probability of going from node 1 to node G, and the expected time needed to graduate are found to be equal to 0.491 and $10.18 \simeq 11$ semesters respectively, using equations (4) and (7).

7. Computer Program for Solution of GERT Network

This is a microcomputer based program developed in Basic. This program is working, at the moment, for 20 nodes (which is reasonably a large number for almost all practical purposes) with few limitations. The limitations are:

- maximum number of paths in the network Is 150,
- maximum number of nodes in one path is 13,
- maximum number of first order loops in the network is 25,
- maximum number of nodes in one loop is 10,
- maximum order of loop is 6.

The limitation may be overcome by increasing memory space. The flow chart for the above program is shown in Fig. 5.

Note :

In order to overcome the constraints of number of nodes in one path a separate routine is included to enumerate the paths for this network only.

Input to the Program

The first card of the input data file indicates the initial source node and the final sink node. Each branch of the network must be described by a separate card. The card must contain the following in the same order shown below:

- node beginning branch,
- node terminating branch,
- probability of realizing the branch,
- the coefficient of distribution (time of realization),
- type of distribution and the subsequent columns for describing the distribution parameters.

At the time of execution the program will ask for the data file name with the prompt: Name of data file please.

In response to the prompt, the user simply keys in the name of data file describing the network and press ENTER.

The result will be shown on the screen. The hard copy will contain the following results:

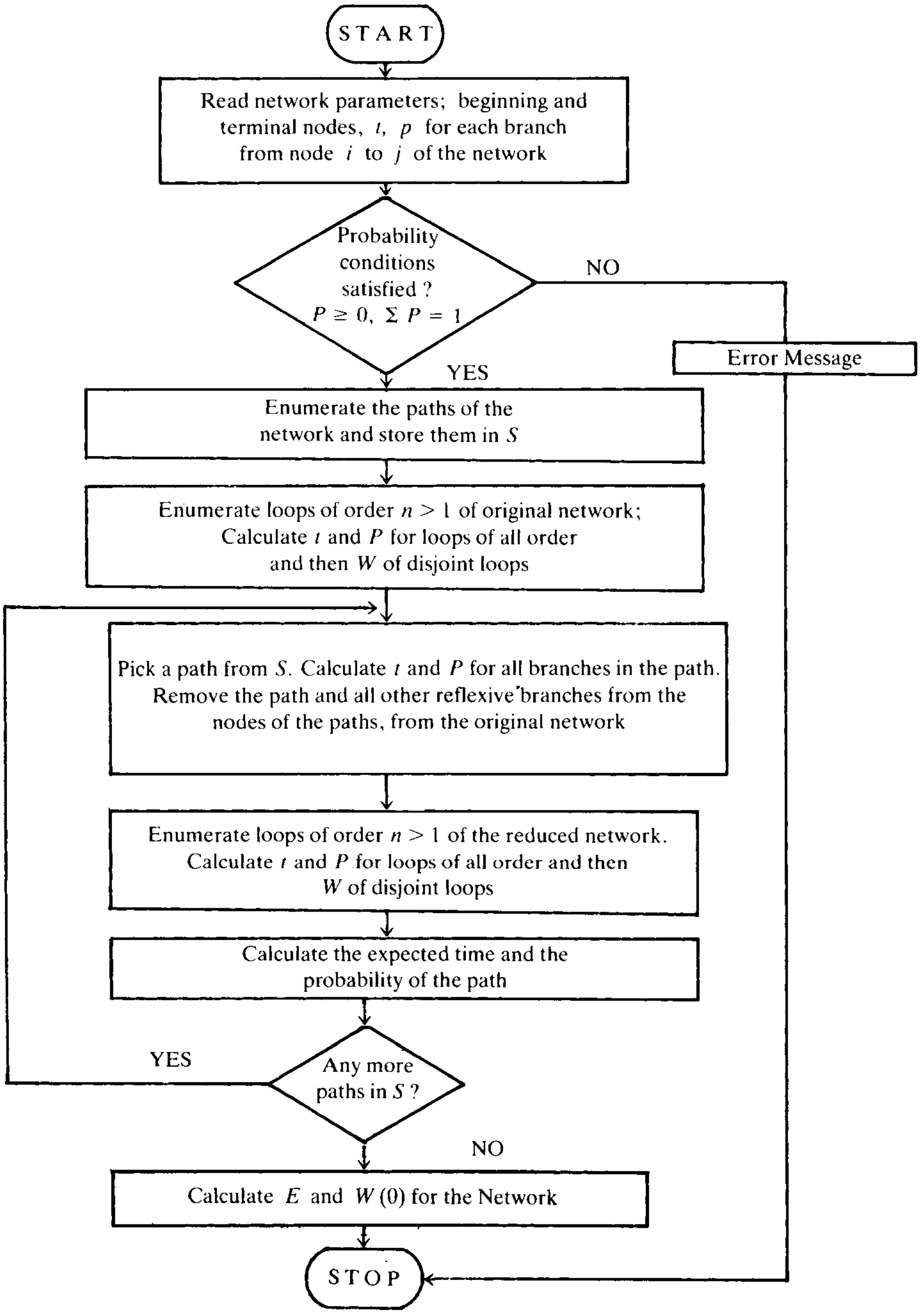

FIG. 5. Flow chart for GERT program.

