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Decision Feedback Equalizers Using Radial Basis Function Networks

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Abstract. Decision feedback equalizers (DFE)s are used extensively in practical communication systems. They are more powerful than linear equalizers especially for severe inter-symbol interference (ISI) channels with deep frequency null. In this paper, radial basis function (RBF) network is used to implement DFE. Advantages and problems of this system are discussed and its results are then compared with DFE using multilayer perceptron net (MLP). Results indicate that the implemented system outperforms both the least-mean square (LMS) algorithm and MLP given the same signal-to-noise ratio.

***Keywords*.** Nonlinear equalization, neural networks, radial basis function, decision feedback equalizers, ISI channels.

Introduction

Equalization is a technique used to remove inter-symbol interference (ISI) produced due to the limited bandwidth of the transmission channel [1]. When the channel is band-limited, symbols transmitted through will be dispersed. This causes previous symbols to interfere with the next symbols, yielding the ISI. Also, multipath reception in wireless communications causes ISI at the receiver. Thus, equalizers are used to make the frequency response of the combined channel-equalizer system flat.

Two classes of equalizers are known: linear and nonlinear equalizers. An example of the latter type is the decision feedback equalizer DFE. The equalization process can be divided into two modes- a training mode and a decision-directed mode. In the first mode, the equalizer is trained to produce the expected output, by sending a training sequence and the coefficients of the equalizer are adjusted to produce the required output at each sampling time. In the second mode, the equalizer is operated on the channel to be equalized to estimate the channel output. The second mode is the normal operating mode in a practical communication system. Since equalization technique is simply deciding on a symbol from signals available in the signal space, (1 or –1 for the binary phase-shift keying (BPSK) system), it can be considered as a classification problem [2]. It accepts the delayed received samples as inputs, and outputs its decision which is one of the possible signals. In the M-ary case, the system has M different possible classes at the equalizer output.

Artificial neural nets ANNs are able to perform complex nonlinear classification problems, and hence they can be used as equalizers. Most ANNs use the mean square error (MSE) as the cost function to be minimized by the network. Problems encountered using ANNs in equalization are the slow rate of convergence and the possibility that the net does not reach the true minimum mean square error MSE [3]. In this case, the net will not be able to optimize its parameters to the least MSE. Two ANN models are used in this paper, namely MLP and RBF nets.

Several approaches using ANNs in equalization have been proposed in the last few years. Kirkland in 1992 used feedfoward ANNs in equalizing a multipath fading channel [4]. In the same year, Peng modified the activation function of the MLP to be suitable for phase-amplitude modulation (PAM) and quadrature-amplitude modulation (QAM) schemes [5]. In 1994, Kechriotis used recurrent ANNs in equalizing different linear and nonlinear channels [6]. Chang, in the same year, introduced a neural-based DFE to equalize indoor radio channel [7]. He also used a wavelet ANN trained with recursive least squares (RLS) algorithm to equalize a nonlinear channel. Al-Mashouq used a feedforward NN to combine both equalization and decoding at the receiver [8]. This method performed better than the cascaded equalizer-decoder pair. Mulgrew investigated the implementation of DFEs using RBF nets in 1996 [9]. In 1997, a new algorithm for training recurrent NN was proposed [2]. It was called the discriminative least squares (DLS) and it was faster to converge than the RLS and LMS algorithms.

In this paper, an RBF net is used as a DFE. The paper disscusses architectures of the DFE and the RBF net. Then, the use of RBF net to implement a DFE is presented. Simulation results are then discussed. Finally, conclusions and suggestions for future work are presented.

Decision Feedback Equalizers (DFE)s

A schematic diagram of a DFE is shown in Fig. 1. An (n,m) DFE denotes an equalizer with n tapped delayed inputs and m feedback signals. So, m output samples are fed back to the input through a feedback filter in addition to the input samples. This feedback helps the system to decorrelate the noise that is produced by the ISI at the final output [10]. DFEs are usually implemented using LMS or RLS algorithms [1]. In all cases, the input-output relation is expressed in the following equation [11]:

$$I_k = \sum_{j=1}^{n} c_j\, x_{k-j} + \sum_{j=1}^{m} g_j\, y_{k-j} + c_e\, e_k \qquad (1)$$

where I_k is the output of the filter, x_k is the received signal, y_k is the decided symbol at the equalizer output. Also, c_j, g_j and c_e are the coefficients of the feedforward and feedbackward filters. The error signal e_k, is the difference between the equalized signal y_k and the output of the equalizer I_k. The subscript (*k-j*) in both filters indicates that the samples are shifted in the line at each sampling interval. Both the feedforward and the feedbackward filters are considered as finite impulse response (FIR) filters. Equation (1) describes the function of the combined filters as an infinite impulse response (IIR) filter.

