J. King Saud Univ., Vol. 11, Comp. & Info. Sci., pp. 69-83 (A.H. 1419/1999)

Queue Size Distribution of an ATM Switch with VBR Video Traffic

Nasser-Eddine Rikli
Department of Computer Engineering, College of Computer & Information Sciences
King Saud University, P.O. Box 51178, Riyadh 11543, Saudi Arabia

(Received 11 June 1996; accepted for publication 28 October 1997)

Abstract. Many analytical tools have been devised to analyze CBR video coders, however only few have been applied to the VBR case. In this paper, we present a semi-Markov process model for approximating the stochastic equilibrium queue length distribution for VBR video sources multiplexed onto a constant capacity ATM channel. The sources are assumed to have two rates, and generate cells that pass through pre-buffers before joining a common queue on a FIFO basis. The model is based on a state process defined by the average rate of change of the queue length as a function of the number of active mini-sources of each type, and is solved using matrix-geometric techniques. An Access Control mechanism uses these results inside its policing function to control access to the network at call setup. Numerical results are obtained for various conditions, and compared to the ones obtained from simulation and fluid-flow models, and good agreement was achieved.

Keywords: VBR Video; Semi-Markov; Matrix-geometric; ATM; Queue distribution; Call admission control.

1. Introduction

Statistical multiplexing of video signals has proven to be a difficult problem when it comes to bandwidth-allocation and traffic-control [1, 2, and 3]. Depending on the time when a decision is to be made, these difficulties may be classified into two groups:

1. *At call setup:* when a new source requires service, it has to be decided whether to accept it or reject it, whether to allocate for it the requested bandwidth or decrease it to an acceptable level. This, will depend on the expected effect of adding the new source on the ATM link, i.e. if accepting a new source will degrade the quality of service (QoS) of any customers being already serviced;
2. *During connection:* a *congestion control mechanism*, based on some *policing function*, has to be continuously monitoring some appropriate traffic parame-

ters (for each source) in order to detect deviations from the expected behavior, and act according to a certain control policy.

Our goal here, is to deal with the first difficulty. Many papers have addressed this subject either directly or indirectly. In [1], the problem of bandwidth allocation and traffic monitoring for ATM networks is addressed. The solution is based on forcing the video codecs to transmit only with a limited number of rates. The transmission capacity for a new connection is allocated according to the equilibrium distribution, while its long-term behavior is controlled by monitoring the holding and recurrence times of the corresponding Markov chains. Other works may be found in [3, 4, and 5].

The latter situation did not yet get robust implementations, as it is very difficult to keep up pace with the very high speed channel. In [2], the role of feedback in flow-control for high-speed broadband ISDN (B-ISDN) using ATM is studied. A single bottleneck switch that uses a threshold mechanism based on its buffer level, sends periodic feedback in a form of a single congestion indicator bit. The source receives this bit after a fixed delay and uses it to switch between two rates (similar to our source model). Explicit solution for the stationary distribution of this system is obtained using spectral decomposition.

In this paper, we will apply *Matrix-Geometric* computational techniques to gather relevant statistical information data on the current status of the network link, and predict its behavior in case a new source requests a connection Fig.1. The only requirement assumed, in the prediction process, is that the new source has known statistical characteristics.

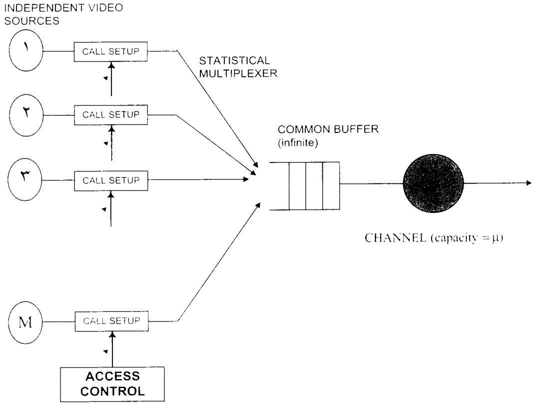

Fig. 1. Controlled statistical-multiplexer feeding an infinite buffer and a single server with fixed capacity.

