J. King Saud Univ., Vol. 21, *Science* (Special Issue), pp. 21-24, Riyadh (2009/1430H.)

On Minimizing Unused Bandwidths in Series Parallel Networks

Brahim Chaourar[1], Ahmed Redha Mahlous[2] and Rod J. Fretwell[3]
[1]*Al Imam University, College of Sciences, P. O. Box 90950, Riyadh 11623, Saudi Arabia*
Correspondence address: *P. O. Box 287574, Riyadh 11323, Saudi Arabia*
bchaourar@hotmail.com
[2]*School of Informatics, University of Bradford, United Kingdom*
ARMahlou@Bradford.ac.uk
[3]*School of Informatics, University of Bradford, United Kingdom*
R.J.Fretwell@Bradford.ac.uk

Keywords: computers networks, Quality-of-Service routing, virtual sub-networks, series parallel graphs, packing of spanning trees, polyhedra.

Abstract: Given a computers network G=(V, E) and a bandwidth vector on its links, the Minimizing Unused Bandwidths Problem (MUB) is to find a minimum number of spanning trees sub-networks working as parallel services and a maximum possible bandwidth for each one such that the remaining total unused bandwidth is minimum. MUB is an open problem.
In this paper a polynomial algorithm is presented to solve MUB problem in a special kind of network topology called series parallel networks. A comparison between the presented algorithm MBSTP and the two other algorithms PLSP and MSTP is presented toward the end of the paper to show the effectiveness of the presented algorithm in designing a computer network.

1. Introduction

Sets and their characteristic vectors will not be distinguished. We refer to Bondy and Murty book (Bondy and Murty 1982) and to Schrijver book (Schrijver 1986) respectively about graph theory and polyhedra terminology and results.

A computers network can be represented by a graph G=(V, E) where V is the set of nodes (computer devices) and E is the set of edges (links). The metric of bandwidths can be represented by a vector b on the links set E. Each service we want to provide to customers is a pair of a spanning tree T of G because it should connect all nodes and be minimal for this property in order to optimize resources, and a fixed bandwidth b_T for all links of T because if it is not the case some bandwidths will be not used.

The remaining of the paper is organized as follows: in section 2, the definition of series parallel networks is given in detail. Section 3 introduces the problem to solve in detail. In section 4, the presented algorithm and its mathematical motivation is given. Section 5 gives the simulation results and analysis. In section 6, the conclusion is given to show the importance of the work.

2. Series Parallel Network definition

Let's give some definitions:

A series extension of a network consists of creating a node on a link between its two adjacent nodes. A parallel extension of a network consists of creating a new link parallel to another existing link. A series parallel network is a graph obtained by starting from one link and applying recursively series and/or parallel extensions. For any series parallel graph G, we denote by s(G) the number of series extensions that we have used to create G. For any vector x defined on the links set E and any subset of links H, $x(H) = \sum_{e \in H} x(e)$. The x-maximum spanning tree is a spanning tree T of G such that x(T) is maximum. This problem is polynomial (Kruskal 1956) and we will use Prim or DJP algorithm (Gabow et al. 1986) to solve it when needed. The x-maxBalanced spanning tree (MBST) is a spanning tree T of G such that the difference between the maximum node degree and the minimum node degree is minimum. If the network is Hamiltonian the balanced spanning tree is a Hamiltonian path.

3. Statement of the Problem

We are interested to study the following problem in series parallel networks:

Minimizing Unused Bandwidths in Computers Networks:

Instance: Given a network G=(V, E) and an integer bandwidths vector b on E.

Question: Find spanning tree sub-networks (pre-computed parallel services) $T_1, \ldots, T_k$ and corresponding integer bandwidths $b_1, \ldots, b_k$ such that:

i) The unused bandwidths $b-(b_1T_1+b_2T_2+ \ldots +b_kT_k)$ is minimum ;
ii) Each sub-network T_j has a maximum possible bandwidth b_j .

We denote by bMax(G, b) the maximum total bandwidths for parallel services and bMin(G, b) the minimum total unused bandwidths. If there is no confusion, we use the notations bMax and bMin. Note that: x(E)=[bMax (|V|-1)]+bMin.

It is not difficult to see that ii) is equivalent to minimizing the number of sub-networks (parallel services) with maximum bandwidth for each one.