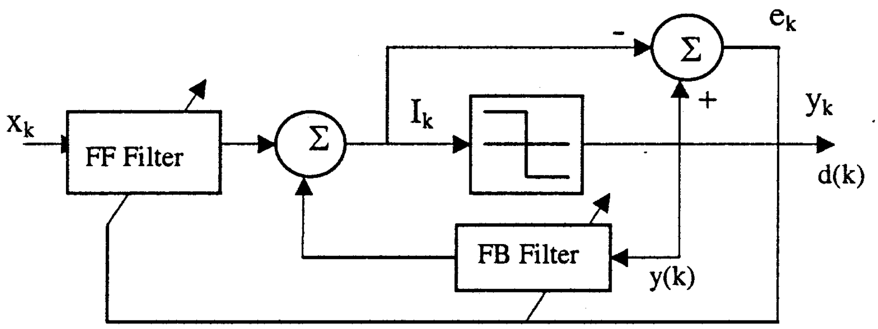


Fig. 1. DEF using two FIR filters, one as feedforward and another as feedbackward.

FF filter

Since the DFE is considered to be a nonlinear equalizer, it is used more often than linear equalizers, especially for the case of severe-ISI channels. These channels are characterized in their frequency response by the existence of frequency nulls that make them totally nonlinear and produce disturbed output [10].

The performance of DFEs depends on the number of the delayed inputs and the number of the feedback signals from output to input. It can be improved by feeding an error signal (the difference between the expected output and the produced output) back to the input in addition to the normal feedback signals [11].

Radial Basis Function (RBF)

RBF nets are well suited to solve interpolation problems. Such problems are stated as follows: given a set of input vectors $\{x_i\}$ and the corresponding output vectors $\{y_i\}$, find the appropriate transfer function that can fit noisy input vectors to produce the most appropriate output according to the given input/ouput vector pairs [9]. It is clear that the equalization problem is a typical interpolation problem.

A general architecture of an RBF net is shown in Fig. 2. It consists of two layers with the activation functions in the first layer are radial, and in the output layer are linear. The activation function of the first layer is called the basis function. It is a radial function characterized by being monotonically increasing or decreasing from a center value [9]. Examples of radial functions are the thin plate spline, multi-quadratic, inverse multi-quadratic and the Gaussian functions [9]. The Gaussian function is most commonly used because of its smooth characteristics. It is given by [3]:

$$h(x) = e^{\frac{-(x-c)^2}{r^2}} \qquad (2)$$

where *c* is the center of the function and *r* is its spread constant. The center and the spread constant control the location and the spread of the decision region of the radial function, respectively. The spread constants should be chosen such that the functions cover their areas and some of the adjacent areas in the space, increasing the ability of the ANNs to generalize for noisy patterns [12]. The output of the RBF net is given by:

$$X = \sum_{j=1}^{p} x_j. \quad \text{where} \quad y = \sum_{j=1}^{n} w_j\, h_j(X) \qquad (3)$$

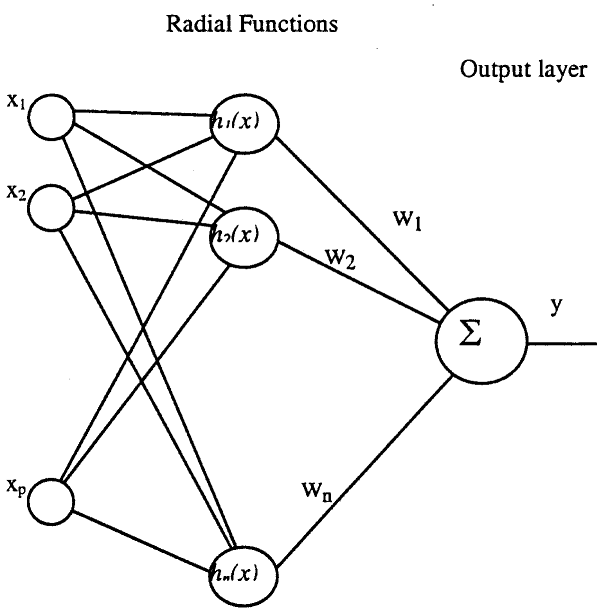


Fig. 2. General architecture of an RBF net.

