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On the Estimation of the Time Required for Emptying Cylindrical Reservoirs Connected to Pipes

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Abstract. The problem investigated in this paper is the accurate evaluation of the time required for emptying a reservoir of constant cross-sectional area in which a pipe is fixed.

In the analysis, a friction coefficient equation is considered which represents the different flow regimes on a Moody diagram satisfactorily. The approximate equation for estimating the time of emptying a reservoir, considering fully rough turbulent flow is derived and discussed. A computer program for estimating the accurate and approximate times of emptying reservoirs is presented. On the basis of the computer program a number of graphs are provided and the factors affecting the above estimation are studied. It is found that in some cases the estimated approximate time is in error by more than 25%.

Nomenclature

The following symbols are used in this paper

A	=	quantity defined by Eq. (13)
A_o	=	quantity defined by Eq. (16)
A_1	=	cross-sectional area of reservoir
a	=	cross-sectional area of pipe
B	=	quantity defined by Eq. (14)
C	=	constant defined by Eq. (5)
C_1	=	quantity defined by Eq. (24)
C_2	=	quantity defined by Eq. (25)

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C_3	=	quantity defined by Eq. (30)
C_4	=	quantity defined by Eq. (29)
C_5	=	quantity defined by Eq. (28)
C_6	=	quantity defined by Eq. (27)
C_7	=	quantity defined by Eq. (26)
D	=	diameter of reservoir
d	=	pipe diameter
F	=	quantity defined by Eq. (19)
f	=	friction coefficient
f_a	=	approximate friction coefficient
g	=	acceleration due to gravity
H	=	head acting on a pipe
H_1	=	initial head
H_2	=	final head
h_f	=	friction head loss
h_m	=	head loss in fittings
k	=	coefficient representing pipe minor losses
L	=	pipe length
Q	=	pipe diacharge
Q_{a1}	=	approximate pipe discharge at initial head
Q_1	=	pipe discharge corresponding to initial head
Q_{a2}	=	approximate pipe discharge corresponding to final head
Q_2	=	pipe discharge corresponding to final head
R	=	Reynolds' number
T	=	time of emptying a reservoir
T_a	=	approximate time of emptying a reservoir
V	=	average pipe velocity
ε	=	equivalent absolute roughness of pipe
ν	=	kinematic viscosity of liquid.

Introduction

In practice unsteady flow in a pipe connected to a reservoir may occur when it is needed to empty the reservoir for maintenance or for cleaning purposes. In addition, failure of the power used in filling a reservoir may cause the aforesaid unsteady pipe flow. In analyzing such pipe flow problems and estimating the time required to empty a reservoir the friction coefficient, f, is usually assumed to be a constant which depends only upon the pipe roughness[1, p.56]. The abov assumption means that the flow is wholly rough throughout the emptying process [2, p.91, 3, p.233, 4, p.396]. In fact the friction coefficient generally depends on both Reynolds' number and the pipe roughness.

Moreover pipe flow may be laminar if Reynold' number is less than 2000 when the acting head in the reservoir is sufficiently small.

Hathoot [5] provided a solution for the above mentioned problem taking into account the effect of variation of the friction coefficient. In his analysis, Hathoot considered the Swamee and Jain equation [6] which covers a significant portion on the Moody diagram and neglected the effect of pipe fittings. However, Hathoot [7] considered and further analyzed the effect of pipe fitting on the time of emptying a reservoir. The analyses mentioned above were limited to the range of validity of the Swamee and Jain friction equation. In this investigation a technique is presented for estimating the time required for emptying cylindrical reservoirs, using a friction coefficient equation which completely covers the Moody diagram [8].

Theory

a) General

In the case of steady flow through a pipe many equations have been presented to estimate the friction head loss [9, p.213, 10, p.172]. However the most reliable of them is that of Darcy-Weisbach [11].