1. Description of network.
2. The expected time with the corresponding probability.

Output of the Program

A sample output is given in Table 3.

TABLE 3. Solution of GERT problem.

Network parameters

From	To	Probability	Activity time
1	2	0.8378	0.00
1	3	0.0644	0.00
1	4	0.0515	0.00
1	5	0.0330	0.00
1	6	0.0076	0.00
1	7	0.0030	0.00
1	8	0.0020	0.00
1	9	0.0007	0.00
2	3	0.9730	1.00
3	4	0.8980	1.00
4	5	0.9310	1.00
5	6	0.9390	1.00
6	7	0.9370	1.00
6	19	0.0050	1.00
7	8	0.9410	1.00
7	19	0.0120	1.00
8	9	0.9040	1.00
8	19	0.0550	1.00
9	10	0.9070	1.00
9	19	0.0500	1.00
10	11	0.8310	1.00
10	19	0.1240	1.00
11	12	0.7070	1.00
11	19	0.2320	1.00
12	13	0.7070	1.00
12	19	0.2210	1.00
13	14	0.5870	1.00
13	19	0.3030	1.00
14	15	0.5540	1.00
14	19	0.2640	1.00
15	16	0.6020	1.00
15	19	0.2340	1.00
16	17	0.5320	1.00
16	19	0.3450	1.00
17	18	0.2030	1.00
17	19	0.4730	1.00
18	19	1.0000	1.00

N.B: Probability of 1.000 and the respective time of 0.00, if any, indicates the involvement of a dummy node.

The Expected time to tranverse the above Network from 1 (0) to 19 (G) is 10.1851 and the corresponding probability is 0.4910.

8. Conditional Probability Distribution

In Fig. 4, one can easily observe that nodes 5 to 17 are directly connected to node G implying that students are graduating after 5,6,7,..., and 17 semesters. The probability that a student who enrolls in the college will graduate is given by (4) with $s = 0$. The break down of this probability, as given in Table 4, can easily be derived.

TABLE 4. Graduation probability breakdown.

Semester	Graduation probability
5	0.00396
6	0.00859
7	0.03798
8	0.03137
9	0.07001
10	0.10989
11	0.07330
12	0.07100
13	0.03633
14	0.01585
15	0.01585
16	0.01156
17	0.00495

Normalizing the probability given in Table 4 (*i.e.*, dividing these by 0.49173) we get a conditional probability distribution which is given in Table 5.

TABLE 5. Conditional probability distribution.

Semester	Probability
5	0.0081
6	0.0175
7	0.0772
8	0.0638
9	0.1424
10	0.2216
11	0.1491
12	0.1444
13	0.0739
14	0.0363
15	0.0322
16	0.0235
17	0.0101

From Table 5 one can see that almost 66 percent of students graduate after studying from 9 to 12 semesters in the college. One can also infer easily that around 3 percent of the graduates have spent from 16 to 17 semesters in their studies.

9. Future Forecast

Table 6 gives detailed breakdown of number of students graduating semesterwise as well as departmentwise.

TABLE 6. Semester and departmentwise Graduates.

Year	Semester	Departments																		
		CE	EE	ME	IE	NE	MNE	AR	PE	UPR.	BM	LA	COMP	PM	TH.E	AERO	EL	ENVM	CHE	Total
1980	I	54	19	19	9	4	1	-	-	-	-	-	-	-	-	-	-	-	-	106
	II	9	0	0	0	0	1	-	-	-	-	-	-	-	-	-	-	-	-	10
																				116
1981	I	57	23	9	3	5	1	1	-	-	-	-	-	-	-	-	-	-	-	99
	II	8	2	1	0	2	-	-	-	-	-	-	-	-	-	-	-	-	-	13
																				112
1982	I	86	23	35	25	13	2	14	-	9	-	8	2	1	1	-	-	-	-	219
	II	18	8	5	7	0	0	0	-	0	-	0	0	0	0	-	-	-	-	38
																				257
1983	I	81	8	9	17	8	6	27	5	7	-	6	11	4	2	5	4	1	4	205
	II	19	1	2	2	1	0	0	0	1	-	1	3	0	0	1	1	0	2	34
																				239
1984	I	52	19	6	26	10	10	17	11	7	4	5	10	3	2	5	5	1	-	193
	II	18	1	2	1	0	2	1	2	0	1	1	3	0	0	1	3	0	-	36
																				229
1985	I	45	-	-	14	18	12	16	16	10	4	2	12	8	15	8	14	0	0	194
	II	11	-	-	4	0	5	2	3	0	0	1	3	1	3	0	2	0	0	35
																				229
1986	I	82	-	1	19	14	8	10	10	3	9	8	15	7	15	4	17	-	13	235
	II	26	-	0	4	1	0	1	1	0	1	0	3	1	1	0	6	-	1	46
																				281

One can easily see that the bulk of number of students graduating every year is in the first semester of the year. Also one can easily see that, except for the year 1980 and 1981, the total number of graduates does not differ much from year to year. The smaller number of graduates in 1980 and 1981 may be due to beginning of the programme, and hence lower enrolment.