The basic idea behind the RBF development is Cover's theorem [3]. It says that complex pattern-classification problems are more likely to be linearly separable in high-dimensional than in low-dimensional space. Using Gaussian radial functions in the RBF net converts problems into new ones in higher dimensional space.

The RBF net is trained by presenting the training data vectors and the corresponding output vectors to the net, and it will compute its weight matrix that minimize the cost function C given by [3]:

$$C = \sum_{i=1}^{p} \{ y_i - \sum_{j=1}^{n} w_j h_j(X_i) \}^2 \qquad (4)$$

These calculations are repeated by adding one basis function at a time until the required MSE is reached.

When the RBF is trained using the exact interpolation method, the number of basis functions needed is the same as the number of examples used in training. This makes the ANN need more computations because of the large number of basis functions used [3,12]. The training process used in this paper is the one used in MATLAB. It uses the minimum number of basis functions that are able to solve the problem undertaken with the required MSE [3]. Of course, for a given number of training examples, the number of basis functions used in this method is less than the number of training examples [3,12]. This improves the generalization abilities of the RBF net because using a number of basis functions equal to the number of examples makes the ANN unable to draw decisions for noisy examples; that may be encountered later during the operation mode [12].

The Implemented System

The implemented RBF-based DFE consists of a tapped-delay line that has 5 taps. At each sampling interval, the signals in the line are shifted by one location and a new received signal is put at the first tap. The RBF net is trained using 500 training samples with their corresponding outputs. It is initialized with one neuron whose activation function is Gaussian with a spread constant of 0.7. Each time, the RBF computes the weight matrix and adds one neuron if the MSE is still high. This process is repeated until the required MSE is obtained. The hidden layer consists of 170 and 300 basis functions for the DFE and linear equalizers, respectively. These numbers are the minimum numbers of basis functions needed to solve the equalization problem in each case and to have a MSE of 10^{-4}.

The RBF-based DFE is compared with an MLP-based DFE that consists of a (9,3,1) MLP. This means that there are 9, 3 and one neurons in the input, hidden and output layers, respectively. The 9 input signals constitute a delay line of 9 taps. Both the hidden and the output layers have activation functions of the tan-sigmoid shape. The MLP net is initialized using the first training example from the channel. The training process then continues using the back-propagation algorithm with a variable training rate. Upon receiving a new training example, it computes the MSE and updates its coefficients accordingly. This process is repeated recursively until the required MSE, which was set to10^{-4}, is achieved.

The two RBF and MLP-based DFEs are used to equalize two channels that are of practical importance. The first is a linear channel that introduces small distortion to its input [1]. The second is a severe-ISI channel whose frequency response has a deep null [10]. The latter type is faced often in practical communication systems and is very difficult to equalize using linear equalizers. However, they can be equalized efficiently using nonlinear equalizers such as DFEs. The two channels used are shown in Fig. 3.