The total head loss in a pipe may be given by:

$$\begin{aligned} H &= h_f + h_m \\ &= \frac{v^2}{2g}\left(\frac{fL}{d}+k\right) \end{aligned} \tag{1}$$

in which h_f is the friction head loss, f the friction coefficient, L the pipe length, v the average velocity, g the acceleration due to gravity, d the pipe diameter, h_m the minor losses, and k is the sum of pipe fitting coefficients [12,13][14, p. 243].
For convenience, Eq. (1) may be put in the form:

$$H = CQ^2\left(\frac{fL}{d}+k\right) \tag{2}$$

in which Q is the pipe discharge and C is given by:

$$C = \frac{8}{g\pi^2}\, d^4 \tag{3}$$

For steady flow through a pipe connected to a reservoir the hed loss H is actually the liquid head in the reservoir which acts on the pie, Fig. 1.

b) Unsteady flow

In the case of emptying a cylindrical tank, although the flow is unsteady the instantenous discharge, Q, is interrelated to the acting head, H, by Eq. (2). The differential equation describing such unsteady flow is:

$$QdT = -A_r\, dH \tag{4}$$

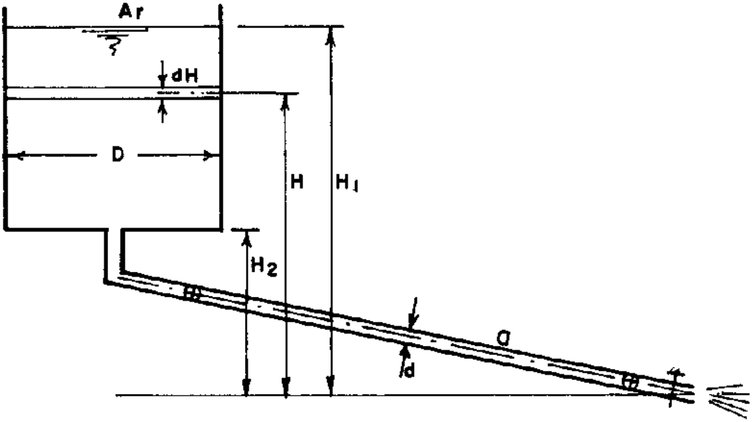

Fig. 1. Definition sketch of a cylindrical tank with a pipe.

in which T is the time, and A_r the cross-sectional area of the reservoir. The time required to lower the water surface from H_1 and H_2 above the pipe is given by:

$$T = -\int_{H_1}^{H_2} \frac{A_r}{Q}\, dH \tag{5}$$

Referring to Eq. (2) it is evident that Q cannot be replaced by an explicit function of H. This is attributed to the fact that the friction coefficient, f, is a function of Reynolds' number which, in turn, is a function of Q.

c) Changing the integration limits

For convenience the integration limits H_1 and H_2 are to be replaced by the corresponding discharges Q_1 and Q_2, respectively. Also dH is to be replaced by a term containing dQ and hence it is necessary to establish the correlation between them. Differentiating Eq. (2) with respect to Q, simplifying and separating variables:

$$dH = CQ\left[\frac{QL}{d}\left(\frac{df}{dQ}\right) + 2\left(\frac{fL}{d} + k\right)\right]dQ \tag{6}$$

Combining Eqs. (5) and (6):

$$T = -\int_{Q_1}^{Q_2} CA_r \left[\frac{QL}{d}\left(\frac{df}{dQ}\right) + 2\left(\frac{fL}{d} + k\right) \right] dQ \tag{7}$$

Before evaluating (df/dQ) it is convenient to discuss the friction coefficient and mathematical representation of the different zones on the Moody diagram.

Friction Coefficient

A number of equations were presented to represent one or more of the different flow regimes on Moody diagram [6,8]. It has been found that the Churchill equation covers all the turbulent zones and extends to the laminar zone. Churchill equation provides the friction coefficient explicitly as:

$$f = 8\left[\left(\frac{8}{R}\right)^{12} + \frac{1}{(A+B)^{1.5}}\right]^{1/12} \tag{8}$$

in which R is Reynolds' number given by

$$R = \frac{4Q}{\pi d \upsilon} \tag{9}$$

where v is the kinematic viscosity of the liquid, A and B are given by:

$$A = \left\{ 2.457 \ln\left[\frac{1}{\left(\frac{7}{R}\right)^{0.9} + 0.27\left(\frac{\epsilon}{d}\right)} \right] \right\}^{16} \tag{10}$$

and

$$B = \left(\frac{37530}{R}\right)^{16} \tag{11}$$

in which ε is the equivalent absolute roughness of the pipe material.

