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FM Spectrum and Necessary Bandwidth for Colour TV and Sound Subcarriers

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There has been a great demand in communication requirements over the past years and broadcast systems in particular are expanding in the high frequency band. For example in terrestial microwave radio system, the TV audio is transmitted in the form of an FM subcarrier placed above the video baseband. Efficient use of radio spectrum and equipment has recently become the subject of increased attention and led common carriers to use more than one subcarrier above video for transmitting FM programs, data and service. In this paper, we will define the RF bandwidth needed for the transmission of video and up to four subcarriers in terms of a given amount of interchannel distortion and subcarrier deviations. Results are presented in the form of design curves.

Nomenclature

FM	Maximum modulating frequency	(Hz)
ΔF	Peak frequency deviation	(Hz)
B_N	Necessary bandwidth	(Hz)
D_l	Mean square frequency deviation due to the luminance signal	$(Hz)^2$
D_{ch}	Mean square frequency deviation due to the chrominance signal	$(Hz)^2$
B_l	Bandwidth of the luminance signal	(Hz)
B_{ch}	Bandwidth of the chrominance signal	(Hz)
$P_l(f)$	Power spectrum of the luminance signal	(Watt/Hz)
P_{ch} (f)	Power spectrum of the chrominance signal	(Watt/Hz)
P_{video} (f)	Power spectrum of the total video signal	(Watt/Hz)
f_s	Chrominance subcarrier frequency	
m_l	Modulation index of the luminance signal	
m_{ch}	Modulation index of the chrominance signal	
W_l (f)	FM power spectrum of the luminance signal	
W_{ch} (f)	FM power spectrum of the chrominance signal	
W_{vl} (f)	FM power spectrum of the total signal	
Θ_i	Phase of the i-th subcarrier where $\Theta_i = \int \omega_{si}(t)\,dt$	
A_i	Magnitude of the i-th subcarrier	
ω_{si}	Frequency of the i-th subcarrier	
X_i	Modulation index of the i-th subcarrier	
Θ	$= \tan^{-1} \dfrac{2 B_{ch} \cdot f_s}{(f_s^2 - B_{ch}^2)}$	
$J_{n_r}(x_r)$	Bessel function of order n_r and argument X_r	

Introduction

Recent interest in the transmission of multichannel sound has led many researchers to study ways and means of implementing their ideas. The immediate goal however is the efficient use of the frequency spectrum for that particular transmission.

There are three methods by which sound associated with video can be transmitted: [1-4].

(a) Multiplexed channels using frequency dividion multiplexing FDM, this is highly reliable and of high quality but expensive:

(b) Sound can be included in the blanking periods of the T.V signal which provides an economical way of sound transmission but sound can easily be lost;

(c) Sound signal frequency modulates a subcarrier placed above the video band, other carriers and/or chrominance signals can also be transmitted in the form of FM subcarriers placed above video. A study of the necessary bandwidth in the presence of these composite signals will be presented.

The bandwidth necessary for transmitting these composite signals can be determined by the use of the well known Carson's rule which states that the necessary RF bandwidth is given by

$$B_N = 2\,[\,F_m + \Delta F\,] \tag{1}$$

Carson's rule, however, has its own limitation as it may result in a waste of the RF spectrum since it does not take into account the subcarriers with low power. In addition, narrow radio beams, nearly equal output powers and cross polarization can be used to reduce adjacent channel interference. As the bandwidth decreases distortion increases, on the other hand, reducing bandwidth means that channels may be put closer together which increases mutual interference. This trade off between spectrum and system performance is not reflected either by the rule.

Carson's rule has been the subject of study by many authors [5-9], however, none of the literature treats the problem of video and subcarrier transmission. In this respect we will present a study of the necessary bandwidth required to transmit a colour T.V. signal and subcarriers, taking into account the interchannel modulation and subcarrier deviation. Results are presented in the form of design curves of signal-to-band limiting distortion noise versus bandwidth, different subcarrier powers and frequencies are considered.

