Using Sine of the Phase Function in Computer-aided Frequency-domain Performance Optimization of Linear Networks

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Abstract. This paper concerns itself with the interesting problem of how to tame the discontinuities inherent in the phase output function of many frequency-domain linear networks to enable incorporating this function into the design specifications in a computer-aided optimization exercise. To surmount this problem, a new technique is described and a report is made of how this is integrated into a previously-reported CAD package of programs. An example is devised to demonstrate the reliability of the proposed method, and results of the optimization exercise are presented.

Nomenclature

P	:	Penalty function
p	:	(k x 1) vector of designable parameters
Ω	:	Design-frequency range
ω	:	Any design frequency point in Ω
W^{φ}	:	Weighting function associated with the phase of the output voltage gain
φ	:	Actual phase output function
S^{φ}	:	Specified phase output function
W^{G}	:	Weighting function associated with the modulus of output voltage gain
G	:	Modulus of the actual voltage gain
S^{G}	:	Modulus of the specified voltage gain
g	:	(k x 1) gradient vector

$R(\omega,p)$:   Real part of the actual voltage gain

$I(\omega,p)$:   Imaginary part of the actual voltage gain

Introduction

For some linear electrical networks, phase output functions which exceed 2π radians in the range of their operating frequencies are not well-behaved. These functions present a challenging difficulty in the face of a designer, who aims at using them as criteria in the course of design optimization of such networks. This statement is basically true because of the serious irregularities, in the value of the used penalty function, which arise each time the value of the phase output function crosses the frontier between zero and 2π radians from either side. Such discontinuities in the phase output function of the optimized network are difficult to deal with. They do not only hinder the convergence of the optimization process when search (nongradient) methods are employed, but also deprive the process of accurate gradient information when it is critically needed.

In order to circumvent these dangers, which threaten the convergence of the optimization process of the aforementioned class of networks to phase output specifications in the frequency domain , we propose the replacement of the nonprincipal phase angle (NPA) with its sine (SNPA) in calculations of the penalty function.

The optimization routine in our CAD package is a gradient, unconstrained quadratic-programming minimization algorithm based on Fletcher [1], Gill and Murray [2], and Lanca [3].

The proposed SNPA technique is now implemented in our previously-reported CAD package of programs [4,5] intended to minister to the need of such an interactive design package on a personal micro-computer. So far as our CAD package stands, its user is offered the following three options upon using the phase of the output voltage-gain function as a performance measure.

a)   The principal phase angle (PA); (0 to $\pm$ π rad.)

b)   The nonprincipal phase angle (NPA); (0 to 2π rad.)

c)   Sine of the nonprincipal phase angle (SNPA)

The availability of these options increases the flexibility of our design package considerably. To exploit this flexibility, a preliminary analysis run is mandatory to decide which of these options may be more promising in the intended optimization exercise.

Moreover, the user is enabled amidst the optimization exercise to switch from one option to another simply by activating a suitable sense switch simulated by a software flag. This is a convenient arrangement, by virtue of which the user may choose the most suitable option according to his assessment of the intermediate results of the specific

optimization exercise. The theoretical foundations of the proposed SNPA technique of using sine of the phase of voltage-gain function in performance optimization, are exposed in the next section. Details and results of a devised design example are then given, and some concluding remarks are drawn.

Theory

The nodal analysis (NA) technique [6] provides a relatively simple formulation of the network equations, especially when a small machine is used. This is largely due to the restricted availability of core storage for program coding in mini-and micro-computers. The nodal analysis, however, is strictly valid only for ac and dc analyses of linear(ised) networks. But since we are interested in linear(ised), lumped, time-invariant networks in the frequency domain, we are saved the need of looking for a formulation technique other than the nodal analysis. If this need was there, the modified nodal analysis (MNA) technique [7, 8, 9] is available for consideration. Nevertheless, the fact that no difference is there between the NA and MNA techniques, as far as the class of networks addressed by the current package for their analyses and CAD purposes is concerned, nullifies the need to the MNA formulation technique.

Thus, for two-port linear(ised) networks that can be described, or modelled, using RLC elements, voltage-controlled current sources and unbalanced, grounded, three-terminal transmission lines, an algorithm for its analysis and optimization in the frequency domain has been programmed in FORTRAN-77.

