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RF Spectrum of Simultaneous FDM and PSK

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The simultaneous transmission of both frequency division multiplexed voice channels and phase shift keying data over microwave radio is considered. The power spectrum of the hybrid analog-digital system is evaluated and the band limiting distortion is found. For a 960 FDM channel calculation example is provided to show the effect of transmitting data with voice on the necessary bandwidth. It is believed that the analysis can be utilized by microwave engineers for frequency allocation and channel assignment.

Nomenclature

$\phi(.)$	Gaussian random signal
s(.)	Frequency modulated signal
$R_s(.)$	Autocorrelation function of the modulated signal s(.)
$R_\phi(.)$	Autocorrelation of the modulating signal
ω	Angular frequency variable = $2\pi f$
$W_s(.)$	Power spectrum of s about the carrier
p^2	Mean square phase modulation = (rad.)^2
$\delta(.)$	Kronker delta function
$s_\phi(.)$	Baseband spectrum applied to phase modulator
S(.)	Baseband spectrum
$S_b(.)$	Preemphasized baseband spectrum
$\Delta^2 F$	Mean square phase deviation (MHz)^2
$P_e(f)$	Preemphasis characteristic
f_0	Bit rate/2 MHz
f_d	Subcarrier frequency MHz
W	RF power spectrum

Introduction

In recent years digital modulation techniques have been playing an increasingly important role in the transmission of information over terrestrial microwave, satellite, and cable systems. This fact coupled with the ever decreasing available RF spectrum has motivated the common carriers to use the empty lower baseband of existing FDM−FM systems and the band above the video and FDM to transmit data and FM programs. Several methods of modulating and transmitting this hybrid information have been presented in the literature. Most prominent are the methods of Data Under Voice and Data Above Voice or Video known as DUV and DAV respectively, in which a one Tl stream of 1.544M bit/sec. is transmitted simultaneously with the analog voice or video. A critical review of these methods can be found in [1−4]. While noise analysis and performance evaluation of these hybrid systems were treated, the published material does not discuss the problem of spectral occupation and necessary bandwidth for transmitting the analog and digital signal over microwave radio links. To calculate the necessary bandwidth, Carson's rule is often used as a rule of thumb. As the radio spectrum gets more and more congested, however some of its limitations are beginning to show up [5].

In this paper we consider the evaluation of the power spectral density of a sinusoidal carrier frequency modulated by FDM voice channels and phase shift keyed data above the voice baseband. We start by deriving the FM modulation spectra of the FDM and PSK signals individually. The spectrum about a carrier which is modulated by the sum of the two statistically independent phase disturbances is then found by convolving the two spectra [6]. For 960 FDM voice channels an example calculation is given to illustrate the method. The power content of different order side band can be readily found and the out of band power is plotted for different bandwidths.

The FM Spectra

Consider a sinusoidal carrier phase modulated by a Gaussian random signal $\phi(t)$ as

$$s(t) = \cos(\omega t + \phi(t)). \tag{1}$$

The correlation function of s(t) is then given by [6]

$$R_s(\tau) = \exp[-R_\phi(0) + R_\phi(\tau)], \tag{2}$$

where
$R_\phi(\tau)$ is the autocorrelation of the phase modulating signal and $R_\phi(0)$ is the mean square phase modulation. The power spectrum of s(t) about the carrier is the Fourier transform of Eq.(2):

$$W_s(f) = \int_{-\infty}^{\infty} \exp[-R_\phi(0) + R_\phi(\tau)]e^{-j\omega\tau}d\tau \tag{3}$$

which can be written as

$$W_s(f) = [\delta(f)\ S_\phi(f) + \frac{1}{2!} S\ (f) \overset{2}{*} S\ (f) + \frac{1}{3!} S\ (f) \overset{3}{*} S_\phi(f) + \ldots].e^{-p^2} \tag{4}$$