This computational technique, i.e. *Matrix-Geometric*, was first introduced by Neuts in 1979 [6]. Then, it has been utilized extensively in a wide spectrum of applications. In the case of packetized voice, it has been used to approximate the stochastic equilibrium queue length distribution for a packet voice statistical multiplexing system [7]. It has been also applied to bandwidth allocation strategies for B-ISDN networks [8], to integrated video, voice, and data communications systems [4], as a numerical method for ATM models [5], and to multicast switching [8].

2. Model Description

2.1 Source model

Each source is assumed to be operating in either one of two modes and characterized with the following parameters:

- A_h: high-rate generation rate to differentiate between the two modes of operation Fig.2 b;
- N : number of quantization levels in either mode;
- A_l : low-rate generation rate representing the magnitude of the quantization levels Fig.2 a.

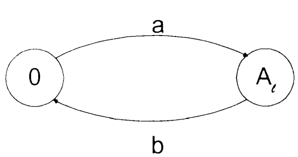

Fig. 2a. Low-rate mini-source.

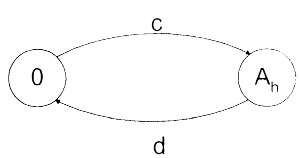

Fig. 2b. High-rate mini-source.

For the resulting aggregate source model, let's denote by:

- M : the number of independent video sources;
- N_1 $(= N\,M)$: aggregate mini-sources of low-rate;
- N_2 $(= M)$: aggregate mini-sources of high-rate.

Each active mini-source generates packets with exponentially distributed lengths, independently of the other mini-sources, and according to a Poisson process. The parameters a and c represent the exponential rates with which the low-rate and the high-rate mini-sources turn ON, and the parameters b and d are the corresponding rates for turning OFF.

When active, a low-rate mini-source delivers A_l *cells/sec*, and a high-rate mini-source delivers A_h *cells/sec*. These cells are then statistically multiplexed onto an ATM link forming an aggregate rate $\lambda_{jk} = j\,A_l + k\,A_h$ Fig.1. The ATM link is assumed to have a known constant capacity C *cells/sec* (i.e., one cell is removed from the queue, while non-empty, every $1/C$ *sec*).

Matching equations

The basic statistical measures (mean, variance, autocorrelation) of the video source model described above, may be derived in terms of the mini-source parameters. Then, a set of measurements obtained in [9] are used to match the model parameters to these values. A complete description of this process may be found in [10], and is omitted here for brevity purposes.

2.2 Process description

A continuous time Markov chain process S may be defined by the states (i,j,k), where:

i: the number of cells in the queue;

j: the number of active high-rate mini-sources;

k: the number of active low-rate mini-sources.

By assuming that C cannot be written as a linear combination of the phase parameters j and k (since $\lambda_{jk} = j\,A_l + k\,A_h$), two cases arise:

1. underload ($\lambda_{jk} < C$): the queue (if non-empty) decreases at an average rate of $(C - \lambda_{jk})$ *cells/sec*, or equivalently the queue decreases by one cell every $1/(C - \lambda_{jk})$ *sec*;
2. overload ($\lambda_{jk} \geq C$): the queue increases at an average rate of $(\lambda_{jk} - C)$ *cells/sec*, or equivalently the queue increases by one cell every $1/(\lambda_{jk} - C)$ *sec*.

A *phase-process* P may be derived by defining the continuous time Markov chain generated by using as states the number of active low-rate and high-rate mini-sources, i.e. the pair (j,k) [6] and [11].

2.3 System model

It can be shown, in a manner similar to [7], that process S may be represented by a Markov Chain *Embedded* at instants of:

- Phase Change;
- Queue Increments;
- Queue Decrements.