The Base Band Spectra

The video luminance signal has an energy spectrum with a narrow passband and can therefore be approximated by a Gaussian noise process at the output of an RC integrating network. The spectrum is thus given by [10]

$$P_l\,(f) = \frac{D_l}{\pi B_l} \quad \frac{1}{1 + (f\,/\,B_l)^2} \tag{2}$$

Whereas the chrominance signal power spectrum can be written as

$$P_{ch}\,(f) = \frac{D_{ch}}{\pi B_{ch}} \quad \frac{1}{1 + ((f - f_s)/B_{ch})^2} \tag{3}$$

The input signal into the FM modulator is the sum of the video and audio subcarriers and can be written as

$$V_{in} = V\,(t) + \sum_i A_i \cos \Theta_i \tag{4}$$

We will neglect the subcarrier deviation by the audio signal since its contribution to the necessary bandwidth is small. Hence, equation (4) becomes

$$V_{in} = V\,(t) + \sum_i A_i \cos \omega_{si}\, t \tag{5}$$

and the modulated signal can be written as

$$V_0 = \sqrt{2} \cos (\,\omega_c t + V_1\,(t) + V_2\,(t)) \tag{6}$$

where the carrier amplitude is taken as $\sqrt{2}$ for unit power and

$$V_1\,(t) = \int_0^t V\,(t)\,dt \tag{7}$$

$$V_2\,(t) = \sum_i X_i \sin \omega_{si}\, t \tag{8}$$

where V_1 (t) and V_2 (t) are the video and subcarrier voltages respectively, at the modulator input.

Assuming that video and audio signals are statistically independent phase disturbances, it can be shown that the spectrum about a carrier which is phase modulated by the sum of two independent signals is the convolution of the spectra about two independent carriers as shown in [11]. The last statement can be written as

$$W_{v0} = W_{v1}\,(f) * W_{v2}\,(f) \tag{9}$$

where * means convolution and W_{v0} is the FM spectrum of the total output signal.

The FM Spectrum

The FM spectrum of the total signal is obtained by convolving the individual spectra. To find the FM spectra of the luminance and chrominance components we start by calculating the modulation indexes of the individual signals as

$$m = \frac{\text{rms frequency deviation}}{\text{bandwidth of frequency deviation}} \tag{10}$$

$$m_l = \frac{\sqrt{D_l}}{B_l} \quad \text{and} \quad m_{ch} = \frac{\sqrt{D_{ch}}}{B_{ch}} \tag{11}$$

for most T.V. systems $D_l \sim 1$ and $B_l \sim .025$ giving $m_l \approx \frac{1}{.025} = 40$, thus at this high value of m, the quasi-static approximation can be used and the FM spectrum for the luminance signal is given by [12]

$$W_l(f) = \frac{1}{\sqrt{2\pi D_l}} \exp(-f^2/2D_l) \qquad (12)$$

It should be noted that the quasi static approximation gives a spectrum accurate only for low frequencies and falls off much faster at the tails. An assymptotic approximation [13] of the spectrum for high frequencies is shown in Appendix A.

The FM spectrum of the chrominance signal can be shown to be

$$W_{ch}(f) = A\Big\{\frac{m^2_{ch} B_{ch}}{f^2 + (m^2 B_{ch})^2} + \frac{m^2_{ch}(1+m^2_{ch}) B_{ch} \cos\Theta}{2}$$

$$\Big[\frac{1}{(f-f_s)^2 + (1+m^2_{ch})^2 B^2_{ch}} + \frac{1}{(f+f_s)^2 + (1+m^2_{ch})^2 B^2_{ch}}\Big]$$

$$+ \frac{(m^2_{ch}\cos\Theta)^2}{2\,!} B_{ch} \frac{(2+m^2_{ch})}{4}\Big[\frac{2}{f^2 + (2+m^2_{ch})^2 B^2_{ch}} +$$

$$\frac{1}{(f-2f_s) + (2+m^2_{ch})^2 B^2_{ch}} + \frac{1}{(f+2f_s)^2 + (2+m^2_{ch})^2 B^2_{ch}}\Big]\Big\} \qquad (13)$$

and

$$A = \frac{1}{\pi} \exp(-m^2_{ch} \cdot \cos\Theta) \qquad (14)$$

and we get

$$W_{vl}(f) \cong \frac{e^{-m^2_{ch}\cos\Theta}}{\sqrt{2\pi D_l}} \Big\{ e^{-\frac{f^2}{2D_l}} + \frac{m^2_{ch}\cos\Theta}{2}$$

$$\Big[e^{-\frac{(f-f_s)^2}{2D_l}} + e^{-\frac{(f+f_s)^2}{2D_l}}\Big] + \frac{(m^2_{ch}\cos\Theta)^2}{2\,!}$$

$$\frac{1}{4}\Big\{ e^{-\frac{(f-2f_s)^2}{2D_l}} + e^{-\frac{(f+2f_s)^2}{2D_l}} + 2e^{-\frac{f^2}{2D_l}} \Big\} \qquad (15)$$

The quasi-static approximation of (13) yields a satisfactory estimate of the FM spectrum about the carrier, however, at the tails of the spectrum a more conservative estimate is found using the assymptotic properties of Fourier Transform, and is given by

$$W_{vl}(f) \underset{f\to\infty}{\sim} \frac{B_l D_l}{\pi f^4} + \frac{B_{ch} D_{ch}}{2\pi}\Big[\frac{1}{(f-f_s)^4} + \frac{1}{(f+f_s)^4}\Big] \qquad (16)$$

In computing distortion we use equation (15) to find the power around the carrier and equation (16) for computing spectrum at the tails of the spectrum.