The analysis program of our CAD package employs LU factorisation [10] together with the adjoint network approach to sensitivity calculations for networks including hierarchically-nested subnetworks [11,12]. The resulting package of CAD programs has been successfully used in many optimization exercises and some of these results are already published.

Yet, the package needed the development and implementation of the new SNPA technique to cope with situations outlined in the introduction of this contribution. Computer-aided optimization is mainly a recursive process which proceeds, in the multi-dimensional parameter space, by successive adjustments of the network elements in a direction that minimises the penalty function. Here, the network elements are those pertaining, or pertinent, to the designable parameters, and the chosen penalty function is a form of the well-known weighted least-squares error function given by

$$P(p) = \sum_{\omega \in \Omega} \frac{\Delta\omega}{2} w^{\varphi}(\omega) \, |\varphi(\omega, p) - S^{\varphi}(\omega)|^2 \; \ldots \ldots (1)$$

when the design specifications are in terms of the phase output function; or by

$$P(p)=\sum_{\omega\in\Omega}\frac{\Delta\omega}{2}w^{G}(\omega)\ |G\ (\omega,p)\ -S^{G}(\omega)|^{2}\ \dots\ (2)$$

when the design specifications are in terms of modulus of the voltage gain. Equations (1) and (2) may be combined into one formula to be used when multiple-criterion performance optimization [5] is to be carried out, and the performance criteria used are both phase and modulus of the voltage-gain function. The combined formula in this case may be given by the following equation

$$P(p)=\frac{1}{2}\sum_{\omega\in\Omega}w^{\varphi}(\omega)\ |\varphi\ (\omega,p)\ -S^{\varphi}(\omega)|^{2}+w^{G}(\omega)\ |G(\omega,p)\ -S^{G}(\omega)\ |^{2}\dots\ (3)$$

Note that $\Delta\omega$ in Eq. (3) above is dropped because the discrete frequency points all over Ω are equispaced in our algorithm, whether Ω is divided linearly or logarithmically. This disregard of $\Delta\omega$ does not affect the convergence speed, although P(p) is thus devalued by a factor of $1/\Delta\omega$.

When the proposed SNPA technique is considered, Eq.(1) may be reintroduced as

$$P(p)=\frac{1}{2}\sum_{\omega\in\Omega}w^{\varphi}(\omega)\ |\sin(\varphi\ (\omega,p))\ -\sin(S^{\varphi}(\omega)|^{2}\ \dots\ (4)$$

Since the package uses a gradient-based minimisation method, the gradient vector in our case is calculated as

$$g_{i}=\frac{180}{\pi}\sum_{\omega\in\Omega}w^{\varphi}(\omega)\ \frac{\sin(\varphi\ (\omega,p))\ -\sin(S^{\varphi}(\omega))}{R^{2}(\omega,p)+I^{2}(\omega,p)}\ .\psi\ (\omega,p)\ \dots\ (5)$$

Here, $\psi\ (\omega,p)$ is given by

$$\psi\ (\omega,p)=\cos(\varphi(\omega,p)).\left\{R(\omega,p)\ \frac{\partial I(\omega,p)}{\partial p_{i}}-I(\omega,p)\ \frac{\partial R(\omega,p)}{\partial p_{i}}\right\}\ \dots\ (6)$$

where, $i = 1, 2, \dots, k$, in both Equations (5) and (6), and k is the number of the designable parameters used in the specific optimization exercise. In order to alleviate the burden on the user of this CAD package, he is asked to define the performance

specifications in the SNPA case exactly as in the other PA and NPA cases. This means that the user is required to type the value of each phase output specification point in degrees. This value is allowed either as a spot-on value or as upper and lower bounds at each frequency point selected in Ω. The computer takes over therefrom, and calculates the sine function of both the specified and actual phase output functions to be used in calculating Equations (4- 6).

Application

The above mentioned package of programs, for computer-aided linear-network analysis and optimization in the frequency domain, has been augmented to incorporate the SNPA technique.