Such process, is also known as *Semi-Markov* process. Its state diagram is shown in Fig.3. The expected sojourn time m_{ijk} in any process state (i,j,k) will depend only on the phase state (j,k), and the queue length distribution can be obtained easily once the equilibrium state probabilities p_{ijk} are determined [7].

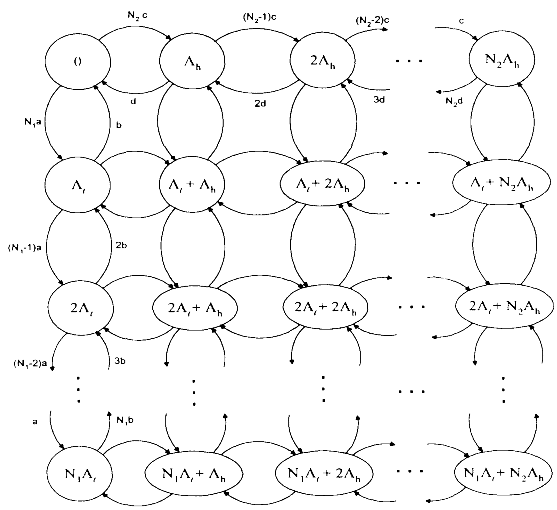

Fig. 3. State-transition-rate diagram for the aggregate source model.

3. Queue Distribution

3.1 Problem formulation

From renewal theory [12], it is well known that:

$$p_{ijk} = \frac{q_{ijk}\, m_{ijk}}{\sum_{l=0}^{\infty} \sum_{m=0}^{N_2} \sum_{n=0}^{N_1} q_{lmn}\, m_{lmn}} \tag{1}$$

where:

p_{ijk} = the equilibrium state probabilities;
q_{ijk} = the equilibrium probabilities of the Embedded Markov Chain;
m_{ijk} =the expected sojourn times.

Let us denote by $P[l,m,n \rightarrow i,j,k]$, the transition probabilities of the Embedded Markov Chain, which will be the $(m, n)^{th}$ element of the column vector $P^{jk}_{l\rightarrow i}$. From Markov chain theory [13], it is well known that:

$$q_{ijk} = \sum_{l=0}^{\infty} \sum_{m=0}^{N_2} \sum_{n=0}^{N_1} q_{lmn}\ P[l,m,n \rightarrow i,j,k] \tag{2}$$

Let us denote also by *level* ℓ the set of states (ℓ,m,n), where: $0 < m < N_1$, $0 < n < N_2$, and by q_i the row vector defined by:
$q_i = [\, q_{i\,00}, q_{i\,10}, q_{i\,20} \ldots, q_{i\,N1\,0}, q_{i\,01}, q_{i\,11} \ldots, q_{i\,N1\,1} \ldots, q_{i\,N1\,N2}\,]$
Thus, (2) may be rewritten as:

$$q_{ijk} = \sum_{l=0}^{\infty} q_l\ P^{jk}_{l\rightarrow i} \tag{3}$$

If we denote by q the partitioned infinite row vector $[q_0, q_1, q_2, \ldots]$, then we get:

$$q = q\,\mathbf{Q} \tag{4}$$

where:

$$\mathbf{Q} = \begin{bmatrix} \mathbf{B}_0 & \mathbf{A}_0 & \mathbf{0} & \mathbf{0} & \cdots \\ \mathbf{B}_1 & \mathbf{A}_1 & \mathbf{A}_0 & \mathbf{0} & \cdots \\ \mathbf{0} & \mathbf{A}_2 & \mathbf{A}_1 & \mathbf{A}_0 & \cdots \\ \mathbf{0} & \mathbf{0} & \mathbf{A}_2 & \mathbf{A}_1 & \cdots \\ . & . & . & . & \cdots \end{bmatrix} \tag{5}$$

is an infinite dimensional square matrix containing the transition probabilities for the Embedded Markov Chain, where all matrices $\mathbf{A}_0$, $\mathbf{A}_1$, $\mathbf{A}_2$, $\mathbf{B}_0$, and $\mathbf{B}_1$ are defined in Appendix A.