and in terms of normalized components we write (4) as

$$W_s(f) = [\delta(f) + p^2\ w_\phi(f) + \frac{(p^2)^2}{2!} w_\phi(f) \overset{2}{*} w_\phi(f) + \ldots].e^{-p^2} \tag{5}$$

where $\overset{n}{*}$ means the convolution process repeated n-1 times; (.) is the unit impulse function, and

$$S_\phi(f) = p^2\ w_\phi(f) \tag{6}$$

and hence

$$\int_{-\infty}^{\infty} w_\phi(f) \overset{n}{*} w_\phi(f)\,df = 1 \text{ for all n} \tag{7}$$

$$\text{and} \int_{-\infty}^{\infty} W_s(f)\,df = 1 \tag{8}$$

For small p^2, only the first few terms of (5) need to be evaluated and the power in the neglected sideband E(n) is found from

$$E(n) = 1 - e^{-p^2} \sum_{i=0}^{n} \frac{(p^2)^i}{(i)!} \tag{9}$$

E(n) can be used as a measure of the accuracy by which the spectrum is estimated. Now we proceed to find power spectrum for the FDM and PSK signals.

1. The FDM Power Spectrum

The FDM baseband signal can be represented by a band-limited Gaussian noise with a flat spectrum given by

$$S(f) = K^2\ (Hz)^2/Hz \quad f_1 < |f| < f_2$$
$$= 0 \text{ otherwise} \tag{10}$$

Assume an idealized preemphasis network is used, with frequency response of the form

$$pe(f) = \frac{f^2}{f^2_2} \tag{11}$$

then the baseband applied to the frequency modulator is given by

$$S_b(f) = k^2 \frac{f^2}{f^2_2} \quad f_1 < |f| < f_2, \tag{12}$$

= 0 otherwise,

and the mean square frequency deviation is defined as

$$\Delta^2 F = 2 \int_{f_1}^{f_2} K^2 \frac{f^2}{f^2_2} df \tag{13}$$

yielding

$$K^2 = \frac{3}{2} \Delta^2 F \frac{f^2_2}{f^3_2 - f^3_1}. \tag{14}$$

The equivalent baseband spectrum which would need to be applied to a phase modulator to give the same result is then,

$$S_\phi(f) = \frac{1}{f^2} S_b(f) = \frac{K^2}{f^2_2} \tag{15}$$

and the mean square phase deviation p^2 can be obtained from (15) by integrating over the baseband:

$$p^2 = 2 \int_{f_1}^{f_2} \frac{K^2}{f^2_2} df = 2\ K^2 \frac{f_2 - f_1}{f^2_2}$$
$$= \frac{3\ \Delta^2 F}{f^2_2 + f_1\ f_2 + f^2_1}. \tag{16}$$

Substituting from (16) into (6), the normalized baseband spectrum W (f) can be written as

$$W_\phi(f) = \frac{1}{2(f_2 - f_1)} \quad f_1 < |f| < f_2, \tag{17}$$

= 0 otherwise.

It is interesting to note that the baseband given by (17) is of rectangular shape, for which an analytical expression of the repeated convolutions in (5) can be readily written. The FDM power spectrum about the carrier $W_{FDM}(f)$ is found by substituting (17) into (5).

2. The PSK Spectrum

The baseband spectrum of a subcarrier modulated by the phase shift keyed random bit stream is given by [7]

$$S_{PSK}(f) = \frac{D^2}{2\ fo} [\text{sinc}^2(\frac{f - f_d}{fo}) + \text{sinc}^2(\frac{f + f_d}{fo})] \tag{18}$$

where

$fo = \frac{1}{2}$ bit rate,

D^2 = the PSK subcarrier power at the frequency moduator,

f_d = the subcarrier frequency,

and

$$\text{sinc } x = \frac{\sin \pi x}{\pi x}$$

The power spectrum about the carrier of a sinusoidal carrier frequency modulated by (18) is given by [8]

$$W_{PSK}(f) = 2 \int_0^{\infty} \phi(\tau) \cos \pi 2 f \tau \, d\tau, \qquad (19)$$

where

$$\phi(\tau) = e^{-k(\tau)} \qquad (20)$$

and

$$k(\tau) = \int_{-\infty}^{\infty} S_{PSK}(f) \frac{(1 - \cos \pi 2 f \tau)}{f^2} df \qquad (21)$$