If we choose b as a sum of spanning tree sub-networks, that is bMin is zero, then the problem is known as the integer cover conjecture for spanning trees and it is an open problem (Chaourar 1993, Chaourar 2002, de Pina and Soares 2003). But it is polynomial for a special kind of topology network called series parallel networks (Chaourar and Mahlous 2006).

This problem has applications in Quality of Service routing (Chaourar and Mahlous 2006) and an overview on the subject can be found on several references (Apostolopoulos et al. 1999, Breslau et al. 1993, Garcia-Luna-Aceves 1989, Gawlick et al. 1995, Gibbens et al. 1988, Guerin et al. 1996, Guerin and Orda 1999, Jaffe 1984, Jin and Nahrstedt 2002, Lee et al. 1995, Ma and Steenkiste 1997a, Ma and Steenkiste 1997b, Matta and Shankar 1995, Matta and Shankar 1996, Partridge and Castineyra 1993, Pornavalai et al. 1997, Rosen et al. 1991, Salama et al. 1997, San and Langandorfer 1996, Steenstrup 1995, Van Mieghem and Kuipers 2003, Wang and Crowcroft 1996).

The main result of this paper is a polynomial algorithm which gives "in practice", that is under simulations, an exact solution and solves the problem for series parallel networks and any integer bandwidths vector.

4. Description of MBSTP

We will now describe MBSTP and MSTP:

MBSTP (respectively MSTP) algorithm:

Input: A computer networks G=(V, E) and a bandwidths vector b.

Phase 1: Find T=b-maxBalanced (respectively b–maximum) spanning tree in G using the there below algorithm (respectively using Prim's algorithm).

If there is no spanning tree then Go to Phase 4.

Phase 2: Find b_T=b–minimum link in T.

Phase 3: Compute new bandwidths vector $b:=b-b_T T$.

Go to Phase 1.

Phase 4: Compute unused bandwidths bMin=b(E).

So MBSTP and MSTP work in a same manner with the difference that the MBSTP uses b-maxBalanced spanning trees (using the there below algorithm) and MSTP uses b-maximum spanning trees (using Prim's algorithm).

Note that the complexity of MBSTP (respectively MSTP) is at most the complexity of finding a maximum spanning tree in the network. Since we have use Prim's algorithm for this purpose, the worst case of the heuristic is O(|V| log |V|) (Prim 1957). The memory complexity is O(|E|) because we can store series parallel networks by the way they are constructed starting from one edge and indicating the nature of each extension (parallel or series: 0 or 1 for example).

Our motivation for using MBSTP is the following result due to Chaourar (2002):

Theorem: If the network is series parallel then MBSTP solves MUB in a polynomial time.

We will give now a polynomial time algorithm to find a b-maxBalanced spanning tree:

Description of the algorithm:

Given a graph G=(V, E) and a weight function w defined on E.

We suppose that the edges are sorted according to their decreasing weights:

$w(e_1) \geq w(e_2) \geq \ldots \geq w(e_m)$.

$E_1=E$;

For j=1, 2, ..., m

While $G_j=(V, E_j)$ is connected do

Find T_j the w-maximum spanning tree in G_j (using Prim's algorithm)

$d_j=w(e_j)-\min\{w(e) : e \text{ belongs to } T_j\}$

$E_{j+1}=Ej-\{e_j\}$

End While

End For

The w-maximum balanced spanning tree is the best balanced from the T_j's, j=1, …, m.

Running Complexity: O(|E| |V| log |V|)

Theorem: The previous algorithm gives the w-maximum balanced spanning tree.

Proof:

Let T be the w-maximum balanced spanning tree, $e=e_k$ and $f=e_t$ be respectively the maximum and the minimum w-edges of T, d=w(e)-w(f) and $F=\{e_k, e_{k+1}, \ldots, e_t\}$.

It is clear that T is the w-maximum spanning tree in (V, F). So it is also the w-maximum spanning tree in G_k. Then $T=T_k$ and we are done.

Another algorithm (PLSP) was used firstly by Chaourar and Mahlous (Chaourar and Mahlous 2006) when the total of unused bandwidths is zero. It gives an exact optimal solution. We will use it in our simulation with MBSTP and MSTP.

5. Simulation Results and Analysis

In order to do simulations, we have implemented a random generator of series parallel networks and bandwidths vectors from nodes number n=5 to n=25 and links number m=5 to 38. To avoid isolated results, we generate, for fixed n and m, at least 5 networks and bandwidths vectors. In order also to uniform, we have generate bandwidths within the range 20–40.