The expressions of the expected sojourn times m_{ijk} may be found in terms of the parameter Γ_{jk} given by:

$$\Gamma_{jk} = \left[(N_1 - j)c + jd + (N_1 N_2 - k)a + kb\right] \tag{6}$$

Using (6) and arrival rate λ_{jk}, we get:

$$m_{ijk} = \begin{cases} 1/\Gamma_{jk} & ; i = 0, \lambda_{jk} < C \\ \left[1 - \exp\left\{-\dfrac{\Gamma_{jk}}{C - \lambda_{jk}}\right\}\right] \Big/ \Gamma_{jk} & ; i > 0, \lambda_{jk} < C \\ \left[1 - \exp\left\{-\dfrac{\Gamma_{jk}}{\lambda_{jk} - C}\right\}\right] \Big/ \Gamma_{jk} & ; i \geq 0, \lambda_{jk} > C \end{cases} \tag{7}$$

3.2 Computational method

Equation (4) can be solved readily using the *Matrix-Geometric* techniques developed by Neuts [6]. The solution is summarized by theorem 1.3.2 in [6], which requires the matrix $\mathbf{A} = \mathbf{A}_2 + \mathbf{A}_1 + \mathbf{A}_0$ to be irreducible.

From the form of matrix **A** derived in Appendix A, it can be proven easily, that:

*Theorem: All elements of the matrix **A** adjacent to the principal diagonal are non-zero (i.e. all states communicate and form one recurrent class). All the other elements are zero.*

This, will lead us to:

*Corollary 1: The matrix **A** is irreducible.*

Corollary 2: Results of theorem 1.3.2 in [6] can be used in solving (4).

Using the results in Appendix A, the denominator of equation (1) can be rewritten as:

$$\sum_{i=0}^{\infty} \sum_{j=0}^{N_1} \sum_{k=0}^{N_2} q_{ijk}\, m_{ijk} = \sum_{j=0}^{N_1} \sum_{k=0}^{N_2} \left[m_{0jk}\, q_{0jk} + m_{1jk} \left(q^*_{jk} - q_{0jk} \right) \right] \tag{8}$$

where each q^*_{jk} represents the equilibrium fraction of transitions into the phase state (j,k) (i.e., $q^*_{jk} = \sum_{i=0}^{\infty} q_{ijk}$), and is computed by:

$$q^*_{jk} = \left[q_0 \left(\mathbf{I} - \mathbf{R} \right)^{-1} \right]_{jk} \tag{9}$$

where $\mathbf{R}$ is the minimal non-negative solution of the matrix equation:

$$\mathbf{R} = \mathbf{A}_0 + \mathbf{R}\,\mathbf{A}_1 + \mathbf{R}^2\, \mathbf{A}_2 \tag{10}$$

and q_0 is a positive left invariant eigenvector of the matrix $\mathbf{B}_0 + \mathbf{R}\,\mathbf{B}_1$, i.e verifying:

$$q_0 = q_0 \left(\mathbf{B}_0 + \mathbf{R}\,\mathbf{B}_1\right) \tag{11}$$

and normalized by:

$$q_0 \left(\mathbf{I} - \mathbf{R}\right)^{-1} \underline{e} = 1 \; ; \; \underline{e} = \left[1 \; 1 \; . \; . \; . \; 1\right]^T \tag{12}$$

The rest of the equilibrium probabilities for the Embedded Markov Chain will be computed using $q_i = q_0 \mathbf{R}^i$.

The stochastic equilibrium probabilities p_{ijk} will be then computed using (1), and the queue length distribution will be computed by summing over all values of j and k.

4. Model Validation

The results presented will be showing the loss probability as a function of the queue size in milliseconds, i.e. the probability that the queue size is larger than a certain value.