The power spectrum of a carrier phase modulated by a group of sine waves can be found in many books [14] on modulation and can be written as

$$W_{v2}(f) = \sum_{n_1=-\infty}^{\infty} \cdots \sum_{n_N=-\infty}^{\infty} \prod_{r=1}^{N} J^2_{n_r}(X_r) \cdot \delta(f - \sum_{r=1}^{N} n_r f_r) \qquad (17)$$

where $\delta(f) = 0$ for $f \neq 0$; $\delta(0) = 1$. To find the power spectrum of the total signal V_0 we substitute from equation (17) into equation (8) to get

$$W_{v0}(f) = \int_{-\infty}^{\infty} W_{vl}(\alpha) W_{v2}(f-\alpha)\, d\alpha$$

$$= \int_{-\infty}^{\infty} W_{vl}(\alpha) \sum_{n_1=-\infty}^{\infty} \cdots \sum_{n_N=-\infty}^{\infty} \prod_{r=1}^{N}$$

$$J^2_{n_r}(x_r) \quad \delta(f - \sum_{r=1}^{N} n_r f_r - \alpha)\, d\alpha \qquad (18)$$

Interchanging summation and integration we can write (18) as

$$W_{v0}(f) = \sum_{n_1=-\infty}^{\infty} \cdots \sum_{n_N=-\infty}^{\infty} \prod_{r=1}^{N} J^2_{n_r}(x_r) W_{vl}(f - \sum_{r=1}^{N} n_r f_r) \qquad (19)$$

From equation (19) it is clear that the power spectrum is made up of a video spectrum having the power $\prod_{r=1}^{N} J^2_0(X_r)$, and a set of sidebands centered around the subcarrier frequencies.

Necessary Bandwidth

The FM spectrum given by (19) extends at both sides of the carrier to ∞. To limit the spectrum to the pre-assigned RF channel, filters are used at the transmitter, receiver, or both. If we assume that only one filter is used, with ideal characteristics and half bandwidth B Hz, then the distortion in the RF signal due to band limiting is given by

$$\text{Distortion} = \frac{\text{Total signal-Band limited signal}}{\text{Total signal}} \qquad (20)$$

and since the total power is normalized to unity, equation (20) can be written as

$$\text{Distortion} = 1 - \text{Band limited signal} \qquad (21)$$

and considering the filter of 2B bandwidth we have

$$\text{Distortion} = 1 - \int_{-B}^{B} W_{v0}(f)\, df \qquad (22)$$

Equation (22) is used to plot the curves of Figs. 1-6. The six curves of Fig. 1 are plotted for one subcarrier having different powers relative to the video power. In the top curve, a subcarrier power of 32dB below video is considered and the lower most curve is for a subcarrier power of −17dB below video. Figures 2-6 show different practical situations of one subcarrier, two subcarriers and four subcarriers of low deviation and high deviation. To show how the formulas (15-19) are used for bandwidth calculation, we consider one, two and four subcarriers transmission above the baseband of 525 NTSC colour T.V. signal with colour subcarrier frequency of 3.58 MHz. The FM subcarriers deviation due to the baseband signal is about 24 KHz peak (low deviation) or 58 KHz peak (high deviation). Different subcarrier powers are considered such that

$$\Delta = 20 \log \left(\frac{\Delta F_s}{\Delta F_{vid}} \right) = -17, -20, -23, -26, -29, -32 \text{ dB} \qquad (23)$$

where ΔF_s and ΔF_{vid} are the subcarrier and video deviation respectively. Assuming video deviation of 4MHz, the corresponding values of ΔF_s can be calculated. Now let us consider the bandwidth B_N calculation using two forms of Carson's rule such that

$$B_{N1} = 2\,(Fm + \Delta F_v) \qquad (24)$$

and

$$B_{N2} = 2\,(Fm + \Delta F_p) \qquad (25)$$

where

$$\Delta F_p = \sum_i \Delta F_i \qquad (26)$$

$$\Delta F_v = \sum_i (\Delta F_i^2)^{1/2} \qquad (27)$$

ΔF_v and ΔF_p are the total deviations assuming voltage addition and power addition of the individual deviations, respectively. The bandwidth B_{N3} is found from curves of Fig. 1 for −50dB bandlimiting distortion. Table 1 shows the calculations for one subcarrier at 5.3 MHz.