To evaluate the power spectrum $W_{PSK}(f)$ we carry out the integral in (21) and substitute in (20). The exponential function of (20) is then expanded and Fourier transformed term-by-term. For small data subcarrier power, only the first few terms of the series need being considered. Taking the first two terms of the expansion for $\exp(-k(\tau))$ we can write the PSK spectrum as

$$W_{PSK}(f) \simeq C_1 \text{ sinc}^2\left(\frac{2f}{fo}\right) + C_2 \left[\text{sinc}\left(\frac{f-f_d}{fo}\right)\right.$$

$$\left. +2 \text{ sinc}^2\left(\frac{f}{fo}\right) + \text{sinc}^2\left(\frac{f+f_d}{fo}\right) \right], \qquad (22)$$

where

$$C_1 = \frac{2}{fo} e^{-m^2} (1-m^2),$$

$$C_2 = \frac{m^2}{4(1-m^2)} C_1,$$

$$m^2 = \frac{D^2}{f^2_d}, \qquad (23)$$

and the total power in the PSK spectrum is

$$\int_{-\infty}^{\infty} W_{PSK}(f)\, df \cong 1. \qquad (24)$$

3. *The Total Spectrum*

The power spectral density of a sinusoidal carrier frequency modulated by the FDM and PSK signals is then obtained by convolving $W_{FDM}(f)$ and $W_{PSK}(f)$ as

$$W_t(f) = W_{FDM}(f) * W_{PSK}(f) \qquad (25)$$

$$\simeq e^{-p^2} W_{PSK}(f) + e^{-m^2} \bar{W}_{FDM}(f)$$

$$+ \frac{m^2}{2} [\bar{W}_{FDM}(f-f_d) + \bar{W}_{FDM}(F+F_D)]$$

$$+ \frac{(m^2)^2}{(2!)} [W_{FDM}(f-2f_d) + 2W_{FDM}(f)$$

$$+ \bar{W}_{FDM}(f+2f_d)] + \ldots \qquad (26)$$

where

$$\bar{W}_{FDM}(f) = W_{FDM}(f) - e^{-p^2} \delta(f).$$

From equation (26) we note the following limiting cases:

(i) $p^2 = 0$ $\quad W_{FDM}(f) = \delta(f)$

$$W_t(f) = W_{PSK}(f)$$

(ii) $m^2 = 0$ $\quad W_{PSK}(f) = \delta(f)$

$$W_t(f) = W_{FDM}(f)$$

and the total power is again

$$\int_{-\infty}^{\infty} W_t(f)\, df = 1 \qquad (27)$$

Equation (26) can be used to calculate necessary bandwidth for quality transmission ofthe hybrid baseband by finding the bandwidth containing a certsin percentage of the total power (99.5%, say). On the other hand the out of band power must be found in order to evaluate interference levels in the neighbouring channels.

Example

A calculation example is given for the case of 960 FDM voice channels and 1.544 Mb/s data stream above the voice baseband:

f_2 = 4.188 MHz

f_1 = 0.316 MHz

$\Delta^2 F$ = 1.2144 $(MHz)^2$

f_d = 6 MHz

2fo = 1.544 MHz

D^2 = variable such that the data loading is 11,8,5,2,−1 dBmo, with test tone rms deviation of 200 KHz.

D^2 can then be calculated from the relation

$$10 \log \left(\frac{D}{0.2}\right)^2 = 11,8,5,2-1,$$

yielding D^2 = .505, .2528, .1264, .0632, 0315$(MHz)^2$, respectively, and from (16) p^2 = .192 $(rad)^2$.