The new package has been put to trial using a devised network consisting of a 0.5 meter-long pair of unbalanced, grounded, transmission lines with an L-C tank circuit connected across its receiving-end terminals, as shown in Fig.1.
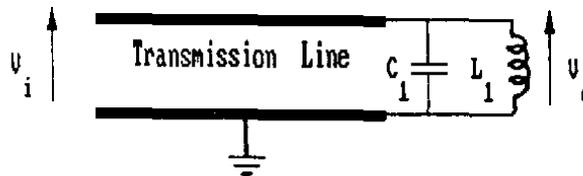
Fig. 1. Circuit example to test the SNPA technique

Although being topologically simple, this network is considered a convenient test vehicle for the presented SNPA technique since its phase output function has been found to assume values between 6π and π radians over the design frequency range of 1.0 - 5.0 MHz. Fig.2 shows both the actual and specified phase output functions at the start of the optimization exercise, while in Fig.3 sine of both phase output functions is shown. The error, or penalty, function here is proportional to the square of the hatched area in Figs. 3, 4, and 5. The two-stage progress of the optimization exercise is represented in Figs. 4, 5, and 6. Figure 6 depicts the phase output functions while Figs. 4 and 5 show their sine functions at the start and end of each optimisation stage.
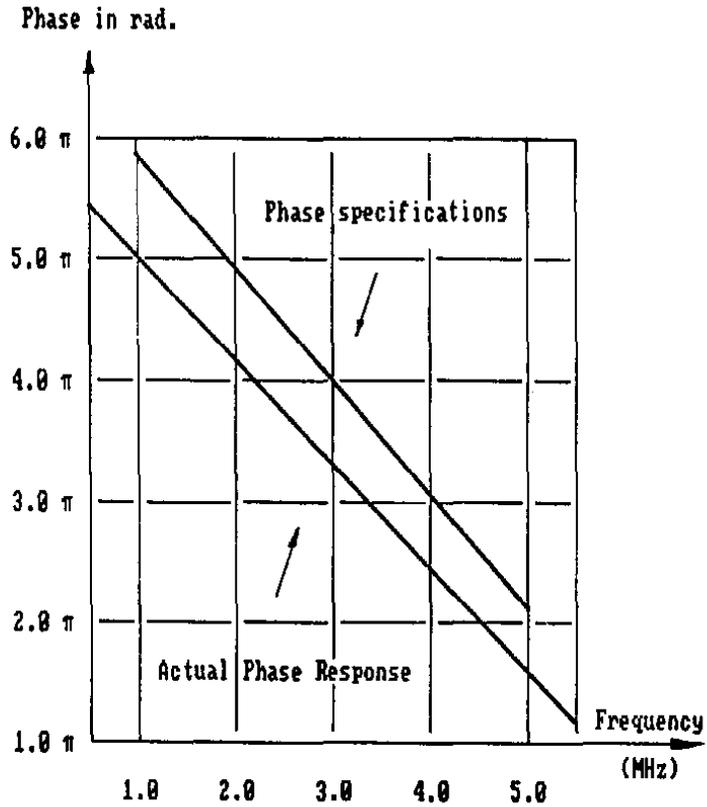
Fig. 2. Actual speified phase output functions at start of the design exercise
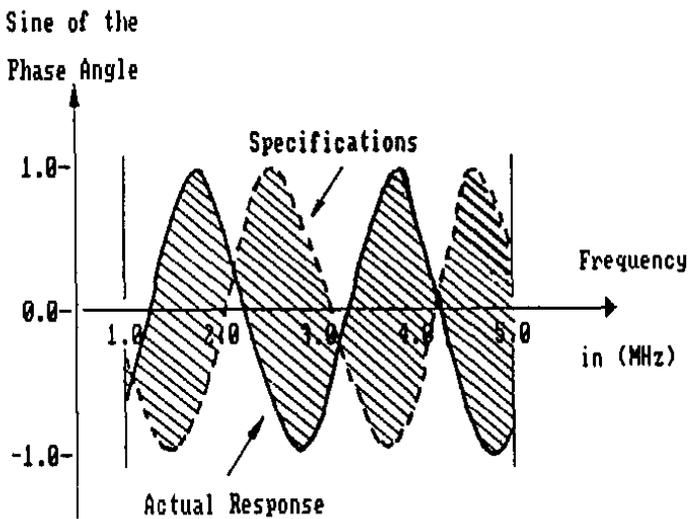
Fig. 3. Sine of the phase functions of Fig. 2 [P(p) is proportional to the hatched area]