We will use the three algorithms PLSP, MBSTP and MSTP to do two simulations. In the first simulation, we will use series parallel networks with bMin=0 and the three algorithms mentioned above are used for comparison. Since PLSP was proved optimal, then it was used as a reference. The second simulation is used with general networks with bMin≠0 and the two algorithms MBSTP and MSTP are used for comparison.

The results for the two simulations are shown in Tables 1 and 2. For each algorithm, we give the (rounded mean) total of unused bandwidths.

Table 1. bMin=0

n	m	PLSP	MBSTP	MSTP
5	9	0	0	0
7	12	0	0	5
10	17	0	0	4
13	19	0	0	2
15	23	0	0	10
17	26	0	0	5
20	28	0	0	14
23	34	0	0	1
25	38	0	0	2

It is clear from the first simulation results that MBSTP is optimal which ensures the results obtained in the theory.

Table 2. bMin≠0

n	m	MBSTP	MSTP
5	9	5	6
7	12	2	10
10	17	12	25
13	19	4	4
15	23	10	12
17	26	1	6
20	28	6	35
23	34	23	26
25	38	15	24

From the second simulation represented by Table 2, we found that MBSTP is absolutely better than MSTP which confirm our idea that MBSTP is optimal in "practice".

6. Conclusion and Future Work

Actually, we can use MBSTP in series parallel extensions of a network to know the optimal solution.

From the simulation given above it was found that the use of MBSTP algorithm will give a direction when designing a computers network or when we make extension of a current network to a bigger one.

Further directions for our problem can be:

- proving mathematically the optimality or near optimality of MBSTP ;
- studying more general classes of graphs topology;
- considering other type than spanning tree for virtual sub-networks ;
- and considering heuristics for general cases based on MBSTP.

References

Apostolopoulos, G., Kama, S, Williams, D., Guerin, R., Orda, A., and Przygienda, T. 1999. "QoS Routing Mechanisms and OSPF Extensions", RFC 2676, August 1999. Available at: http://www.faqs.org/rfcs/rfc2676.html.

Bondy, J. A., and Murty, U. S. R. 1982. "Graph Theory with Applications", Elsevier, New York, 1982.

Breslau, L., Estrin, D., and Shenker, S. 1993. "A Simulation Study of Adaptive Source Routing in Integrated Services Networks", Technical Report, University of South Carolina, Computer Science Department, USA, September 1993.

Chaourar, B. 1993. "Hilbert Bases and Combinatorial Applications", PhD thesis, Joseph Fourier University, Grenoble, France, 1993.

Chaourar, B. 2002. "On Greedy Bases of Matroids", European Journal of Combinatorics, 23, 2002, 769-776.

Chaourar, B., and Mahlous, A. R. 2006. "Optimizing QoS Resources in Series Parallel Networks", Saudi 18th national Computer Conference, March 2006, Riyadh, Saudi Arabia.

Gabow, H. N., Galil, Z., Spencer, T. and Tarjan, R. E. 1986. "Efficient algorithms for finding minimum spanning trees in undirected and directed graphs", Combinatorica, 6(2), 1986, 109-122.

Garcia-Luna-Aceves, J. 1989. "A Unified Approach to Loop-Free Routing Using Distance Vectors or Link States", Proceeding of Association for Computing Machinery Special Interest Group (ACMSIG) on Data Communication Sigcomm'89,

Texas, USA, September 1989. Available at:
http://portal.acm.org/citation.cfm?id=75268&coll=portal&dl=ACM.

Garey, M., and Johnson, D. 1979. "Computers and Intractability: A Guide to the theory of NP-Completeness", Freeman and Co., New York, 1979.

Gawlick, R., Kalmanek, C., and Ramakrishnan, K. G. 1995. "On-line Routing for Permanent Virtual Circuits", Proceedings of INFOCOM'95 14th Annual Joint Conference of the IEEE Computer and Communications Societies, Boston, MA, USA, 1, 278-288.

Gibbens, R. J., Kelly, F. P., and Key, P. B. 1988. "Dynamic Alternative Routing - Modelling and Behaviour", Proceedings of 12th International Teletraffic Congress, Bonatti editors, Elsevier, Turin & Amsterdam, 1988, 1019-1025.
Available at: http://www.statslab.cam.ac.uk/~frank/PAPERS/darmb.html.