4.1 Quantization levels N

The choice of the number of quantization levels N as a free parameter was justified by our results. Indeed, the variation in the survivor function* versus the buffer size with N as a parameter was negligible for any $N \geq 2$.

4.2 Packet size

Similar observation holds for the packet size, i.e. the variation in the survivor function versus the buffer size with the packet size as parameter was negligible. This means that the results presented will hold not only for ATM networks, but also for other networks.

4.3 Computation of matrix R

The iterative computation of matrix **R** using (10) was the essential factor in determining the convergence speed of the algorithm. Moreover, we noted that the execution of the program was slower as the number M of multiplexed sources increased.

* the survivor function is defined as: Prob{queue size > x} = 1-F(x) = Prob(loss given that the buffer size = x), where F(x) is the distribution function given by: F(x) = Prob{queue size < x}

5. Results

We will show in what follows the most salient results of our study. The results presented will be in terms of the probability of loss as a function of the buffer size, which is given in *msecs*. Note that this probability is equivalent to the probability of having the queue size larger than the buffer size.
The parameters used are chosen to give a clear picture of the trend in the results. This, of course, holds for all the parameters used, i.e. the utilization, the holding time, and the activity-ratio, where:

- *utilization = total arrival rate over the channel capacity;*
- *holding time = average time spent in either mode;*
- *activity ratio = average arrival rate in high-rate mode over average arrival rate in low-rate mode.*

5.1 Comparison with other models

In Fig.4, we have compared our results with those in [9] and [10] for a utilization of 75%, a mean-ratio of 2.5, a holding-time of 1.5 sec, and for a single video source. We noticed that the two analytic models (i.e. *fluid-flow* and *matrix-geometric*) are in good agreement, while the simulation is close to the model results for buffer sizes less than 1000 *msec*.

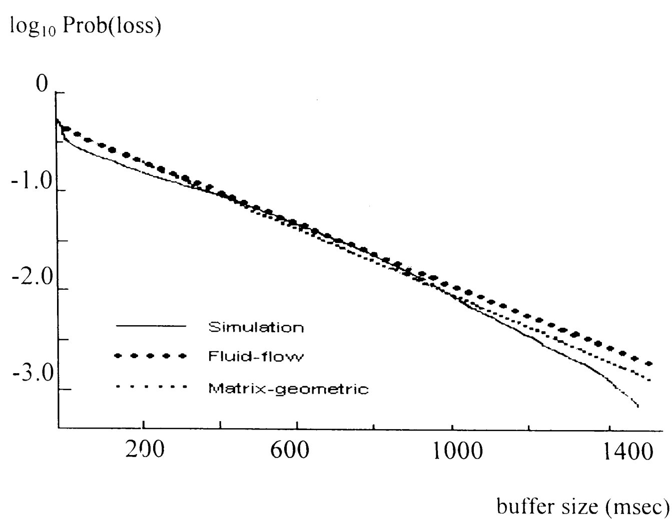

Fig. 4. Comparison to fluid-flow model and simulation.

When we make the same comparison for a higher number of video sources, we found out that the obtained results become practically identical with the other analytical models, while the difference with those obtained using the simulation become quite significant.

This is due to the assumption that we made on the operation of the queueing system, i.e. the queue length cannot be increasing when the total rate of the mini-sources is less than the channel capacity *C*. This will over-estimate the probability that the queue is empty, when compared to the simulation's. In fact, the queue length will be fluctuating positively around zero when $\lambda_{ij} < C$ (since we have a single server).

5.2 Smoothing-effect

The smoothing effect of statistical multiplexing is shown in Fig. 5. The data was obtained for a utilization of 75%, a holding-time of 1.5 sec, a activity-ratio of 2.5, and number of levels N = 5. As the number of multiplexed sources is increased the system performance improves compared to a single source with the same characteristics. This shows once again the significant gain obtained by using the statistical multiplexing technique.