Sine of the

Phase Angle

Specifications

1.0-

0.0-

1.0   2.0   3.0   4.0   5.0

Frequency

in (MHz)

-1.0-

Actual Response
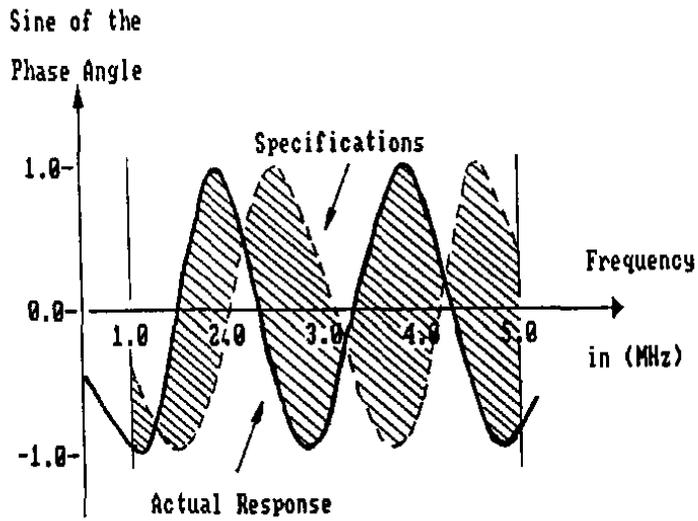
Fig. 4. Progress of the first stage of design

Sine of the

Phase Angle

Specifications

1.0-

0.0-

1.0   2.0   3.0   4.0   5.0

Frequency

in (MHz)

-1.0-

Actual Response
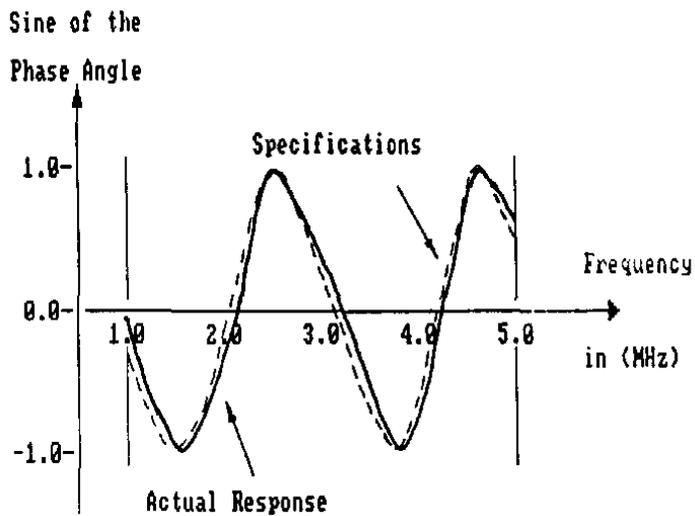
Fig. 5. Progress of the second stage of design

Phase in rad.
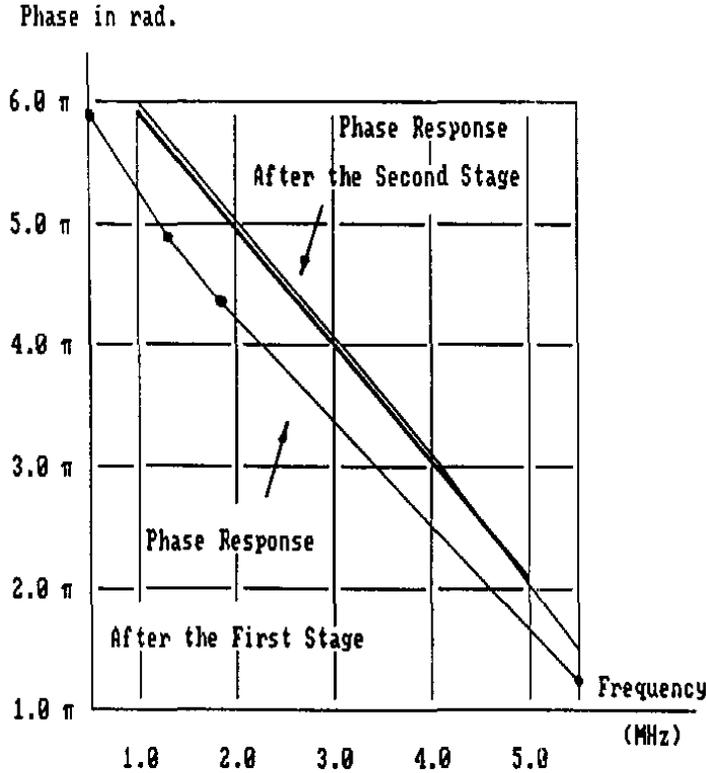
Fig. 6. Progress of the 2-stage CAPO exercise

