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Local Non-similarity Solutions for Mixed Convection Flow with Lateral Mass Flux over an Inclined Flat Plate Embedded in a Saturated Porous Medium

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Abstract. The problem of mixed convection along an inclined wall, acting as a source or sink with lateral mass flux, embedded in a porous medium is solved by the local non similarity method. The wall surface temperature, the flow free stream velocity and lateral surface velocity vary in a power law along the wall. The two components of buoyancy force are retained. The inclination of the plate ranges from horizontal to vertical. Numerical solutions are carried out up to the third level of truncation and results are presented for a wide range of mass flux parameter at different wall temperature distributions and inclination angles.

Nomenclature

C	=	specific heat of convicted fluid
f	=	dimensionless steam function defined by Equation (6)
f′	=	dimensionless velocity in streamwise direction
g	=	acceleration due to gravity
G	=	auxiliary velocity function, $(= \partial/\partial\xi)$
K	=	permeability of the porous medium
k	=	thermal conductivity of fluid
m	=	constant defined in Equation (4)
n	=	constant defined in Equation (5)
P	=	coefficient of the first derivative in the ordinary differential equations
Pe_x	=	Peclet number, $Pe_x = u_\infty x/\alpha$
Pe_x^*	=	modified Peclet number, $Pe_x^* = V_w x/\alpha$

q_w	=	local heat flux at the wall
Ra_x	=	local Rayleigh number, $Ra = \rho_\infty \beta g (T_w - T_\infty) x^3/\mu\alpha$
T	=	temperature
u	=	Darcy's velocity in x-direction.
υ	=	Darcy's velocity in y-direction.
V_w	=	lateral surface velocity defined in Equation (4)
x	=	coordinate along the streamwise direction.
y	=	coordinate normal to the streamwise direction.

Greek symbols

α	=	equivalent thermal diffusivity
β	=	coefficient of thermal expansion
δ_T	=	thermal boundary layer thickness
η	=	dimensionless similarity variable defined by Equations (8)
η_T	=	values of η at the edge of the thermal boundary layer
ξ	=	injection or withdrawal parameter defined by equations (9)
θ	=	dimensionless temperature defined by Equation (7)
λ	=	constant defined in Equation (4)
μ	=	viscosity of convective fluid
ν	=	kinematic viscosity of convective fluid
ρ	=	density of convective fluid
ϕ	=	inclination angle from vertical
Φ	=	auxiliary temperature function, $(= \partial\theta/\partial\xi)$
X	=	auxiliary temperature function, $(= \partial\Phi/\partial\xi)$
ψ	=	stream function

Subscripts

∞	=	conditions at the free stream
x	=	local values at x
w	=	condition at wall
f,G,H,θ,Φ,X	=	denote the differential equation associated with the particular dependent variable
m.c.	=	mixed convection
n.c.	=	natural convection

Introduction

As a continuous effort toward a complete understanding of transport phenomena in porous media, the influence of lateral mass transfer on mixed convection over an inclined plate is considered. The interest in this process is due to its applications in geothermal energy, Cheng [1]. Little work has been done to study the effect of lateral surface velocity, although a lot was made to study the convective process over impermeable surfaces. Cheng [2] discusses the problem of aiding and opposing mixed convection flow over an inclined impermeable flat plate. However, he neglects the normal buoyancy force component in the momentum equation and obtains similarity solution for a special case of free stream velocity and wall temperature distributions. As an extension to Cheng's work, Abu Romman [3] considers the same problem but the normal component of buoyancy force was retained. He uses the local non-similarity approximation to solve the problem. To the author's knowledge, the only work that shows the effect of lateral mass transfer on mixed convection is the work made by Lai and Kulacki [4] in which they studied the effect of mass transfer on mixed convection over horizontal plates. Dweik *et al.* [5] studied the effect of lateral mass flux on free convection from inclined plate embedded in a saturated porous medium using the local non-similarity approximation. In this work, the effect of lateral mass flux on mixed convection flow over an inclined flat plate is considered.

Analysis

Consider the problem of convective heat transfer in a porous medium adjacent to an inclined surface as shown in Fig. 1, which is maintained at a temperature different from that of the porous medium. The origin of the coordinate system is placed on the surface where its temperature begins to deviate from that of the ambient temperature with y and x denoting the coordinates perpendicular and parallel to the bounding surface. Figure 1 shows the physical model of two configurations (a) and (b), where (b) is a mirror image of (a). It is shown that in configuration (a) the inclination angle ϕ measured from vertical is positive in clockwise direction and negative in counter clockwise direction, while for configuration (b), ϕ is negative in clockwise direction and positive in counter clockwise direction.

Having invoked the Boussinesq and boundary layer approximations, the governing equations based on Darcy's law are given by Cheng [1]

$$\frac{\partial^2 \psi}{\partial y^2} = \pm \frac{K\rho_\infty g\beta}{\mu}\left[\frac{\partial T}{\partial y}\cos\phi - \frac{\partial T}{\partial x}\sin\phi\right] \tag{1}$$