Table 1. Calculations for one Subcarrier at 5.3 MHz

Δ (dB)	ΔF_s	ΔF_{vid} MHz	ΔF_p MHz	ΔF_v MHz	F_m MHz	B_{N1} MHz	B_{N2} MHz	B_{N3} MHz
−32	.1	4	4.1	4.001	5.3	18.8	18.602	14.6
−29	.141	4	4.141	4.002	5.3	18.882	18.604	15.0
−26	.2	4	4.2	4.005	5.3	19	18.61	15.5
−23	.282	4	4.282	4.01	5.3	19.164	18.62	16.2
−20	.4	4	4.4	4.02	5.3	19.4	18.64	17.2
−17	.565	4	4.565	4.04	5.3	19.73	18.68	18.7

For the case of Fig. 6 in which four high deviation subcarriers are transmitted above the video band, Carson's rule bandwidth is 23.34 MHz. This bandwidth results in a S/N of about 65dB for −32dB subcarriers and about 42dB for subcarriers power of −17dB below video. From the preceeding discussion it is clear that the bandwidth necessary for transmitting the video and subcarriers is strongly related to the subcarrier power, this is mainly due to the interaction between the video and subcarriers spectra causing higher order side band of the video signal to have appreciable power at the subcarrier frequencies. Finally, it can be seen from the figures that upto four subcarriers can be transmitted with the video signal, using as low bandwidth as 18.5 MHz provided that the subcarrier power is reduced to about −27dB and thus, higher quality subcarrier equipment is needed such that an acceptable S/N is attained for the sound channels. Also it can be clearly seen from the figures that increasing the subcarrier power increases the bandwidth required for the same transmission quality and tolerable interference level.

Conclusion

The paper presents new results for the transmission of video and multichannel sound diplexed above the video band. A closed form expression of the RF spectrum of the video and subcarriers signal is presented and used to calculate the signal to bandlimiting distortion, and the out of band powers causing adjacent channel interference. For one, two and four subcarriers, the S/N is plotted versus the bandwidth for the range of subcarriers power of interest. It is concluded that the use of Carson's rule to estimate the necessary bandwidth in the present case leads to waste of spectrum at low subcarrier powers, to high distortion and adjacent channel interference in the case of high subcarrier powers. It is believed that the design curves presentented in Fig. 1-6, cover most practical situations of interest and can be used by radio engineers for reliable bandwidth calculations.

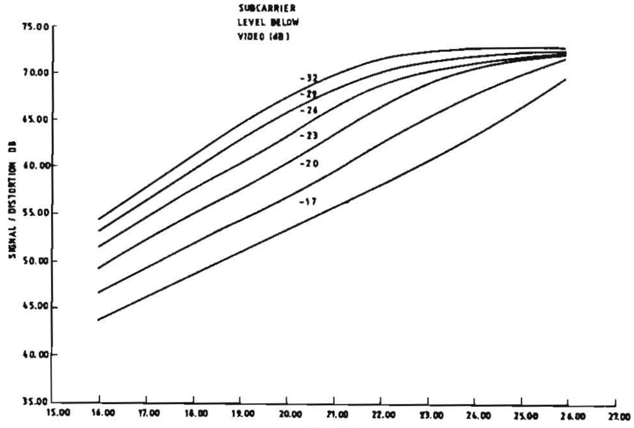

Figure 1: One Subcarrier LO Dev.

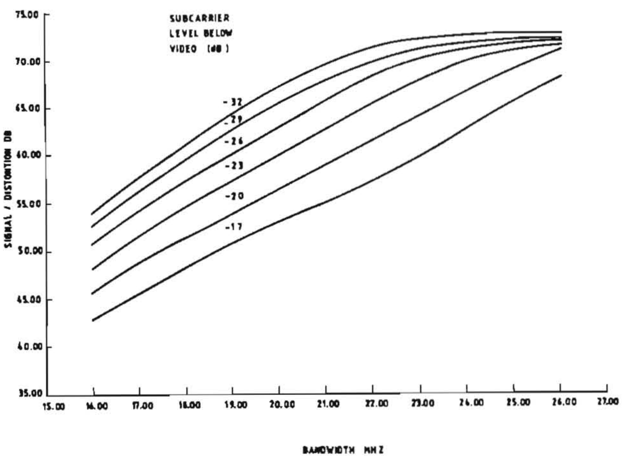

Figure 2: One Subcarrier Hi Dev.