Values of the distributed circuit coefficients of the used transmission line are given in Table 1. The two coefficients L and C are very strictly independent of frequency over the entire range of frequencies from tens, or hundreds, of KHz up to microwave frequencies of several GHz. The other two coefficients of the transmission line, i.e. R and G, on the other hand, increase steadily and considerably with increasing the frequency [12]. However, these two coefficients are considered "constants" over the range of frequencies used for this optimization exercise.

Table 1. Primary constants of the T.L. used in the exercise

R =	100.0 Ohm/meter
L =	1.0 milli-Henry/meter
G =	0.01 Siemens/meter
C =	1.0 nano-Farad/meter

The objective of this design exercise has been swiftly achieved in two interactive stages, with only 1 interaction and 3 function evaluations each. Table 2 summarizes the statistics of this interactive design example, where both L_1 and C_1 of Fig.1 has been chosen as the designable parameters.

Table 2. Progress of the CAPO exercise using SNPA technique

	Start value	Value at end of stage		Units
		1	2	
Design Par. L_1	100.0	99.44	97.73	*H
Design Par. C_1	1000.0	367.34	2.83	pF
Penalty Func. P(p)	(Start/End)	654.06/593.26	593.26/4.0	-
% Reduction in P(p)	-	9.2	99.4	-
No.of Interactions	-	1	1	-
No.of Func. Calls	-	3	3	-

Concluding Remarks

The proposed SNPA technique is evidently very efficient in handling the problem of accomodating "wild" phase responses, like the one exhibited by the network example used in this contribution, into the penalty function.

It may be fair, however, to say that when this SNPA technique is applied to network design problems in which the phase response is well within the range from 0 to $\pm \pi$ radians, convergence to the solution point in the parameter space has been observed to be comparatively slow. Sometimes, even, the design exercise ended in failure. Fortunately enough, the other two methods, using the phase output function directly, are there to be employed.

When added to the work reported in [4] and [5], the work whose theory and application are highlighted in this contribution, heralds an integrated algorithm for multiple-criterion performance optimization of linear(ised) electronic networks to both modulus and phase of the output voltage performance function in the frequency domain. This effort is fundamental for furnishing the CAD area in the small-machine environment with both efficient and reliable techniques for analysis and optimization of this class of networks at a very economical cost.

The package is written in a highly-modular fashion using FORTRAN-77 coding language with an extensive documentation. This fact makes the package portability very convenient for those who are interested in this field of activity. On the other hand, this modularity has been proven to be a relief when modifications of the code were deemed necessary.

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استخدام جيب زاوية الطور كدالة أداء للحصول على التصميم الأمثل للدوائر الخطية في حيز التردد بمساعدة الحاسوب

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ملخص البحث . يختص هذا البحث بمعالجة مشكلة عدم اتصال دوال زاوية الطور في كثير من الدارات الخطية حتى يمكن استخدامها في التصميم الأمثل لهذه الدارات باستخدام الحاسوب .
وقد تم اقتراح طريقة للتغلب على هذه المشكلة، ويصف البحث هذه الطريقة ويقرر كيف تم دمجها في مجموعة من برامج استخدام الحاسوب في التصميم الأمثل . ولقد استخدمت كمثال في التصميم دارة تم اختيارها لتوضح مدى موثوقية الطريقة المقترحة، وتم عرض نتائج هذا التصميم الأمثل .