It is worthy to note that in computing f, A and B, double precision should be used since these quantities internally calculate very large and very small numbers and are therefore sensitive to round-off error [15].

Final Formulation of the Time Integral

Refering to the integral of Eq. (7), the quantity df/dQ is to be evaluated. Differentiation of Eq. (8) with respect to Q yields:

$$\frac{df}{dQ}=\frac{2}{3}\left(A_o\right)^{-11}\left\{\frac{-12}{Q}\left(\frac{8}{R}\right)^{12}+\frac{d}{dQ}\left[\frac{1}{\left(A+B\right)^{1.5}}\right]\right\} \tag{12}$$

in which

$$A_o=\left(\frac{8}{R}\right)^{12}+\frac{1}{\left(A+B\right)^{1.5}} \tag{13}$$

and

$$\frac{d}{dQ}\left[\frac{1}{\left(A+B\right)^{1.5}}\right]=-1.5\left[\frac{1}{\left(A+B\right)^{2.5}}\right]\left[\frac{dA}{dQ}+\frac{dB}{dQ}\right] \tag{14}$$

Differentiation of both A and B with respect to Q yields:

$$\frac{dA}{dQ}=-35.3808\,A^{15/16}\left(\frac{F}{Q}\right)\left(\frac{7}{R}\right)^{0.9} \tag{15}$$

in which

$$F=\left(\frac{7}{R}\right)^{0.9}+0.27\left(\frac{\epsilon}{d}\right) \tag{16}$$

and

$$\frac{dB}{dQ}=-\frac{16B}{Q} \tag{17}$$

Substitution of Eqs. (13), (14) (15) and (17) into Eq. (12) yields:

$$\frac{df}{dQ}=-\frac{(8)^{12}}{Qf^{11}}\left\{\left(\frac{8}{R}\right)^{12}-\left[\frac{2}{\left(A+B\right)^{2.5}}\right]\left[2.2113A^{15/16}F\left(\frac{7}{R}\right)^{0.9}+B\right]\right\} \tag{18}$$

Substituting df/dQ as given by Eq. (18) into Eq. (7), simplifying and rearranging:

$$T=\int_{Q_1}^{Q_2} CA_r\left\{\frac{(8)^{12}L}{df^{11}}\left\{\left(\frac{8}{R}\right)^{12}-\left[\frac{2}{\left(A+B\right)^{2.5}}\right]\cdot\left[2.2113A^{15/16}F\left(\frac{7}{R}\right)^{0.9}+B\right]\right\}-2\left(\frac{fL}{d}+k\right)\right\}dQ \tag{19}$$

For convenience Eq. (19) is put in the form:

$$T = \int_{Q_1}^{Q_2} CA_r \left\{C_1\left[C_2 - C_7 C5\right] - C_6\right\} dQ \tag{20}$$

in which

$$C_1 = \frac{(8)^{12} L}{dF^{11}} \tag{21}$$

$$C_2 = \left(\frac{8}{R}\right)^{12} \tag{22}$$

$$C_1 = \frac{2}{(A+B)^{2.5}} \tag{23}$$

$$C_6 = 2\left(\frac{fL}{d} + k\right) \tag{24}$$

$$C_5 = C_4 + B \tag{25}$$

$$C_4 = 2.2113 A^{C_3} F\left(\frac{7}{R}\right)^{0.9} \tag{26}$$

and

$$C_3 = \frac{15}{16} \tag{27}$$

Determination of Integration Limits

As mentioned above the integration limits, H_1 and H_2 of Eq. (5), are replaced by the limits Q_1 and Q_2 of Eq. (7). Therefore it is necessary to find Q_1 and Q_2 corresponding to H_1 and H_2, respectively. According to Eq. (4) the pipe discharge is given by:

$$Q = \sqrt{\frac{H}{c\left(\frac{fL}{d} + k\right)}} \tag{28}$$

However Eq. (28) is implicit since the friction coefficient, f, generally depends upon both Reynolds' number, hence the discharge, and the relative roughness of the pipe. The friction coefficient, f, given by Eq. (8)can be inserted into Eq. (28) which may be solved by trial and error. At the first trial step a preliminary discharge may be estimated by assuming wholly rough turbulent flow. In this case the friction coefficient [3, p.233, 4, p. 376, 14, p. 243] is given by:

$$f_a = \frac{1}{\left(1.14 + 2.0 \log\frac{d}{\epsilon}\right)^2} \tag{29}$$

in which f_a is the approximate friction coefficient. Therefore the approximate discharge limits, Q_{a1} and Q_{a2} are given by:

$$Q_{a1} = \sqrt{\frac{H_1}{C\left(\frac{f_a L}{d} + k\right)}} \tag{30}$$

and

$$Q_{a2} = \sqrt{\frac{H_2}{C\left(\frac{f_a L}{d} + k\right)}} \tag{31}$$

The above Q_a values are then used to evaluate R, and hence f and a second trial is made to find Q_1 and Q_2 and trials are continued till the difference between two successive estimated Q values becomes practically small. In the following the time computed taking into account the variation of f will be refered to as the exact or accurate time.

Estimation of the Approximate Time of Emptying a Reservoir

As mentioned before in estimating the time of emptying a reservoir the friction coefficient was previously assumed constant and the flow was assumed fully turbulent. Equation (7) may be used to estimate the approximate time of emptying a reservoir, taking into account that the friction coefficient is independent of the discharge. Therefore using Eq. (7) and substituting df/dQ = O:

$$T_a = -\int_{Q_{1a}}^{Q_{a2}} 2CA_r\left(\frac{f_a L}{d} + k\right) dQ \tag{32}$$

Integrating and simplifying:

$$T_a = 2CA_r\left(\frac{f_a L}{d} + k\right)\left(Q_{a1} - Q_{a2}\right) \tag{33}$$

Computer Program

It is evident that the integral of Eq. (23) is to be evaluated numerically. A computer program is presented to evaluate the actual time required to empty a cylindrical reservoir. The program is also designed to compare the accurate time with the approximate one taking into account a wide range of variables. In Fig. 2 is shown a detailed flow chart of the program.

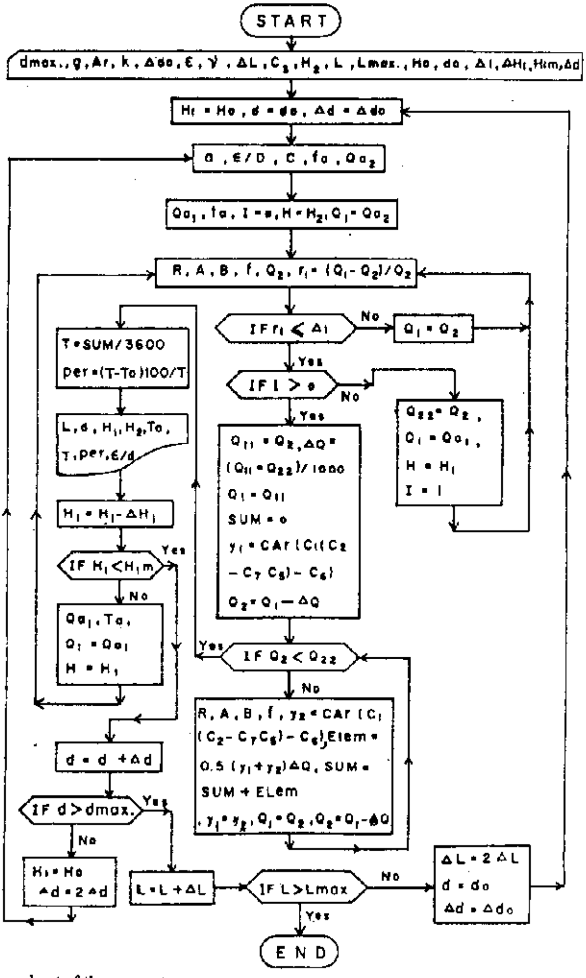

Fig. 2. Flow chart of the computer program.