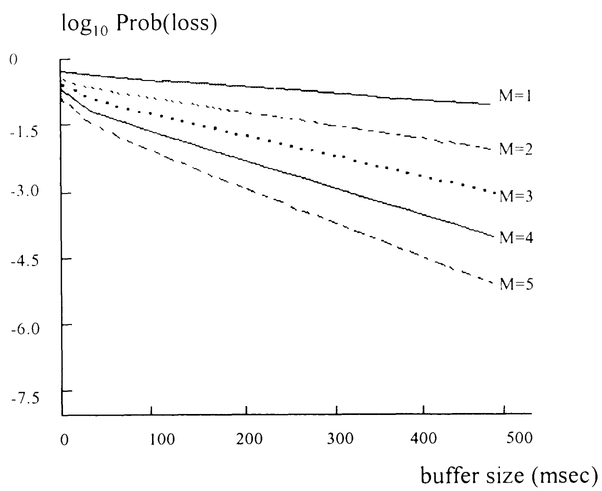

Fig. 5. Smoothing effect.

5.3 Holding-time

The effects of the holding-time are shown in Fig. 6. The data obtained was for a utilization of 75%, a activity-ratio of 2.5, number of levels $N = 5$, and number of sources $M = 5$. A degradation in the survivor function was observed when the *holding-time* was increased. This is due to the fact that the sources that are in the high rate mode are given enough time to build-up the queue size.

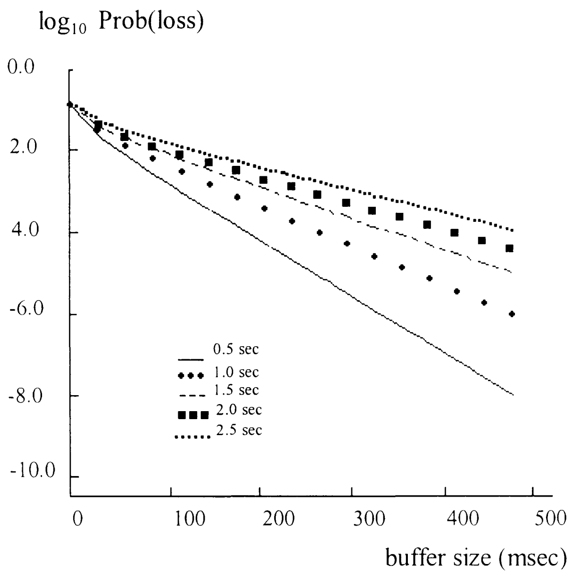

Fig. 6. Holding time effect.

5.4 Activity-ratio

The effects of the activity-ratio are shown in Fig. 7. The data obtained was for a utilization of 75%, a holding-time of 1.5 sec, number of levels N = 5, and number of sources M =5. For the same reasons as with the *holding-time*, we observed a degradation in the survivor function when the *activity-ratio* increases, as this also permits the queue to build up during the high-rate mode period.

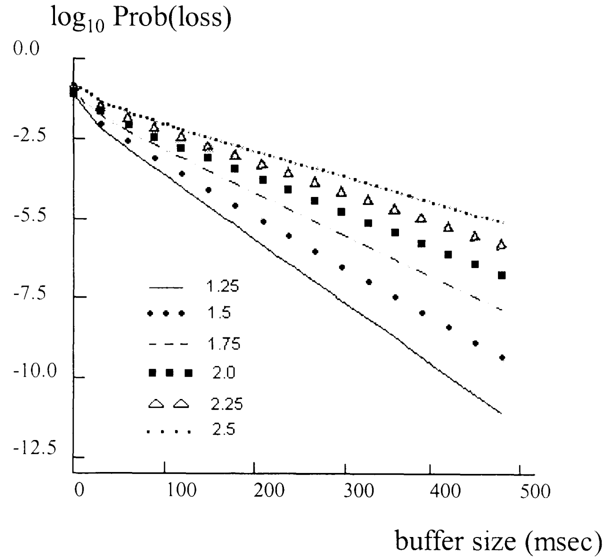

Fig. 7. Activity ratio effect.