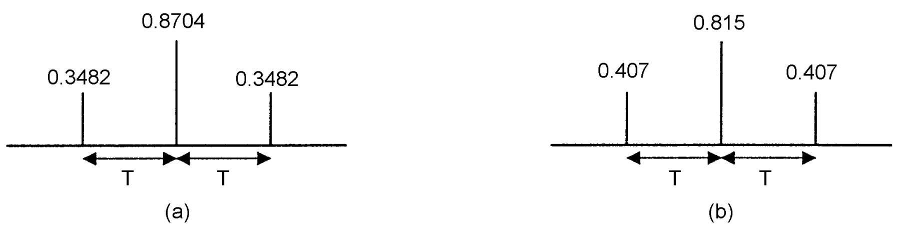


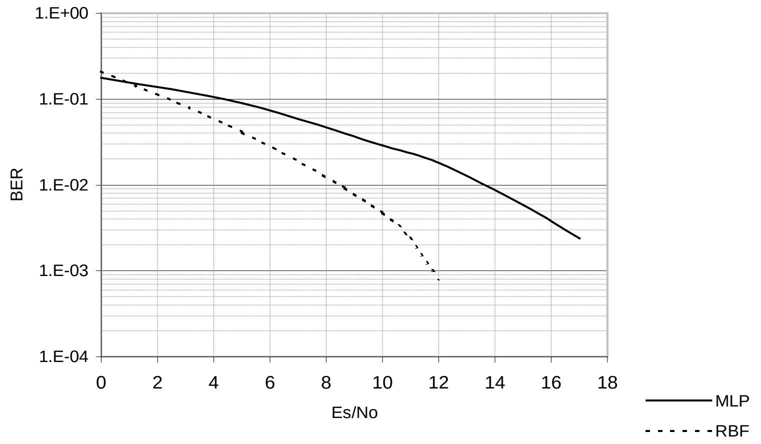
Fig. 3. (a) Channel 1, (b) Channel 2.

Two DFE cases were simulated in this paper. The first case is a DFE in which the detected symbols are used as feedback signals. In the second case, the correct symbols are the signals that are used as feedback signals, which is not possible practically. This is because if the correct symbols are known to the receiver, there is no need for doing communication [10]. However, it is used to find the lower bound of the performance of the DFE used. In summary, (5,0) and (4,1) DFEs are implemented using both MLP and RBF nets.

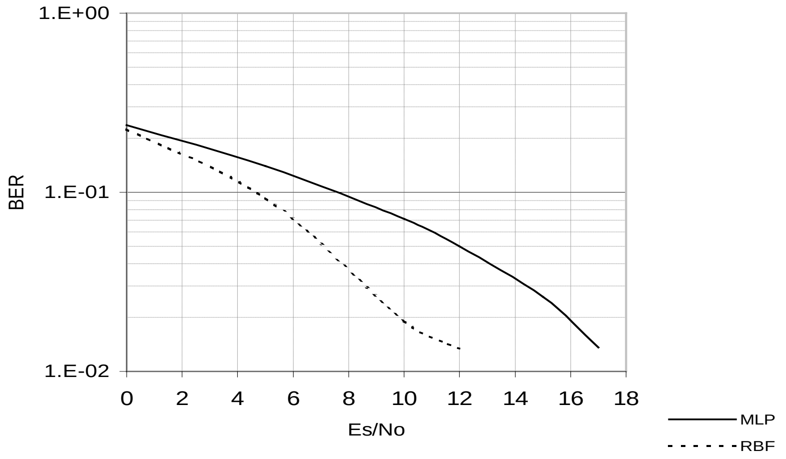
Simulation Results

The results of using linear equalization for channels 1 and 2 are shown in Figs. 4-a and b, respectively. The RBF-based equalizer performance is better than that of the MLP-based by 5 and 4 dBs, for channels 1 and 2, respectively at 10^{-2} bit error rate

(BER). It is clear that channel 2 was not equalized well using linear equalization because of its severe-ISI.



(a)

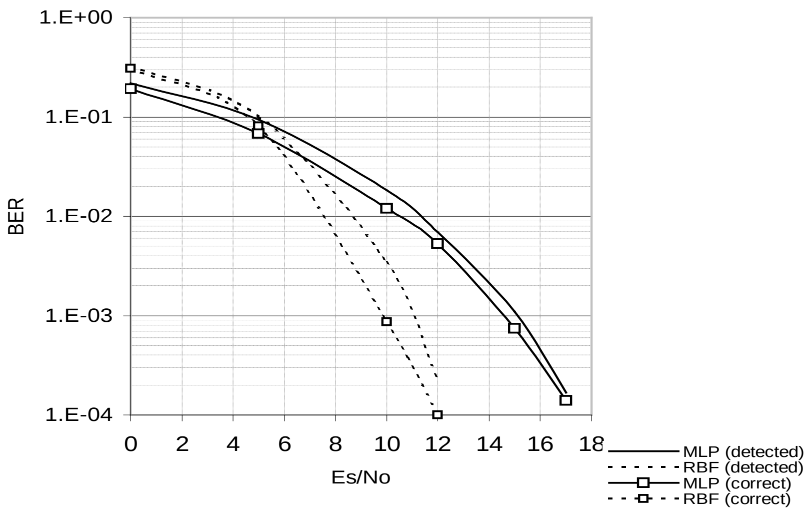


(b)