Analysis of the Results

A wide range of variables are considered, in applying the comuter program of Fig. 2, to discuss and analyze the problem of emptying a cylindrical reservoir. The final acting head is fixed to the value $H_2 = 0.5$ m. On the other hand the initial head is given the values $H_1 = 30$m, 26m, 18m 14m, 10m, 6m and 2m. In all computer runs the equivalent absolute roughness is taken $\varepsilon = 0.0003$m and the kinematic viscosity $\nu = 10^6$ m^2/s (water at 22°C). The pipe diameter ranges between 0.05m and 0.8m whereas pipe length between 100m and 4000m.

In Fig. 3 is shown sketched the exact time versus the head ratio H_1/H_2 for various pipe diameters for a length of 1000m. The coefficient summing up form losses in the pipe is taken k = 10. It is evident from Fig. 3 that time increases as ε/d increases. This should be expected since high ε/d values mean more resistance to the flow and vice versa. It is also clear that curves are steeper at smaller head ratios, in other words $(\Delta T/\Delta\ (H_1/H_2)$ is greater at smaller head ratios. This is because $\Delta T/\Delta\ (H_1/H_2) = \Delta T/\Delta H_1)/H_2$, as H_2 is constant and it is clear that for the same H_1 time is longer at smaller initial heads than that at higher initial ones.

In Figs. 4 through 8 comparisons are made between exact and approximate estimations of the time of emptying cylindrical reservoirs for a variety of conditions. In Figs. 4 through 8 it can be seen that in all cases the time difference percentage increases as the pipe length increases. This may be attributed to the fact that for shorter pipe length resistance to flow is less and discharge is greater and hence Reynolds' numbers are higher. By referring to the Moody diagram it is shown that at higher Reynolds' numbers curves flaten and hence exact and approximate values of the coefficient of friction, f and fa become closer to each other. For given values of L and H_1/H_2it is clear that the time percentage difference increases as ε/d increases. This is because at higher ε/d values d is smaller and Q and hence R are smaller, and as mentioned above as R becomes smaller the difference between exact and approximate friction coefficients increases (Moody diagram).

In all curves of Figs. 4 through 8 the time difference percentage decreases as the initial head ratio H_1/H_2 increases. In fact increasing H_1/H_2 results in increasing the initial head, H_1 which corresponds to a higher value of the initial Reynolds' number. As reported above higher values of Reynolds' number mean closer values of the exact and approximate values of the friction coefficient.