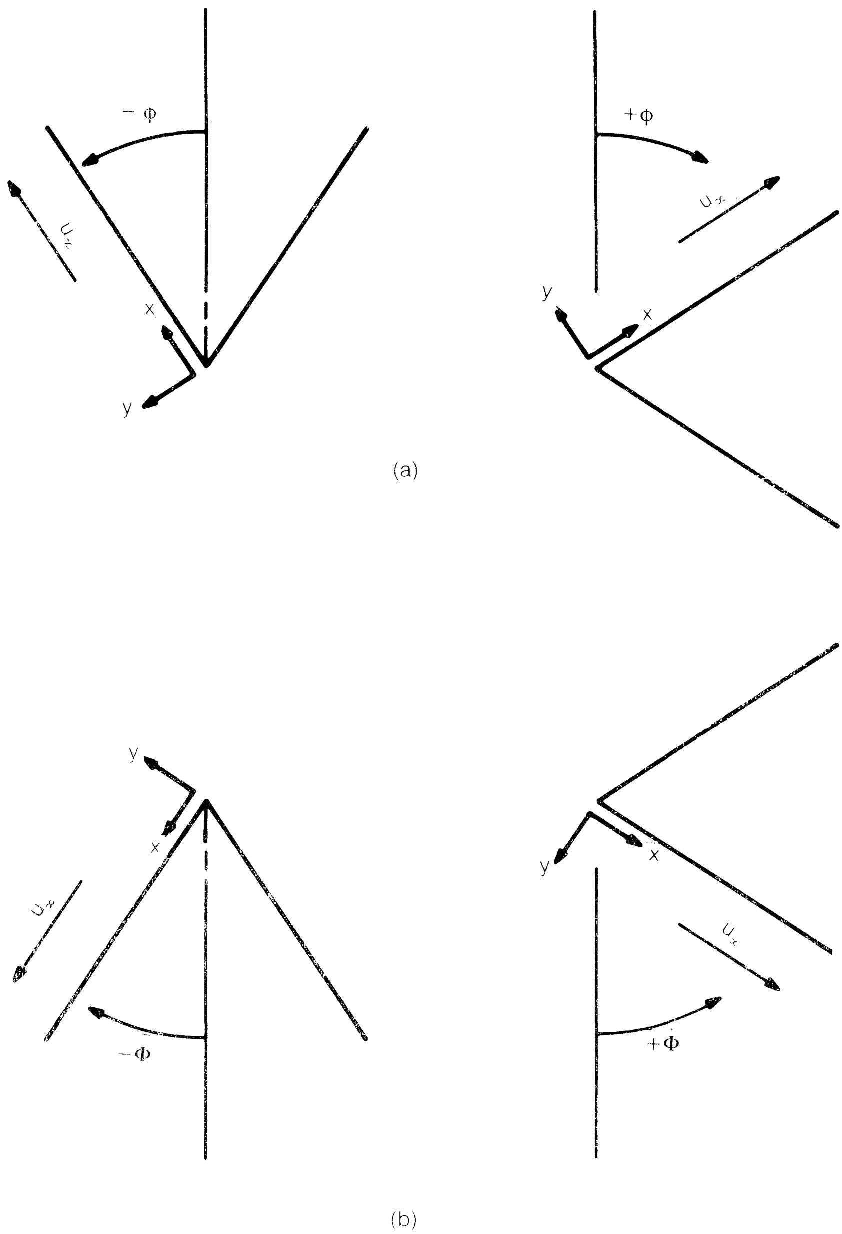

Fig. 1. Coordinate system

$$\frac{\partial^2 T}{\partial y^2} = \frac{1}{\alpha}\left[\frac{\partial T}{\partial x}\frac{\partial \psi}{\partial y} - \frac{\partial T}{\partial y}\frac{\partial \psi}{\partial x}\right] \tag{2}$$

where the stream function ψ is defined as

$$\frac{\partial \psi}{\partial y} = u \ , \ -\frac{\partial \psi}{\partial x} = \upsilon \tag{3}$$

The + ve sign on the RHS of equation (1) indicates aiding flow, while the − ve sign indicates opposing flow. This applies for both configurations (a) and (b). K is the permeability of the porous mediuum; ρ, μ, β are the density, viscosity, and thermal expansion coefficient of the convected fluid; $\alpha = k/(\rho_\infty C)_f$ is the equivalent thermal diffusivity with $(\rho_\infty C)_f$ denoting the product of density and specific heat of the convected fluid, T is the temperature; g is the gravitational acceleration. The subscript ∞ refers to the condition at the free stream. The boundary conditions considered here are:

$$\text{at} \quad y = 0 \ , \ \upsilon = V_w = Cx^m, \ T_w = T_\infty \pm Ax^\lambda \tag{4}$$

$$\text{as} \quad y \rightarrow \infty \ , \quad u = u_\infty = Bx^n, \ \ T = T_\infty \tag{5}$$

where A and B are positive constants; C is a positive constant in case of injection (*i.e.* $V_w > 0$), and negative in case of withdrawal (*i.e.* $V_w < 0$). Aiding flows are attained if the buoyancy force has a component in the free stream direction and opposing flows are attained when the buoyancy force has a component opposite to the free stream direction. Thus, aiding flows are obtained for configuration (a) if the wall is hotter than the free stream temperature (*i.e.* $T_w = T_\infty + Ax^\lambda$), and for configuration (b) if the wall is colder than the free stream temperature (*i.e.* $T_w = T_\infty - Ax^\lambda$). Opposing flows are obtained for configuration (a) if the wall is colder than the free stream and for configuration (b) if the wall is hotter than the free stream.

Equations (1) and (2) subjected to the boundary conditions (4) and (5), in general do not admit similarity solution. Thus, approximate solutions based on the local non-similarity method will now be presented. To this end, the following variables should be first introduced

$$\psi = (\alpha u_\infty x)^{1/2} f(\eta,\xi) \tag{6}$$

$$\theta(\eta,\xi) = \frac{T - T_\infty}{T_w - T_\infty} \tag{7}$$

The dimensionless independent variables η and ξ are defined as

$$\eta(x,y) = \left(\frac{u_\infty x}{\alpha}\right)^{1/2} \frac{y}{x} \tag{8}$$

$$\xi(x) = \frac{C x^{\frac{1}{2}(2m-n+1)}}{(\alpha B)^{1/2}} = \frac{Pe_x^*}{(Pe_x)^{1/2}} \tag{9}$$

where Pe_x and Pe_x^* are the local Peclet number and the modified local Peclet number defined by $Pe_x = \dfrac{(u_\infty)_x x}{\alpha}$ and $Pe_x^* = \dfrac{(V_w)_x x}{\alpha}$, respectively. It follows from equation (9) that $\xi > 0$ for $V_w > 0$ and $\xi < 0$ for $V_w < 0$. Note that $\xi = 0$ corresponds to the case of impermeable surface considered by Abu Romman [3].