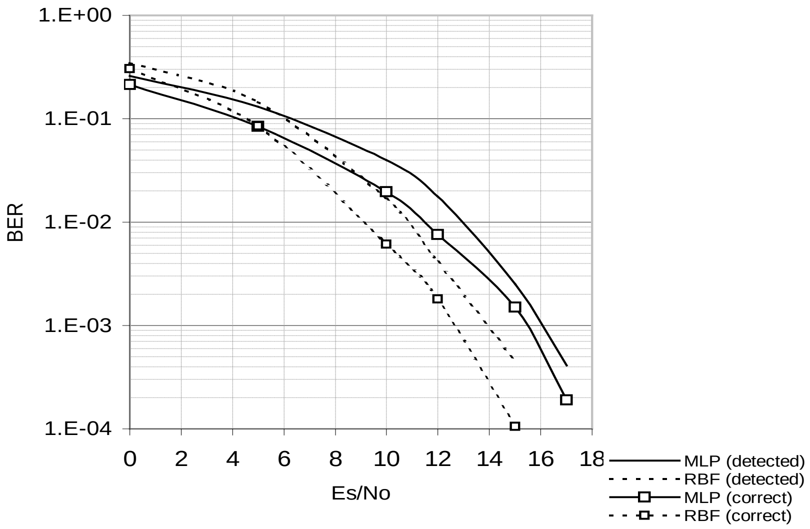
Figs. 4. Performance of linear equalization of (a) channel 1 (b) channel 2.

Figure 5-a shows the performance of both MLP and RBF-based (4,1) DFEs for channel 1. It is clear that the RBF-based equalizer outperforms the MLP-based one by about 4 dBs at 10^{-3} BER. Figure 5-b shows the same information as part (a) for channel 2. Also, the RBF-based DFE outperforms the DFE based on MLP by about 2 dB. Of course, the overall performance for channel 2 is worse than that of channel 1 because channel 2 is more severe. In both channels, the DFE based on RBF is better than the one

based on MLP even when the correct symbol is fedback in the MLP and the detected one is feedback in the RBF. This means the former DFE is better than the latter always, since feeding back the correct symbol is the most ideal case.



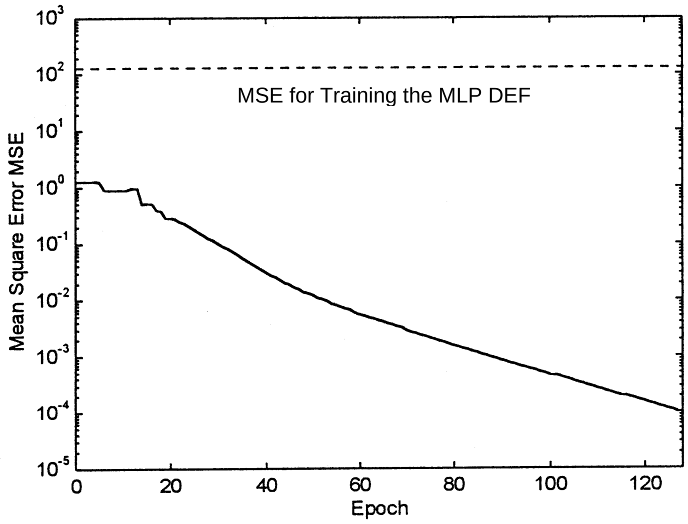
(a)