6. Conclusions

In this paper, we have applied a robust mathematical technique in calculating stationary probabilities for an ATM switch with infinite buffer size. The major advantage of this technique is the stability of its recursive computational algorithm for the matrix **R**. However, this algorithm still needs some improvement regarding its convergence speed, which we think, should not be a formidable task.

The major findings are in the confirmation of the previous work in [9 and 10] concerning the effects of statistical multiplexing, and the performance degradation incurred when either the holding time or the activity level are increased. Also, we have shown once more the versatility of the Matrix-geometric technique by applying it in the analysis of ATM networks carrying VBR video traffic.

A straightforward continuation of this work, can be the generalization of this technique for multi-level VBR video traffic, and in improving the performance of the computational algorithm.

References

[1] Heeke, H. "A Traffic-control Algorithm for ATM Networks." *IEEE Trans. on Circuits and Systems for Video Technology*, Vol. 3, (June 1993), 182-189.

[2] Pazhyannur, R.S. and Agrawal, R. "Feedback-based Flow Control of B-ISDN/ATM Networks." *IEEE Jour. on Selected Areas in Commun.*, 13, No. 7 (September 1995), 1252-1266.

[3] Kim, Y.H. and Un, C.K. "Analysis of Bandwidth Allocation Strategies with Access Restrictions in Broadb and ISDN." *IEEE Trans. on Commun.*, 41, No.5 (May 1993), 771-781.

[4] Stamatelos, G.M. and Hayes, J.F. "An Integrated System for Video, Voice, and Data Communications." *SUPERCOMM/ICC'92*, Vol. 1, (June 1992), 511-515.

[5] Stewart, W. J. "On the Use of Numerical Methods for ATM Models." *IFIP Transactions C (Communication Systems)*, Vol. C-15, (January 1993), 375-396.

[6] Neuts, M.F. "A Versatile Morkovian Point Process." *Jour. of Appl. Prob.*, Vol. 16, (1979), 764-779.

[7] Daigle, J.N. and Langford, J.D. "Queuing Analysis of a Packet Voice Communication System." *IEEE Jour. on Selected Areas in Commun.*, SAC-4, No.6 (September 1986), 847-855.

[8] Chen, X., Lambadaris, I. and Hayes, J. F. "A General Unified Model for Performance Analysis of Multicast Switching." *GLOBCOM '92*, Vol.3, (December 1992), 1498-1502.

[9] Maglaris, B., Anastassiou D., Sen, P., Karlsson, G. and Robbins, J. "Performance Models of Statistical Multiplexing in Packet Video Communications." *IEEE Trans. Commun.*, Vol.36, (July 1988), 834-844.

[10] Sen, P., Maglaris, B., Rikli, N.E. and Anastassiou, D. "Models for Packet Switching of Variable-bit-rate Video Sources." *IEEE Jour. on Selected Areas in Commun.*, 7, No. 5 (June 1989), 865-869.

[11] Stern, T.E. "A Queueing Analysis of Packet Voice." Conference Record of the 1993 IEEE Global Telecommunications Conference 1, San Diego, (1993), 71-76.

[12] Kohlas, J. *Stochastic Methods of Operations Research*, Cambridge University Press, Cambridge, 1982.

[13] Karlin, S. and Taylor, H. *A First Course in Stochastic Processes*, 2nd ed. New York: Academic Press, 1975.