Substituting equations (6)-(9) into equations (1), (2), (4) and (5) yields:

$$f'' = \pm\Omega\ [\theta'\cos\phi - \frac{1}{(Pe_x)^{1/2}}(\theta'\eta\frac{(n-1)}{2} + \frac{1}{2}(2m-n+1)\xi\frac{\partial\theta}{\partial\xi} + \lambda\theta]\ \sin\phi \tag{10}$$

$$\theta'' - \lambda\theta\ f' + \left(\frac{n+1}{2}\right) f\theta' = \frac{1}{2}(2m-n+1)\xi[f'\frac{\partial\theta}{\partial\xi} - \theta'\frac{\partial f}{\partial\xi}] \tag{11}$$

with boundary conditions given by:

$$\text{at}\ \ \eta = 0\ ,\theta(0,\xi) = 1\ ,f(0,\xi) = \frac{-2\xi}{(n+1)}[1 + (2m-n+1)\frac{\partial f}{\partial\xi}] \tag{12}$$

$$\text{as}\ \eta \rightarrow \infty\ ,\ \theta(\infty,\xi) = 0\ ,\ f'(\infty,\xi) = 1 \tag{13}$$

where '' denoting $\partial/\partial\eta$, $Gr_x = g(T_w - T_\infty)_x\, \beta kx/\nu^2$ is the modified Grashof number at x, $Re = (u_\infty)_x\, x/\nu$ is the local Reynolds number and $\Omega = (Gr/Re)_x = gA\beta k/\nu\beta x^{\lambda-n}$ is the mixed convection parameter at x. This parameter is a measure of the relative importance of free to forced convection; ϕ is the inclination from vertical; λ is the exponent in equation (4); n is the exponent in equation (5); m is the exponent in equation (4); f is the stream function and θ is the dimensionless temperature. The presence of ξ and its derivatives shows that the problem is non-similar. However, it is noted that for the special case of $m = (n - 1)/2$, all of the terms involving the partial differention of ξ in equations (10) and (11) drop out. Furthermore, the injection parameter ξ as given by (9) becomes constant and independent of x. In order to obtain similarity solution, the parameters $(\frac{Gr}{Re})_x$ and Pe_x must also be independent of x, and this happens when $\lambda = n = -1$. But at $n = -1$, the boundary condition for f becomes infinity, hence, similarity doesn't exist.

Similarity exists, as a special case, in the case of vertical wall when the normal buoyancy force (the third term of the RHS of equation (10)) vanishes when $m = (n - 1)/2$ and $\lambda = n$. This is also the case for inclined walls when the normal buoyancy is neglected. Another case where similarity exists is the horizontal plate in which similarity is obtained when $m = (n - 1)/2$ and $\lambda = (3n + 1)/2$ which is the case considered by Lai and Kulacki [4]. For impermeable plate (*i.e.* $\xi = 0$) similarity exists when $\lambda = n = -1$ which is the result obtained by Abu Romman [3].

It is also noted that for constant surface velocity (*i.e.* $m = 0$), similarity does not exist.

First Level of Truncation

The method of local non-similarity is well documented in the literature and a full description of the numerical scheme is given by Minkowycz and Sparrow [6]

To obtain the set of equations and boundary conditions for the first level of truncation, delete all the terms involving $\frac{\partial}{\partial\xi}$ in equations (10)-(13), and write the resulting ordinary differential equations in a standard form. Then the equations for the first level of truncation are:

$$f'' = Q_{1f} \tag{14}$$

$$\theta'' + P_{1\theta}\theta' = Q_{1\theta} \tag{15}$$

where

$$Q_{1f} = \pm\Omega[\theta' \cos\phi - \frac{1}{Pe^{1/2}}(\lambda\theta + \frac{n-1}{2}\eta\theta')\sin\phi] \tag{16}$$

$$P_{1\theta} = \frac{1+n}{2}\, f \tag{17}$$

$$Q_{1\theta} = \lambda\theta f \tag{18}$$

The first subscript appended to P and Q denotes the level of truncation, and the second subscript identifies the dependent variable of the differential equation to which P and Q belong.

The boundary conditions are:

$$\text{at } \eta = 0 \;, \theta(0,\xi) = 1 \;, f(0,\xi) = \frac{-2\xi}{n+1} \tag{19}$$

$$\text{as } \rightarrow \eta = \infty \;, \theta(\infty,\xi) = 0 \;, f'(\infty,\xi) = 1 \tag{20}$$

Second Level of Truncation

At the second level of truncation, equations (10)-(13) are retained without approximation. After taking the partial differential of these equations with respect to ξ, deleting all the terms involving $\frac{\partial^2 f}{\partial \xi^2}$ and $\frac{\partial^2 \theta}{\partial \xi^2}$, and substituting for $\frac{\partial f}{\partial \xi} = G$ and $\frac{\partial \theta}{\partial \xi} = \Phi$, we obtain

$$f'' = Q_{2f} \tag{21}$$

$$\theta'' + P_{2\theta}\,\theta' = Q_{2\theta} \tag{22}$$

$$G'' = Q_{2G} \tag{23}$$

$$\Phi'' + P_{2\Phi}\,\Phi' = Q_{2\Phi} \tag{24}$$

where

$$Q_{2f} = \pm\Omega[\theta'\cos\phi - \frac{1}{Pe_x^{1/2}}(\lambda\theta + \frac{1}{2}(2m-n+1)\xi\Phi + \frac{n-1}{2}\eta\theta')\sin\phi] \tag{25}$$

$$P_{2\theta} = \frac{1}{2}(2m-n+1)\xi G + \frac{1+n}{2}f \tag{26}$$

$$Q_{2\theta} = \frac{1}{2}(2m-n+1)\xi f'\Phi + \lambda\theta f' \tag{27}$$

$$Q_{2G} = \pm\Omega[\Phi'\cos\Phi - \frac{1}{Pe_x^{1/2}}(\lambda\Phi + \frac{1}{2}(2m-n+1)\Phi + \frac{n-1}{2}\eta\Phi')\ \sin\ \Phi] \tag{28}$$

$$P_{2\Phi} = P_{2\theta} \tag{29}$$

$$Q_{2\Phi} = \frac{1}{2}(2m-n+1)(\Phi G' + f'\Phi - \theta' G) - \frac{1+n}{2}\theta' G + \lambda(\theta G' + f'\Phi) \tag{30}$$

The boundary conditions are:

$$\theta(0,\xi) = 1\ ,\ f(0,\xi) = \frac{-\xi}{m+1} \tag{31}$$

$$\Phi(0,\xi) = 0\ ,\ G(0,\xi) = \frac{-1}{m+1} \tag{32}$$

and

$$\theta(\infty,\xi) = 0 \quad ,\ f'(\infty,\xi) = 1 \tag{33}$$

$$\Phi(\infty,\xi) = 0 \quad ,\ G'(\infty,\xi) = 0 \tag{34}$$

Third Level of Truncation

To obtain the equations for the third level of truncation, first, introduce new functions H and X

$$H = \frac{\partial G}{\partial \xi} = \frac{\partial^2 f}{\partial \xi^2}, X = \frac{\partial \Phi}{\partial \xi} = \frac{\partial^2 \theta}{\partial \xi^2}$$