Since the college study system is now in its full swing, and the situation has now probably stabilized we shall consider the number of graduates from year 1982 onwards. The numbers of 1980 and 1981 will not be considered to obtain greater accuracy.

Figure 6 shows that the data points are scattered around the mean value $246.6 \simeq 247$. The best forecast in this situation will then be the mean itself. A 99 percent confidence interval around the mean is given by

$$\bar{x} \pm s\, t\, 0.01,\quad n - 1 \,/\, \sqrt{n} \tag{8}$$

where $\bar{x}$, s, n & t have their usual meanings in statistics. Thus 99 percent confidence interval for the mean number of graduates per year is 246.6 ± 37.2.

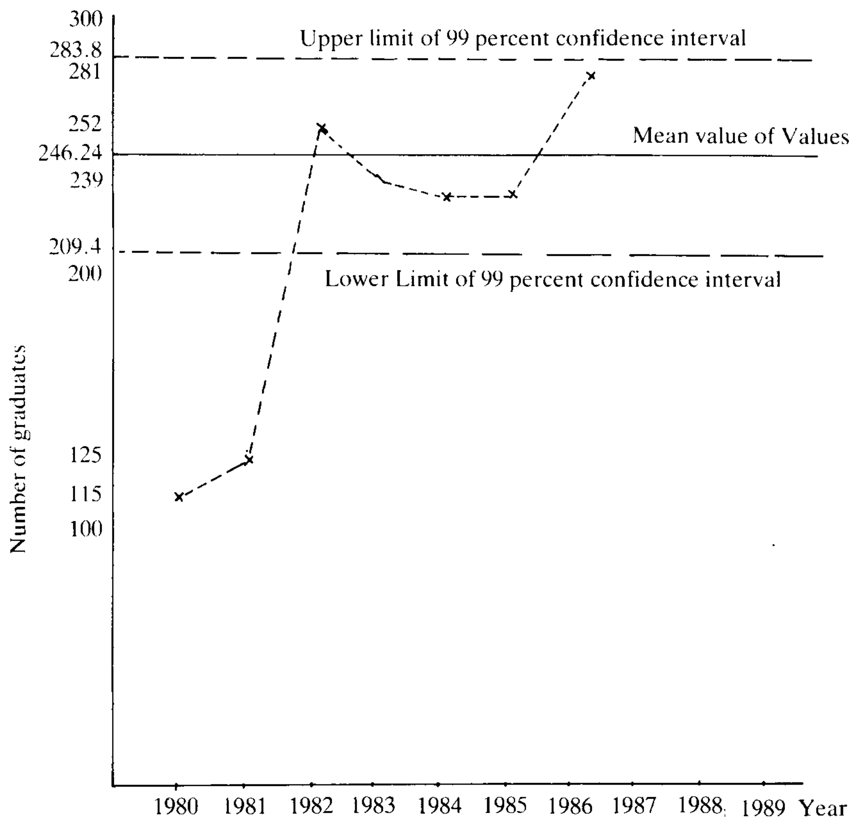

FIG. 6. Scattergram of graduates and confidence interval.

10. Conclusion and Recommendations

The probability of graduation is very low which indicates that quite a good number of students are admitted to the college who either are not well educated at high school level or just merely don't have the aptitude for engineering. The college should design a system for admitting new students to the college which should not totally depend on the overall GPA of the student in high school. The effect of a student's performance in science and maths and English courses in the high school on the college education may be studied to decide the criterion of admission.

It has been found that a student who enters the College of Engineering directly after high school or a transfer student with zero transferable credit will take on the average eleven semesters or five and a half years to earn a B.Sc. in engineering from King Abdulaziz University. This is a reasonable time for the completion of a B.Sc. in engineering, specially if it is realized that most of the students spend the first semester in an English intensive program. However, this total period of study can be reduced if the university decides to offer summer sessions.

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استخـدام طريقـــة جِرْت لبنـاء نمـوذج نمطي لنظـام التعليم العـــالي

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المستخلص . برغم اتساع تطبيق طريقة التقييم والمراجعة بالرسم (جِرْت) على العديد من المشاكل الصناعية والإدارية ، إلا أن تطبيقاتها في المجال التعليمي قليلة ومحدودة . وفي هذا البحث تم تطبيق طريقة جرت على نظام التعليم العالي بالمملكة العربية السعودية بهدف تحديد متـوسط المـدة التي يجب أن يقضيهـا الطالب في الجامعة قبل حصوله على شهادة البكالوريوس ، وكذلك احتمال تخرجه من الجامعة بعد الالتحاق بها ، بجانب إيجاد متوسط عدد الـطلبـة المتخرجين سنويًا بمعامل ثقة يصل إلى ٩٩٪ . وقد تم تطوير برنامج على الحـاسب الشخصي يقوم بهذه المهمة مستخدمًا البيانات الإحصائية التي أمكن الحصول عليها عن خريجي كلية الهندسة بجامعة الملك عبد العزيز .