Guerin R., Orda, A., and Williams, D. 1996. "QoS Routing Mechanisms and OSPF Extensions", IETF Internet Draft, November 1996. Available at: citeseer.ist.psu.edu/guerin96qos.html.

Guerin, R. and Orda, A. 1999. "QoS-based Routing in Networks with Inaccurate Information: Theory and Algorithms", IEEE/ACM Transactions on Networking, 7(3), 1999, 350-364.

Jaffe, J. 1984. "Algorithms for Finding Paths with Multiple Constraints", IEEE Networks, 1(14), 1984, pp. 95-116.

Jin, J. and Nahrstedt, K. 2002. "QoS Service Routing for Supporting Multimedia Applications", Technical Report, University of Illinois at Urbana-Champaign, Department of Computer Science, USA, November 2002. Available at: citeseer.ist.psu.edu/701851.html.

Kruskal, J. B. 1956. "On the shortest spanning tree of a graph and the traveling salesman problem", Proceedings of the American Mathematical Society, 7, 48-50.

Lee, W. C., Hluchyi, M. G. and Humblet, P. A. 1995. "Routing subject to Quality of Service Constraints in Integrated Communication Networks", IEEE Networks, 9(4), 1995, 46-55.

Ma, Q. and Steenkiste, P. 1997a. "Quality of Service Routing for Traffic with Performance Guarantees", Proceedings of IFIP'97 5th International Workshop on Quality of Service, New York, USA, May 1997, 115-126.

Ma, Q. and Steenkiste, P. 1997b. "On Path Selection for Traffic with Bandwidth Guarantees", Proceedings of IEEE International Conference on Network Protocols, Atlanta, GA, USA, October 1997.
Available at: citeseer.ist.psu.edu/ma97path.html.

Matta, I. and Shankar, A. U. 1995. "Type of Service Routing in Datagram Delivery Systems", IEEE Journal on Selected Areas in Communications, 13(8), 1995, 1411-1425.

Matta, I. and Shankar, A. U. 1996. "Dynamic Routing of Real Time Virtual Circuits", Proceedings of IEEE International Conference on Network Protocols, Columbus, OH, USA, 1996, 132-139.
Available at: citeseer.ifi.unizh.ch/matta96dynamic.html.

Partridge, C., Castineyra, I. 1993. "Routing Flows in an Internet", unpublished paper, April 1993.

de Pina, J. C. and Soares, J. 2003. "Improved Bound for the Carathéodory Rank of the Bases of a Matroid", Journal of Combinatorial Theory, Series B, 88(2), 2003, 323-329.

Pornavalai, C., Chakraborty, G., and Shiratori, N. 1997. "QoS Based Routing Algorithm in Integrated Services Packet Networks", Proceedings of 1997 International Conference on Network Protocols (ICNP'97), Atlanta, GA, USA, October 1997, 167-174.

Pulleyblank, W. R. 1983. "Polyhedral Combinatorics", in Bachem, A., Grotschel, M. and Korte, B. (Eds.), "Mathematical Programming, the State of Art", Springer-Verlag, Berlin, 1983, 312-345.

Rosen, J. B., Sun, S. Z. and Xue, G. L. 1991. "Algorithms for the Quickest Path Problem", Computers and Operations Research, 18, 1991, 579-584.

Salama, H. F., Reeves, D. S. and Viniotis, Y. 1997. "A Distributed Algorithm for Delay-Constrained Unicast Routing", Proceedings of INFOCOM'97 16th Annual Joint Conference of the IEEE Computer and Communications Societies, Japan, April 1997, 84-91.
Available at: http://citeseer.ist.psu.edu/article/salama97distributed.html.

San, Q. and Langandorfer, H. 1996. "A New Distributed Routing Algorithm with end-to-end Delay Guarantee", Proceedings of 2nd Protocols Multimedia Systems Workshop, October 1995, 452-58.

Steenstrup, M. 1995. "Routing In Communications Networks", Prentice Hall, Englewood Cliffs, NJ 07362, USA, 1995.

Van Mieghem, P. and Kuipers, F. A. 2003. "On the Complexity of QoS Routing", Computer Communications, 26(4), 2003, 376-387. Available at: http://wotan.liu.edu/docis/dbl/cocomm/2003_26_4_376_OTCOQR.htm.

Wang, Z. and Crowcroft, J. 1996. "QoS Routing for Supporting Resource Reservation", IEEE Journal of Selected Areas in Communications, 14, 1996, 1228-1234.