Equations (10)-(13) are retained without approximation, as are auxiliary equations deduced by taking $\frac{\partial}{\partial \xi}$ of the above equations. Additional auxiliary equations are generated by taking the second derivative of equations (10)-(13) with respect to ξ and deleting any terms containing $\frac{\partial^3 f}{\partial \xi^3}$ and $\frac{\partial^3 \theta}{\partial \xi^3}$. The following six equations are obtained:

$$f'' = Q_{3f} \tag{35}$$

$$\theta'' + P_{3\theta}\theta' = Q_{3\theta} \tag{36}$$

$$G'' = Q_{3G} \tag{37}$$

$$\Phi'' + P_{3\Phi}\,\Phi = Q_{3\Phi} \tag{38}$$

$$H'' = Q_{3H} \tag{39}$$

$$X'' + P_{3X}\,X' = Q_{3X} \tag{40}$$

where

$$Q_{3f} = Q_{2f} \tag{41}$$

$$P_{3\theta} = P_{2\theta} \tag{42}$$

$$Q_{3\theta} = Q_{2\theta} \tag{43}$$

$$Q_{3G} = Q_{2G} - \pm\Omega\frac{1}{2}(2m - n + 1)\xi X\frac{\sin\,\phi}{Pe_x^{1/2}} \tag{44}$$

$$P_{3\Phi} = P_{2\Phi} \tag{45}$$

$$Q_{3\Phi} = Q_{2\Phi} + \frac{1}{2}\,(2m - n + 1)\,\xi\,[f'X - \theta'\,H] \tag{46}$$

$$Q_{3H} = \pm\Omega[X'\cos\phi - \frac{1}{Pe_x^{1/3}}\sin\phi(\lambda X + (2m - n + 1)X + \frac{n-1}{2}\eta X')] \quad (47)$$

$$P_{3x} = \frac{1}{2}(2m - n + 1)\xi G + \frac{n+1}{2}f \quad (48)$$

$$Q_{3x} = \Big[\frac{1}{2}(2m - n + 1)[\xi(2XG' + \Phi H' - 2H\Phi') + 2f'X + 2\Phi G' - 2\theta' H - 2G\Phi'] - \frac{n+1}{2}(2\Phi' G + \theta' H) + \lambda(\theta' H' + 2G'\Phi + f'X) \quad (49)$$

The boundary conditions are:

$$\theta(0,\xi) = 1,\ f(0,\xi) = \frac{-\xi}{m+1} \quad (50)$$

$$\Phi(0,\xi) = 0,\ G(0,\xi) = \frac{-1}{m+1} \quad (51)$$

$$X(0,\xi) = 0,\ H(0,\xi) = 0 \quad (52)$$

and

$$\theta(\infty,\xi) = 0,\ f'(\infty,\xi) = 1 \quad (53)$$

$$\Phi(\infty,\xi) = 0,\ G'(\infty,\xi) = 0 \quad (54)$$

$$X(\infty,\xi) = 0,\ H'(\infty,\xi) = 0 \quad (55)$$

Results and Discussion

Numerical solutions of the equations for the first-, second-, and third level of truncation at selected values of $\frac{Gr}{Re}$, λ,m,n,Pe,ϕ and ξ for aiding and opposing flows were carried out using the integral method described by Minkowycz and Sparrow [6]. The numerical method used in the solution of the problem is a combination of forward integration scheme (Runge - Kutta) and shooting method. Convergence of the solution is easily attainable for first and second level truncation. However, for the third level truncation, convergence becomes more difficult to obtain and needs larger number of iterations. Results for $\xi = -1.8$ to 5 are obtained (not all presented here) with very little difficulty. Divergence, if ever occurred, is overcome by selecting a proper relaxing parameter used in the solution. The results presented here are based on the third level of truncation. The results of greatest practical interest in geothermal applications are the thermal boundary layer thickness and the heat transfer rate. Figures 2 and 3 show the effect of lateral mass flux on dimensionless temperature profiles. It is shown in Fig. 2 that the fluid injection ($\xi > 0$) reduces the temperature gradients at the wall (heat transfer rates), while fluid withdrawal ($\xi < 0$) increases the temperature gradients. It is shown that an isothermal layer of fluid (at T_w) adjacent to the wall exist for large positive ξ (*i.e.* at large injection rate). This is because thermal diffusion in this layer, which is made up of fluid that has been injected, is negligible. Because of this layer, the heat transfer rate (temperature gradient at the wall) approaches zero. This layer is more pronounced for the case of forced convection (*i.e.* Gr/Re = 0) over an isothermal wall with high inclination angle [see Fig. 2(a)], due to the reduction in buoyancy force and the tangential component of buoyancy. A similar layer exists for opposing flows at relatively high Gr/Re.

The effect of Gr/Re is shown in Fig. 2(a-c). It is shown that increaing Gr/Re increases the temperature gradient at the wall due to the increase in buoyancy force. An opposite effect is seen for opposing flows [see Fig. 3(a-c)]. Increasing ϕ decreases the temperature gradient at the wall due to the reduction in the tangential component of buoyancy force, [see Fg. 2(a,d)]. An opposite effect is shown for opposing flows. If the edge of the thermal boundary layer thickness (η_T)is defined as the value of η when θ has a value of 0.01, it follows from equation (8) that the expression for the boundary layer thickness is

$$\frac{\delta_T}{x} = \frac{\eta_T}{(Pe)^{1/2}} \tag{56}$$

Selected values of $-\theta'$ $(0,\xi)$, $f'(0,\xi)$ and η_T at selected values of Gr/Re,λ,m,n,Pe,ϕ and ξ are shown in Tables 1 and 2. It is shown that the injection of fluid increases δ_T, while withdrawal reduces it.