Appendix A

From Markov chain theory, we have:

$$q_{ijk} = \sum_{l=0}^{\infty} \sum_{m=0}^{N_1} \sum_{n=0}^{N_2} q_{lmn} \, P[l,m,n \rightarrow i,j,k] \tag{a-1}$$

and using the previous notation, the row vector q_i may be written as:

$$q_i = \sum_{l=0}^{\infty} q_l \left[P_{l\rightarrow i}^{00} \,,\, P_{l\rightarrow i}^{10} \,,\, . \, . \, . \, . \,,\, P_{l\rightarrow i}^{N_1 0} \,,\, . \, . \, . \, . \,,\, P_{l\rightarrow i}^{N_1 N_2} \right] = \sum_{l=0}^{\infty} q_l \, \mathbf{P}_{l\rightarrow i} \tag{a-2}$$

where $\mathbf{P}_{l\rightarrow i}$ can be derived easily from (a-2), and represents the transition probability matrix from states at level l to states at level i .
By assuming that queue increments or decrements of more than one packet at a time is not possible, we get:

- $\mathbf{P}_{l\rightarrow i} \neq 0$, only for: l = i-1, or l = i, or l = i+1, and i>0.
- $\mathbf{P}_{l\rightarrow 0} \neq 0$, only for: l = 0, or l = 1, and i=0.

Thus, we have:
$q_0 = q_0 \, \mathbf{P}_{0\rightarrow 0} + q_1 \, \mathbf{P}_{1\rightarrow 0}$
$q_i = q_{i-1} \, \mathbf{P}_{(i-1)\rightarrow i} + q_i \, \mathbf{P}_{i\rightarrow i} + q_{i+1} \, \mathbf{P}_{(i+1)\rightarrow i}$, for i>0.

It can be shown that for i>0, the transition probabilities are independent of the *level* they are in. Thus, we have:

- $\mathbf{A}_0 = \mathbf{P}_{(i-1)\rightarrow i} = \mathbf{P}_{0\rightarrow 1}$
- $\mathbf{A}_1 = \mathbf{P}_{i\rightarrow i} = \mathbf{P}_{1\rightarrow 1}$
- $\mathbf{A}_2 = \mathbf{P}_{(i+1)\rightarrow i} = \mathbf{P}_{2\rightarrow 1}$
- $\mathbf{B}_0 = \mathbf{P}_{0\rightarrow 0}$
- $\mathbf{B}_1 = \mathbf{P}_{1\rightarrow 0}$

Equations (4) and (5) then follow readily.

توزيع طول الصف لمقسم ATM مزود بمشفرات فيديو متغيرة السرعة

ناصرالدين ركلي

قسم هندسة الحاسب، كلية علوم الحاسب والمعلومات، جامعة الملك سعود،
ص.ب ٥١١٧٨، الرياض ١١٥٤٣، المملكة العربية السعودية

(قدّم للنشر في ١١/٦/١٩٩٦م؛ وقبل للنشر في ٢٨/١٠/١٩٩٧م)

ملخص البحث. طورت وسائل تحليلية عدة لدراسة خصائص مشفرات الفديو الثابتة السرعة، ولكن القليل من التحاليل طبقت على المشفرات ذات السرعة المتغيرة. يعرض في هذا البحث نمذج شبه ماركوفي لتقريب التوزيع المتزن لطول صف نظام مكون من مقسم ATM ذى معبر مشترك له سرعة ثابة، و يخدم هذا المقسم عدة مشفرات فيديو ذات سرعة متغيرة. يفترض أن هذه المشفرات تعمل بسرعتين، و تولد خلايا تمر على مخزنات أولية منفصلة قبل أن تجتمع فى صف مشترك و تخدم بنظام FIFO يعتمد النموذج المقترح على حالة المعالج المكونة من معدل تغير سرعة طول الصف بدلالة عدد المنابع الصغيرة من كل نوع، و بذلك تمكن من استعمال طريقة Matrix Geometric لحله. يمكن استعمال هذا النموذج كجزء من دالة سياسة التحكم فى الدخول على الشبكة عند مرحلة إعداد النداء. تم الحصول على نتائج رقمية لحالات عدة لتشغيل النظام، و قورنت مع نتائج نماذج أخرى، فكان التطابق بينها ممتازاً.