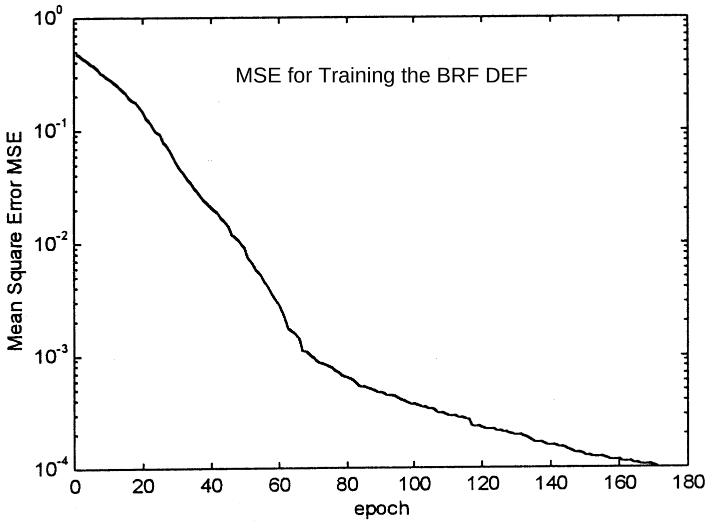
(b)

Figs. 5. (a) Performance of DFE of channel 1, (b) Performance of DFE of channel 2.

Figures 6-a and 6-b show the convergence of both the MLP and RBF-based DFEs, respectively. Both equalizers were able to reach the required MSE but the RBF is faster. On the other hand, the RBF-based DFE needs more computations in the decision-directed modes. This is due to the high number of basis functions in the hidden layer of the RBF system compared to the MLP system. Simulation results showed that increasing the number of neurons in the hidden layer of the MLP will not improve the convergence time or the BER performance. So, the price payed for reducing the BER and speeding up the training process by using the RBF-based DFE, is the more computations required in the decision-directed mode.



(a)



(b)

Figs. 6. (a) Convergence of MLP-based DFE. (b) Convergence of RBF-based DFE.

Conclusion and Discussion

In this paper, linear and DFE equalizers were implemented using both MLP and RBF nets. The above systems were tested for two different channels. Results showed that linear equalizers are not good for severe-ISI channels. Also, it is seen that the RBF-based equalizers perform better than the MLP-based one, especially at high SNR. Moreover, the RBF equalizer converges faster than the MLP in the training mode but needs more computational time in the decision-directed mode, because of its large number of neurons compared with the MLP. Trade-off between fast convergence and performance in one side and the on-line computational time in the other side should be taken into consideration upon designing such systems in practice.

The DFE performs better when the correct symbol is the feedback signal that is an ideal case. They also are efficient in reducing the effect of the deep frequency null of channel 2. According to [1], the MLP- based DFE outperforms the conventional DFE based on LMS and hence does the RBF-DFE implemented in this paper.

Extension of this research is to implement the same concept using different training algorithms that converge faster. Regarding the RBF net, regularization terms can be added to its weight matrix equation. It is claimed in [3] that this can reduce the noise variance in the output signal, which improves the performance. Also, DFE can be implemented using error feedback as in [11] but via the ANN approach.

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تصميم معدلات مرجعة للرموز المقررة باستخدام الشبكات العصبية المصممة باستخدام الدوال المحورية الأساسية

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(استلم في ١٩٩٩/٢/١٠م؛ وقبل للنشر في ١٩٩٩/١١/٩م)

ملخص البحث. تستخدم المعدّلات عند أجهزة الاستقبال لتقليل ما يحدثه تحديد مقدار الطيف المسموح به في أنظمة الاتصالات من أخطاء ناتجة عن تداخل النبضات الكهربائية المرسلة. و تعتبر المعدّلات المرجعة للرموز المقررة (decision feedback equalizers) أكثر تأثيرا من المعدلات الموازنة (linear equalizers) وخصوصا عندما يزيد التداخل بين الرموز.

استخدمت في هذا البحث الشبكات العصبية المصممة باستخدام الدالة المحورية الأساسية

(radial basis function) لتصميم معدل مرجع للرموز المقرّرة. تم مناقشة المزايا والمشاكل التي يتميز بها هذا المعدل ومقارنته بالمعدل المرجع للرموز المقررة والمصمم باستخدام

الشبكات العصبية متعددة الطبقات (multi-layer perceptron). ودلت نتائج المحاكاة بالحاسب على أن النظام المنفذ يفوق الأنظمة السابقة بالنسبة لاحتمال الخطأ تحت نفس الظروف.