The local surface heat flux is given by:

$$q_w(0,\xi) = -k\left(\frac{\partial T}{\partial y}\right)_{y=0} = -k(T_w - T_\infty)\theta'(0,\xi)\left(\frac{u_\infty}{\alpha x}\right)^{1/2} \tag{57}$$

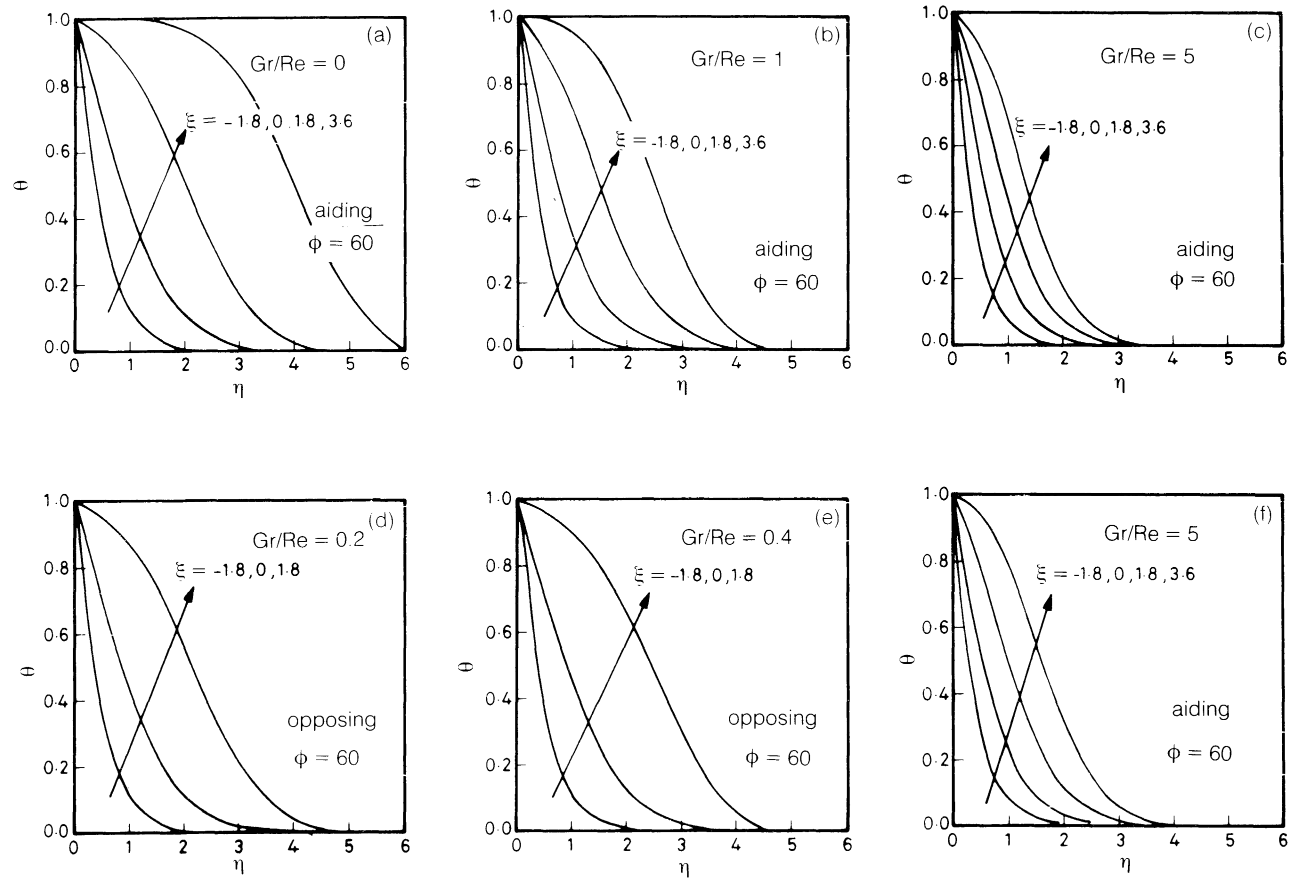

Fig. 2. Dimensionless temperature profiles at $\lambda = 0, m = 0, n = 1/3$ and $Pe = 70$

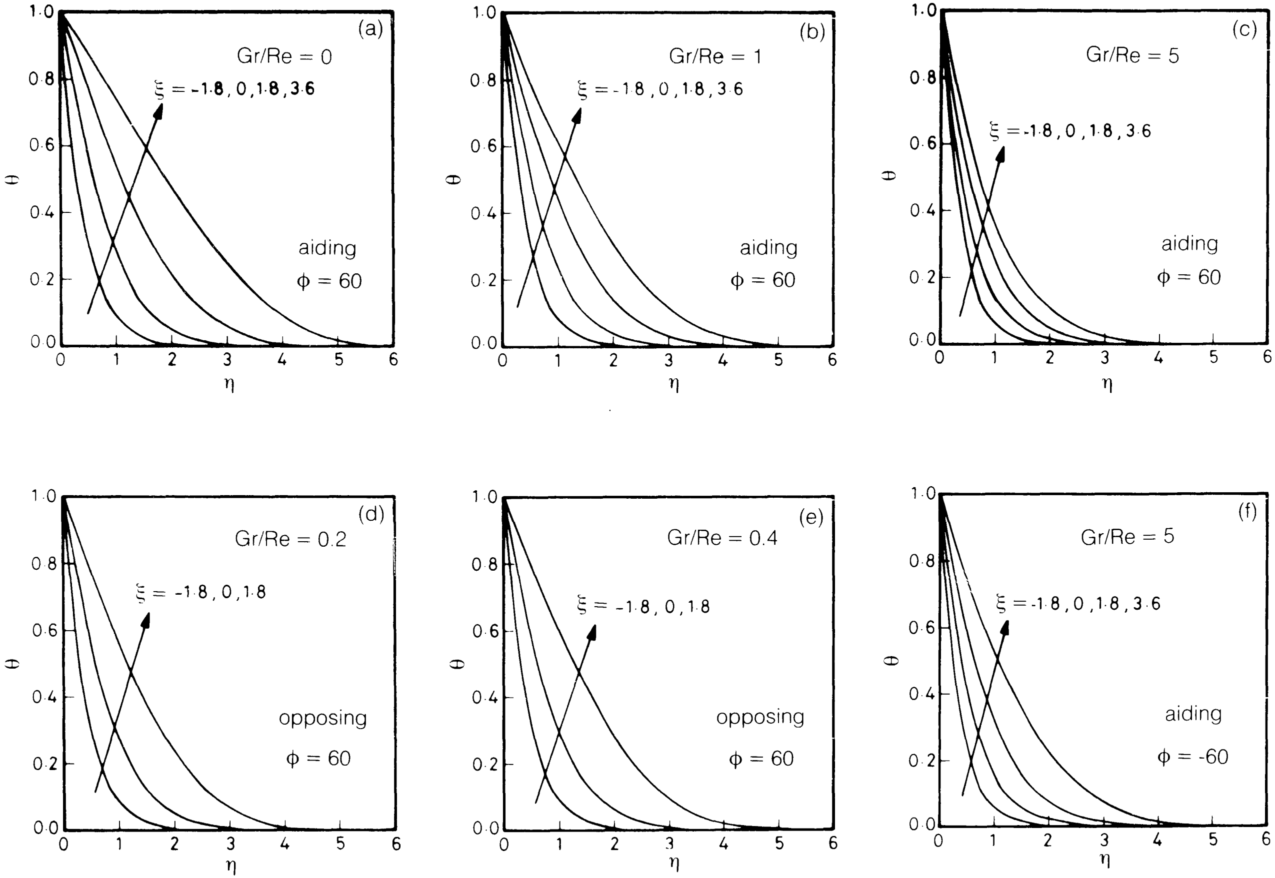

Fig. 3. Dimensionless temperature profiles at $\lambda = 1, m = 0, n = 1/3$ and $Pe = 70$