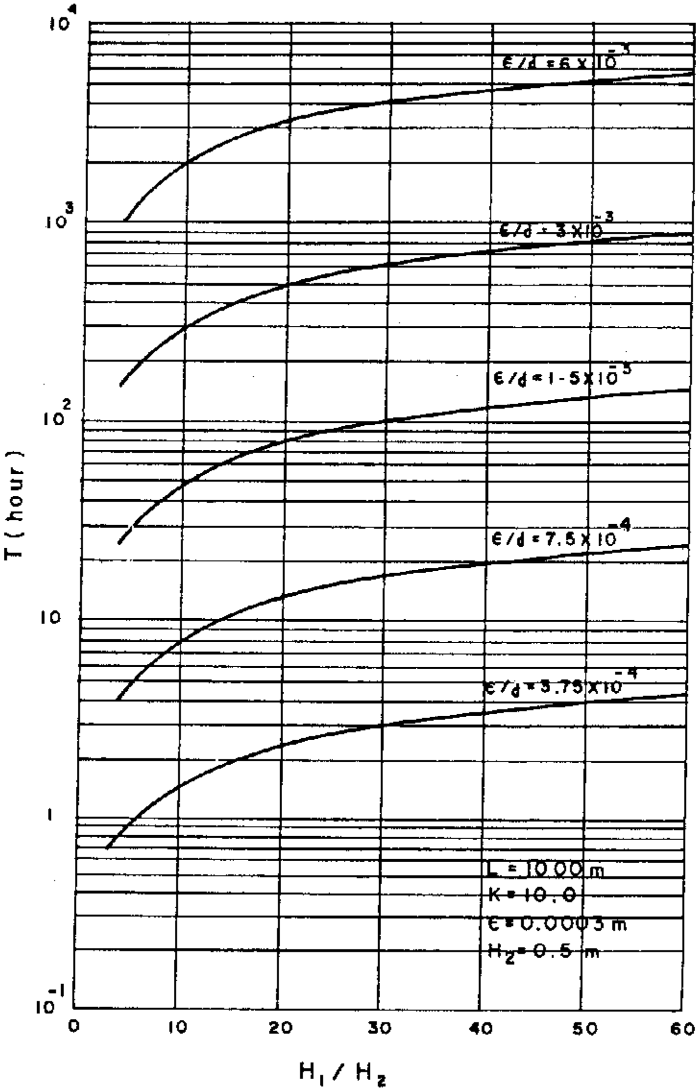

Fig. 3. Exact time of emptying a cylindrical tank versus initial to final head ratios.

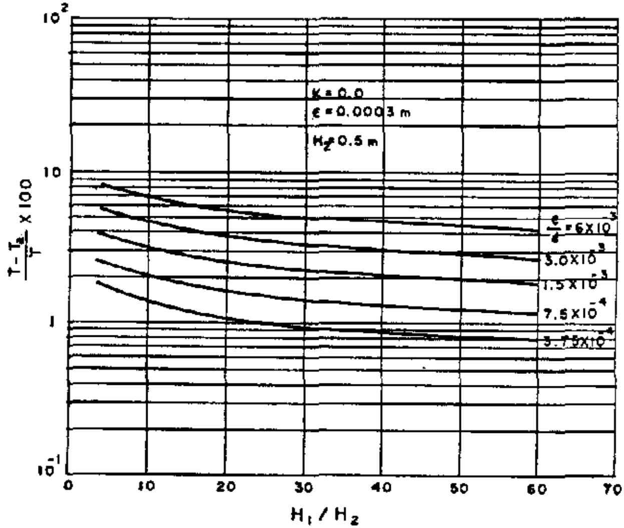

Fig. 4. Time Difference percentage versus initial head ratio (L = 100 m).

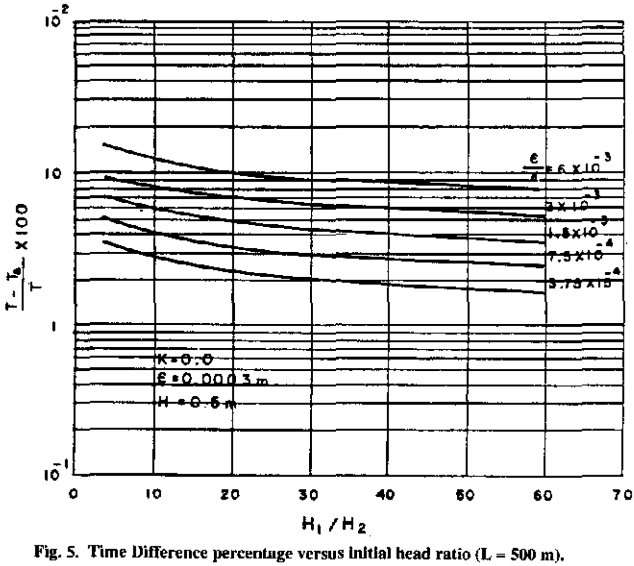

Fig. 5. Time Difference percentage versus initial head ratio (L = 500 m).