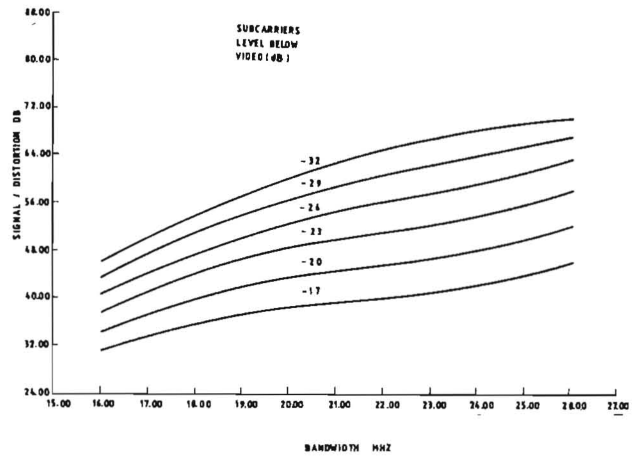

Figure 5: Four Subcarriers Lo. Dev.

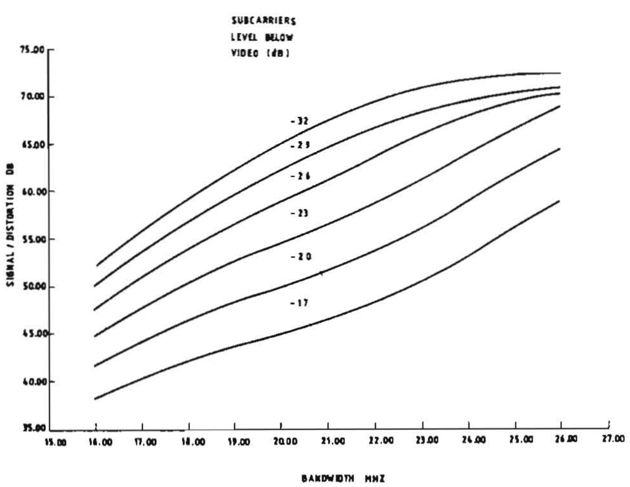

Figure 3: Two Subcarriers Lo Dev.

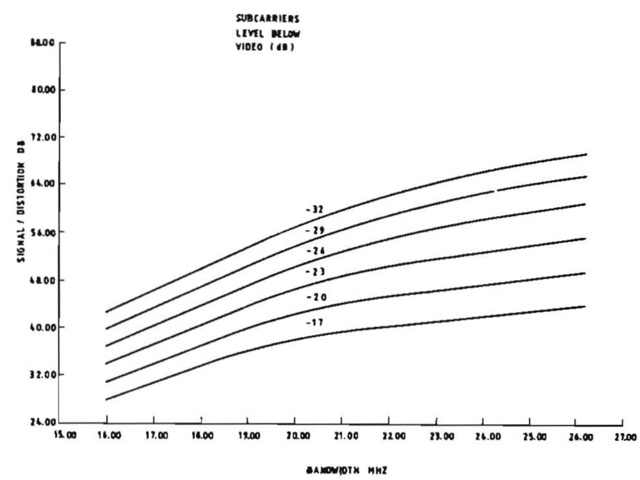

Figure 6: Four Subcarriers Hi Dev.

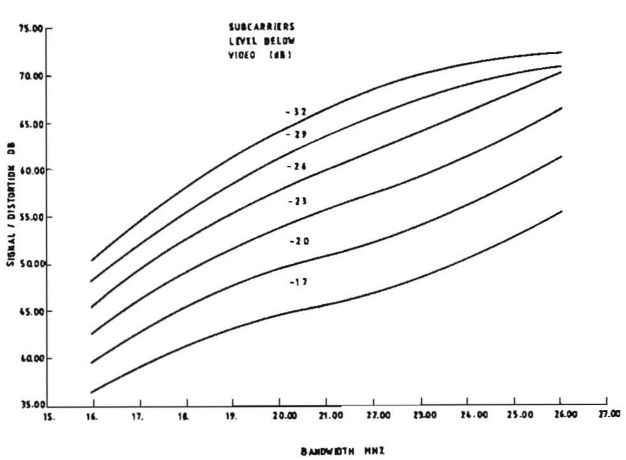

Figure 4: Two Subcarriers Hi Dev.