Table 1. Values of $-\theta'(0,\xi)$, $f'(0,\xi)$ and η_T for $\lambda = 1$, m = 0, n = 1/3 and $\phi = 45$

	$\xi = -1.8$			$\xi = 0$			$\xi = 1.8$		
$\frac{Gr}{Re}$	$-\theta'(0)$	$f'(0)$	η_T	$-\theta'(0)$	$f'(0)$	η_T	$-\theta'(0)$	$f'(0)$	η_T
0	2.387	1.0	1.651	1.170	1.0	2.610	0.5180	1.0	3.810
1	2.573	1.743	1.590	1.436	1.775	2.371	0.7888	1.828	3.310
3	2.899	3.221	1.481	1.847	3.291	2.102	1.190	3.388	2.751
5	3.183	4.695	1.370	2.176	4.784	1.872	1.509	4.904	2.412
8	3.355	6.891	1.271	2.588	7.001	1.661	1.909	7.138	2.101
10	3.778	8.351	1.199	2.828	8.469	1.562	2.142	8.615	1.899
50	6.652	37.21	0.75	5.788	37.38	0.850	5.054	37.577	0.985

Table 2. Values of $-\theta'(0,\xi)$, $f'(0,\xi)$ and η_T for $\lambda = 1$, m = 1, n = 1/3 and $\phi = 45$

	$\xi = -1.8$			$\xi = 1.8$		
$\frac{Gr}{Re}$	$-\theta'(0)$	$f'(0)$	η_T	$-\theta'(0)$	$f'(0)$	η_T
0	2.271	1.0	1.81	0.570	1.0	3.010
1	2.436	1.736	1.760	0.852	1.831	2.750
3	2.749	3.209	1.620	1.263	3.391	2.380
5	3.031	4.678	1.501	1.587	4.906	2.120
8	3.404	6.873	1.401	1.991	7.141	1.870
10	3.628	8.332	1.330	2.227	8.618	1.745
50	6.5187	37.194	0.799	5.152	37.584	0.931

where $\theta'(0,\xi)$ is the dimensionless temperature gradient at the wall. The effect of the surface mass flux on the surface heat transfer can be shown by the heat flux ratio:

$$\frac{q_w(0,\xi)}{q_w(0,0)} = \frac{\theta'(0,\xi)}{\theta'(0,0)} \qquad (58)$$

where $q_w(0,0)$ and $\theta'(0,0)$ are the surface heat flux and dimensionless temperature gradient without mass flux, *i.e.* $\xi = 0$ which is the case considered by Abu Romman [3]. Equation (58) is plotted as a function of the mass flux parameter ξ in Fig. 4. It is shown that the surface mass transfer increases the surface heat flux for the case of

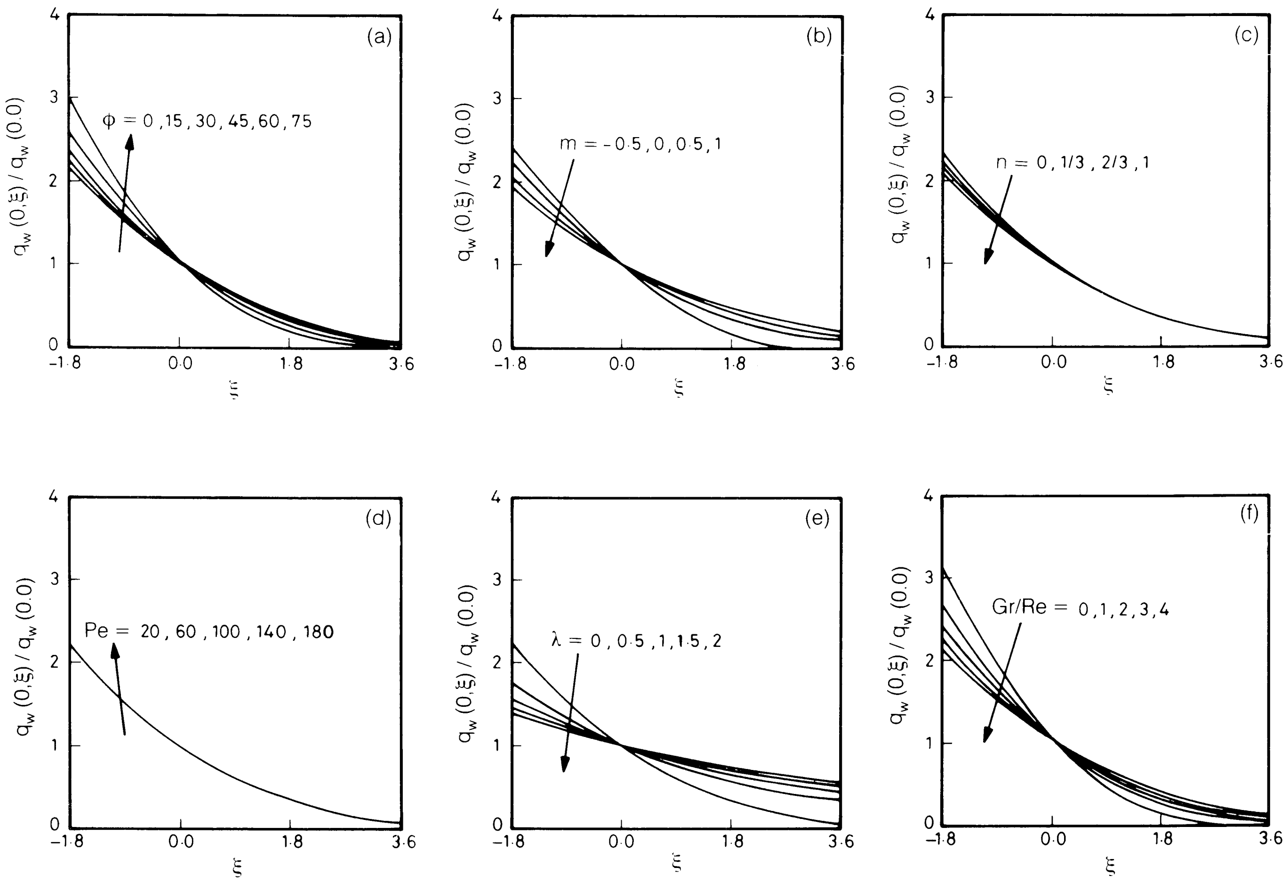

Fig. 4. Effect of mass flux on heat transfer rates for aiding flows at $\lambda = 0$, $m = 0$, $n = 1/3$, $\phi = 45$, Gr/Re =| 3 and Pe = 70

fluid withdrawal ($\xi < 0$) while opposite effect is observed for the case of fluid injection ($\xi > 0$).