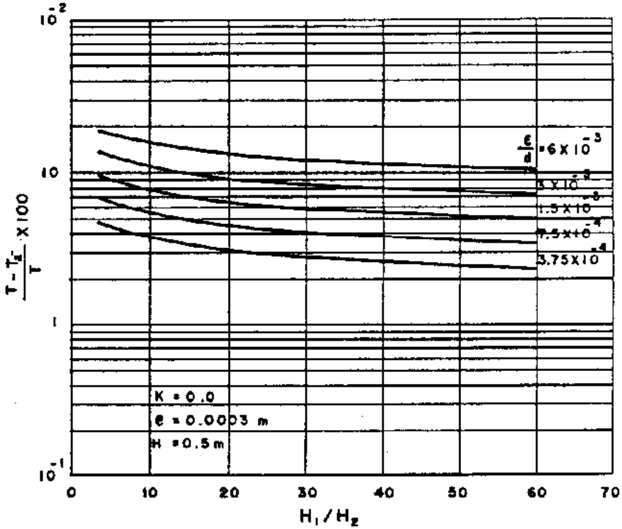

Fig. 6. Time Difference percentage versus initial head ratio (L = 1000 m).

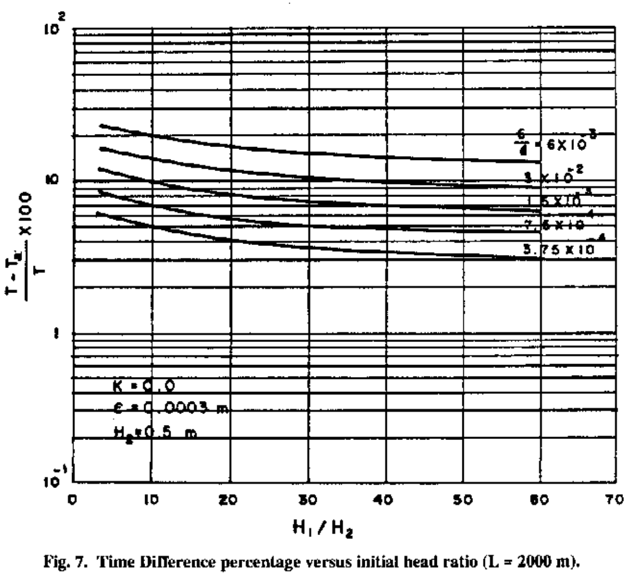

Fig. 7. Time Difference percentage versus initial head ratio (L = 2000 m).

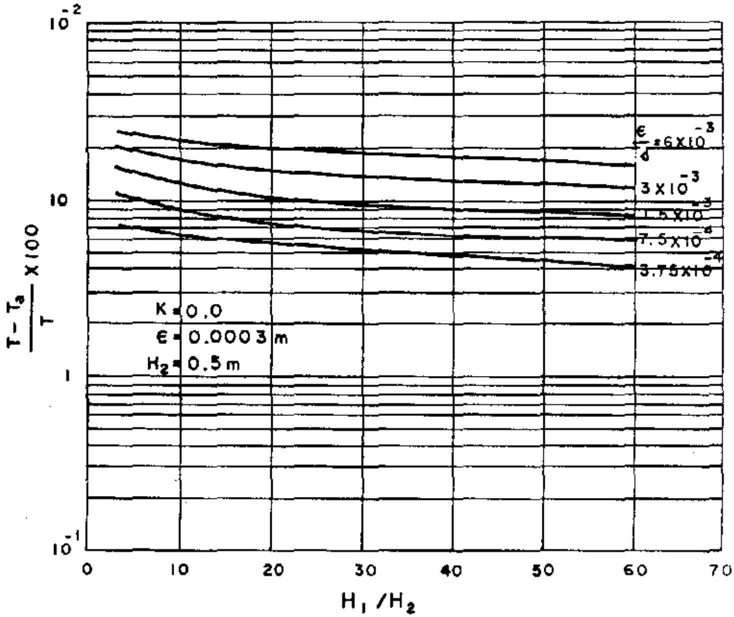

Fig. 8. Time Difference percentage versus initial head ratio (L = 4000 m).