Table 1 shows the side band power distribution and Fig.1 is a plot of bandlimiting distortion versus bandwidth for different data loadings. The necessary bandwidth determined using Carson's rule is about 24 MHz.

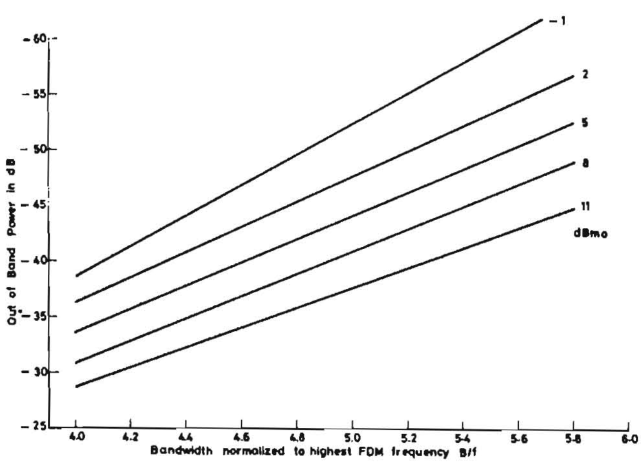

Fig. (1) **Out of Band Power**

Discussion and Conclusions

The use of Carson's rule to estimate the bandwidth necessary for the quality transmission of the baseband considered in this paper results in about 24 MHz, almost independent of the data deviation. However, from Fig.1 we can see that the band-limiting distortion varies almost linearly with the data loading. For a predefined distortion level (−40 dB, say), the necessary bandwidth increases with the data loading from 17 MHz at data power of −1 dBmo to 22 MHz at 11 dBmo. For a distortion level of −50 dB, the necessary bandwidth increases from 20 MHz at −1 dBmo to 27 MHz at 11 dBmo, which means that Carson's rule over estimates the necessary bandwidth at small data loading and leads to interference problems at high loading power.

From the above discussion we conclude that Carson's rule as a design tool, must be modified to reflect the change of the transmission quality with necessary bandwidth, especially for the cases where a low power subcarrier is placed above the signal baseband. We also conclude that decreasing subcarrier power is highly recommended since it will reduce the potential of adjacent channel interference. By using efficient data coding methods, the data carrier can be transmitted at low power without appreciably degrading the signal quality.

Table 1: Sideband Power Distribution

Data Spectrum				FDM Spectrum				Necessary Bandwidth MHz		
Data loading dBmo	Modulation Index m^2	% of RF Power	Neglected sideband power	% sideband power 1st	2nd	3rd	Neglected sideband power	Carson's Rule	for-40dB distortion	for 99.5 or RF power
−1	8.525×10^{-4}	82.53	−65 dB	99.9148	.085177	3.6×10^{-5}	−95 dB	22.35	17.1	13.5
2	1.75×10^{-3}	82.53	−60 dB	99.8252	.174694	1.53×10^{-4}	−90 dB	22.65	18.2	13.8
5	3.5×10^{-3}	8 .253	−52 dB	99.6500	.348777	6.1×10^{-4}	−85 dB	22.9	19.25	14.2
8	7×10^{-3}	82.53	−46 dB	99.3024	.695116	2.43×10^{-3}	−80 dB	23.25	20.8	15
11	1.4×10^{-2}	82.53	−40 dB	98.6098	1.38053	9.66×10^{-3}	−72 dB	23.85	22.3	16

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الطيف الناجم عن الإرسال المتزامن للقنوات الهاتفية والمعلومات الرقمية

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يتعرض هذا البحث لدراسة الإرسال المتزامن للقنوات الهاتفية بطريقة تعديل وتركيب الذبذبات والمعلومات الرقمية بتعديل الطور عبر شبكات الميكروويف، ويقدم البحث بعض النتائج التي قد تفيد مصممي شبكات الميكروويف في اختيار الذبذبة والسعة الملائمتين لهذا النوع الهام من الإرسال.