The horizontal velocity slip at the wall is given by:

$$u(0,\xi) = u_\infty f'(0,\xi) \tag{59}$$

Selected values of $f'(0,\xi)$ are shown in Tables 1 and 2. Thus, the effect of lateral mass transfer on the velocity slip at the wall can be shown by the ratio:

$$\frac{u(0,\xi)}{u(0,0)} = \frac{f'(0,\xi)}{f'(0,0)} \tag{60}$$

where $u(0,0)$ and $f'(0,0)$ are the values for the case without mass transfer, *i.e.* $\xi = 0$. Equation (60) is plotted as a function of the mass flux parameter ξ in Fig. 5. It is shown that fluid injection increases the velocity slip while fluid withdrawal decreases the velocity slip.

In order to establish the criteria for pure free or mixed convection we have

$$q_w(0,\xi) = -k(T_w - T_\infty)\theta'(0,\xi)\left(\frac{u_\infty}{\alpha x}\right)^{1/2} = h(T_w - T_\infty) \tag{61}$$

rearranging:

$$\frac{Nu}{(Pe_x)^{1/2}} = -\theta'(0,\xi)_{m.c.} \tag{62}$$

and from Dweik *et al.* [5] we have:

$$\frac{Nu}{(Ra_x)^{1/3}} = -\theta'(0,\xi)_{n.c.} \tag{63}$$

where $Nu = hx/k$, the subscripts (m.c.) and (n.c.) refer to mixed convection and natural convection respectively and h is the heat transfer coefficient. By noting that $\frac{Gr}{Re} = \frac{Ra}{Pe}$, we have

$$\frac{Nu}{(Pe_x)^{1/2}} = \theta'(0,\xi)_{n.c.}\left(\frac{Gr}{Re}\right)^{1/3}\frac{1}{(Pe)^{1/6}} \tag{64}$$

The definition of ξ for free convection as given by Dweik *et al.* [5] is

$$\xi = \frac{Pe_x^*}{(Ra_x)^{1/3}} \tag{65}$$

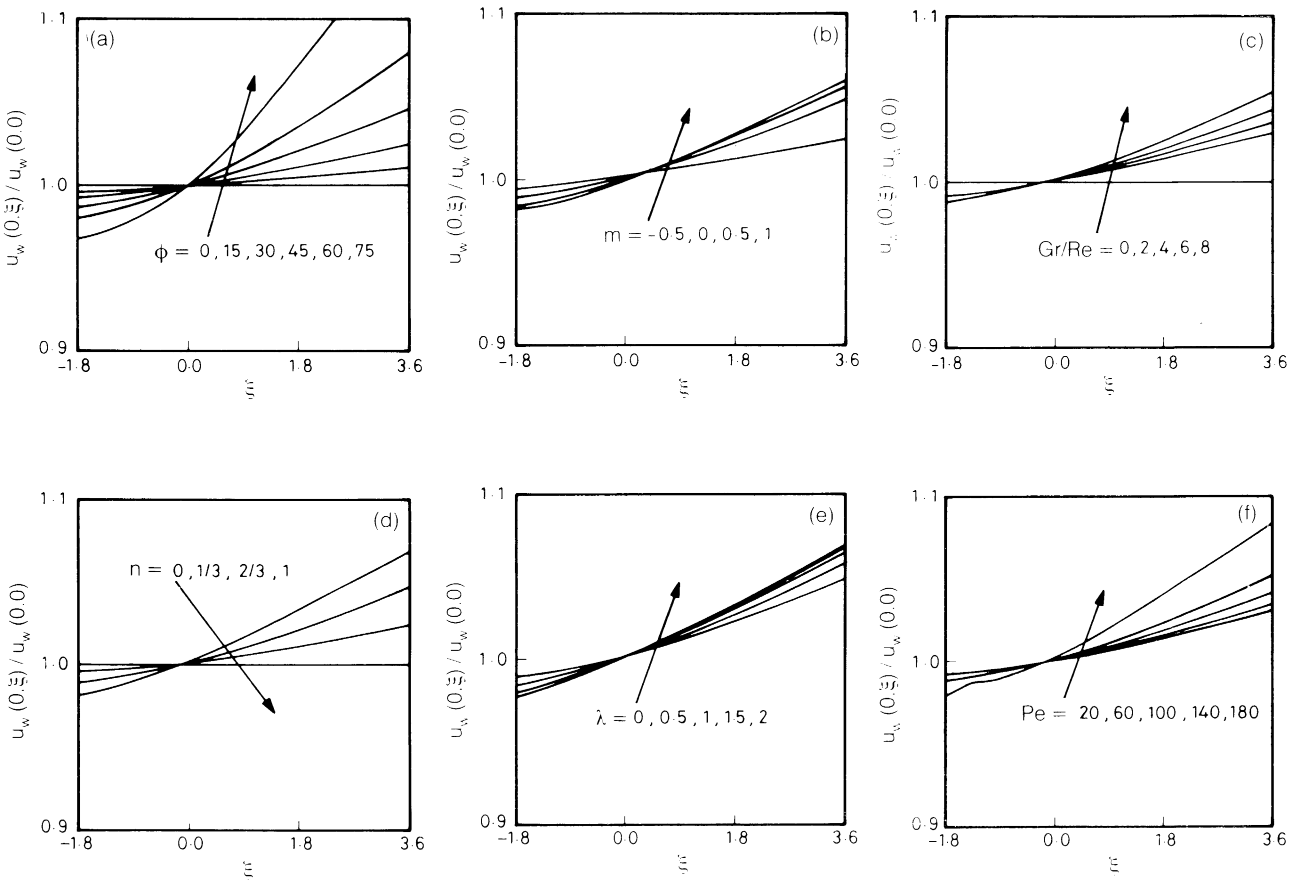

Fig. 5. Effect of mass flux on velocity slip at the wall for aiding flows at $\lambda = 0, m = 0, n = 1/3, \phi = 45, Gr/Re = 3$ and $Pe = 70$