Conclusions

A computer program is developed to provide accurate estimation of the time of emptying a reservoir connected to a pipe. The solution considers an accurate estimation of the friction coefficient. A comparison of the accurate and approximate times of emptying reservoirs shows that time estimates which consider approximate values of the friction coefficient may be in error by more than 25%. Such high error percentages may be found with long pipes, small initial heads and small pipe diameters. On the other hand if the approximate friction coefficient is considered for short pipes, high initial heads, and large pipe diameters, the estimated time may be in error by less than 1%.

References

[1] Morris, H.M. and Wiggert, J.M. Applied Hydraulics in Engineering. 2nd ed. New York: John Wiley & Sons, 1972.

[2] Featherstone, R.E. and Nalluri, C. Civil Engineering Hydraulics. London: Granada, 1982.

[3] Streeter, V. and Wylie, B. Fluid Mechanics. 1st metric ed. New York: McGraw-Hill Co., 1983.

[4] Vennard, J. and Street, R. Elementary Fluid Mchanics. 6th ed. New York: John Wiley & Sons, 1982.

[5] Hathoot, H. "Unsteady Flow Through Pipes." Bul. Faculty of Engg. Alexandria University, 26, 1(1987).

[6] Swamee, P. and Jain, A. "Explicit Equations for Pipe Flow Problems." Journal of Hydraulics Division, ASCE, 102, No. 5 (1976), 657-664.

[7] Hathoot, H.M. "Unsteady Flow Through a Pipe with Fittings." ICID Proc. 6th Afro-Asian Regional Conference, 1-5 Sept. Cairo, Egypt, 2, 1 (1987), 1-11.

[8] Churchill, S.W. "Friction-factor Equation Spans All Fluid-Flow Regimes." Chem. Engg., 84, 24 (1977), 91-92.

[9] James, L.G. Principles of Farm Irrigation System Design. New York: John Wiley & Sons, 1988.

[10] Finkel, H.J. (Ed). CRC Handbook of Irrigation Technology. Vol. 1, Boca Raton, Florida: CRC Press Inc., 1982.

[11] Von Bernuth, R.D. "Simple and Accurate Friction Loss Equation for Plastic Pipe." J. Irrig. & Drain. Engg., ASCE, 116, 2 (1990), 294-298.

[12] Hathoot, H.M. "Minimum Cost Design of Horizontal Pipelines." J. Transport. Engg., ASCE, 110, 3 (1984), 382-389.

[13] Hathoot, H.M. "Minimum Cost Design of Principles." J. Transport. Engg., ASCE, 112 2 (1986), 465-480.

[14] Cheremisinoff, P.N.; Cheremisinoff, N.P. and Cheng, S.L. (Eds.) Civil Engineering Practice. 2-Hydraulics/Mechanics. Lancaster, Pennsylvania: Technomic Pub. Co. 1988.

[15] Scaloppi, E.J. and Allen, R.G. "Hydraulics of Center Pivot Laterals." J. Irrig. and Drain. Engg., ASCE, 119, 3 (1993), 554-567.

عن حساب الزمن اللازم لتفريغ الخزانات الاسطوانية المتصلة بأنابيب

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(استُلم في ٢٠/٨/ ١٩٩٥م؛ قبل للنشر في ١٦/ ٣/ ١٩٩٦م)

ملخص البحث. تمت دراسة الحساب الدقيق للزمن اللازم لتفريغ خزان ذي مقطع ثابت المساحة متصل بأنبوب. تم خلال هذا البحث استخدام معادلة لمعامل الاحتكاك تمثل مناطق التدفق المختلفة على منحنيات مودي تمثيلا مرضيا. تم كذلك تقديم ومناقشة معادلة تقريبية لزمن تفريغ الخزان تفترض أن التدفق من النوع الاضطرابي كامل الخشونة. تم تقديم برنامج للحاسوب يمكن بواسطته حساب الزمن الدقيق والتقريبي اللازمين لتفريغ الخزان. وعلى أساس ذلك البرنامج، تم رسم مجموعة من المنحنيات ، من خلالها درست العوامل المؤثرة على حسابات الزمن. ولقد وجد أنه في بعض الحالات يكون الزمن التقريبي غير دقيق بنسبة تتجاوز ٢٥٪.