Appendix

-Let us assume that we have the autocorrelation function of the luminance signal given by

$$R_{V_l}(\tau) = e^{-k(\tau)} \qquad (A1)$$

where $K(\tau) = 2(\pi\tau)^2 \int_{-\infty}^{\infty} P(f) \left(\frac{\sin \pi f \tau}{\pi f \tau}\right)^2 df \qquad (A2)$

Successive differentiation of equation (A2) yields

$$K^{(n)}(\tau) = (2\pi)^2 \, \Phi^{(n-2)}(\tau), \qquad n \geq 0 \qquad (A3)$$

where $\Phi(\tau)$ is the correlation function of the video base band.

If $\Phi(\tau)$ and its first n−3 derivatives are continuous everywhere, but the $(n-2)^{nd}$ derivative, $\Phi^{n-2}(\tau)$ has a finite number of ordinary discontinuities located at $\tau = \tau_p$, then $K(\tau)$ and the first (n−1) derivatives are continuous for all τ

but the n^{th} derivative $k^{(n)}(\tau)$ has ordinary discontinuities located at the same τ_l, the l^{th} discontinuity in $K^{(n)}$ is given by

$$\Delta_K^{(n)} = k^{(n)}(\tau_l +) - K^n(\tau_l -) \qquad (A4)$$

$$= (2\pi)^2 \; \Delta^{n-2} \qquad n \geq 2 \qquad (A5)$$

where Δ^n denotes the p^{th} discontinuity in $\Phi^n(\tau)$

$$\Delta^n = \Phi^n(\tau_l +) - \Phi^n(\tau_l -) \qquad (A6)$$

From equations (A5) (A6), we obtain

$$\Delta_K^{(3)} = K^{(3)}(0+) - K^{(3)}(0-) = -16\pi^3 B_l D_l \qquad (A7)$$

as the discontinuity in the third derivative of $K(\tau)$ itself and its first two derivatives are continuous everywhere.

Therefore $R_{V_l}(\tau) = e^{-K(\tau)}$ and its first derivatives are continuous for all τ but the third derivative $R^{(3)}_{V_l}(\tau)$ is discontinuous at $\tau = 0$.

$$\Delta^{(3)} = R_{V_l}^{(3)}(0+) - R_{V_l}^{(3)}(0-) = -\Delta_K^{(3)} \qquad (A8)$$

Thus, using equations (A7) & (A8), and under certain conditions for high frequency the Fourier transform of $R_{V_l}(\tau)$ has the asymptotic representation

$$W_{V_l}(f) \simeq \frac{1}{(J2\pi f)^{n+1}} \sum_l \Delta_l^n \; e^{-J2\pi\delta\tau_l} \qquad (A9)$$

$$\simeq \frac{1}{(J2\pi f)^4} \times -16\pi^3 \; B_l \; D_l$$

$$= \frac{B_l D_l}{\pi f^4} \qquad (A10)$$

Similarly, it can be shown that the assymptotic spectrum of the total video signal is given by

$$W_{video}(f) \sim \frac{B_l D_l}{\pi f^4} + \frac{B_{ch} D_{ch}}{2\pi} \left[\frac{1}{(f-f_s)^4} + \frac{1}{(f+f_s)^4} \right] \qquad (A11)$$

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سعة الموجة اللازمة لنقل الصورة وموجات فرعية بتردد عال فوق ذبذبة الصورة

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إن التوسع في استخدام وسائل المواصلات حدا بالباحثين في هذا المجال الى استغلال الذبذبات المتناهية القصر (الميكروويف). وفي شبكات الميكروويف نرى أن الصوت لبرامج التلفاز ينقل على شكل موجة حاملة فرعية بتردد عال فوق النطاق الأساسي للصورة، لذا نجد أن هناك دراسات عديدة تهدف الى استخدام الطيف الراديوي بكفاءة حتى أن تلك الدراسات أسفرت عن استخدام موجات حاملة فرعية فوق ذبذبة الصورة لإرسال برامج إذاعية ومعلومات وخدمات أخرى. وفي هذه الورقة سنقوم بتعريف سعة الموجة الراديوية اللازمة لإرسال الصورة إضافة الى ثلاث موجات فرعية آخذين بعين الاعتبار التداخل بين القنوات. وقد تم عرض النتائج على شكل رسومات بيانية.