and then

$$\frac{\xi_{n.c.}}{\xi_{m.c.}} = \frac{(Pe_x)^{1/2}}{(Ra_x)^{1/3}} \tag{66}$$

Equation (66) provides the corresponding value for $\xi_{n.c.}$ if $\xi_{m.c.}$, Pe, and Gr/Re are given. The values of $\theta'(0,\xi)$ for the forced convection case are found at $\frac{Gr}{Re} = 0$. Equation (64) is plotted in Fig. 6 as a function of $\xi_{m.c.}$ and $\frac{Gr}{Re}$. The limiting cases of free and forced convection are also shown as asymptotes in the same figure. From this figure, we see that the withdrawal of fluid makes free convection occurs at rela tively small values of $\frac{Gr}{Re}$ as compared with impermeable surface; while injection makes the free convection occurs at relatively high values of $\frac{Gr}{Re}$.

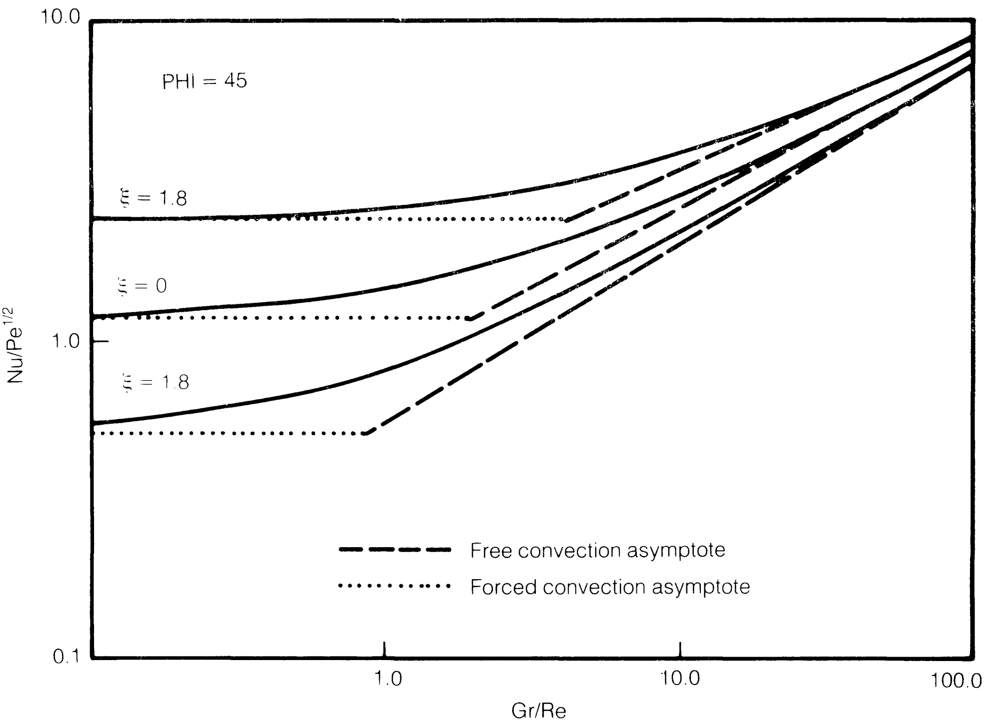

Fig. 6. Free and forced convection asymptotes for aiding flows at $\lambda = 1$, m = 0, and Pe = 70

Table 3 shows a comparison between the work by Lai and Kulacki [4] for selected values of $Ra/(Pe)^{3/2}$,n,m,λ on a horizontal plate. The injection parameter used is $f_w = \frac{2\xi}{1+n}$

Table 3. Values of $-\theta'(0,\xi)$ on a horizontal plate at λ = 0.5, n = 0, and m = − 0.5

$\frac{Ra}{P_e^{3/2}}$	$f_w = -1$	$f_w = 0$	$f_w = 1$
0	0.6337 (0.6337)	0.8862 (0.8862)	1.2008 (1.2009)
0.6	0.8036 (0.8037)	1.0279 (1.0281)	1.3120 (1.3123)
1	0.8860 (0.8862)	1.1018 (1.1020)	1.3741 (1.3745)
2	1.0449 (1.0450)	1.2493 (1.2495)	1.5037 (1.5041)
5	1.3573 (1.3575)	1.5501 (1.5503)	1.7821 (1.7825)
8	1.5721 (1.5724)	1.7607 (1.7610)	1.9831 (1.9836)
15	1.9288 (1.9290)	2.1134 (2.1137)	2.3255 (2.3261)

The values in brackets are those obtained by Lai and Kulacki [4].

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حل غير متشابه للحمل المختلط من سطح مائل مغموس في وسط مسامي
بوجود انتقال المائع باتجاه عمودي على السطح

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(استلم في ٩/٩/١٩٩٣م ؛ قبل للنشر في ٢١/٦/١٩٩٤م)

ملخص البحث. يعالج هذا البحث انتقال الحرارة بوساطة الحمل المختلط من سطح مائل مغموس في وسط مسامي بوجود انتقال المائع باتجاه عمودي على السطح. أعطى البحث أوضاعًا مختلفة من حيث وضع السطح بالنسبة للوسط المسامي، وتمّ حساب معدّلات انتقال الحرارة ومعامل انتقال الحرارة على السطح بافتراض تغيّر درجة حرارة السطح وسرعة المائع باتجاه موازي للسطح مع البعد عن مقدمة السطح ولحالات مختلفة من انتقال المائع إلى أو من خلال السطح باتجاه عمودي عليه.

خلافًا لكل الأبحاث السابقة فقد تّم ابقاء كلا المركبتين لقوة الطفو في معادلات الحركة. وقد استخدمت طريقة الحل الغير متشابه في حل المعادلات الناتجة وعرضت النتائج لقيم مختلفة من أرقام رايلي وبيكلي كما تمّ عرض النتائج لزاوية ميل السطح بحيث تتغير من ٩٠ درجة، أي سطح عمودي، إلى صفر، أي سطح أفقي.