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Induction Machine Transient Model Using Complex Variables and Trapezoidal Integration

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Abstract. This paper presents a model which predicts, with a high degree of accuracy, the dynamic response of induction machines. The model is simple, fast, accurate and numerically stable. Using the complex form of the induction machine equations along with trapezoidal integration and the fact that mechanical transients are much slower than electrical transients, the differential equations that describe the dynamic behavior of the symmetrical induction machine were transformed into algebraic equations which could be easily solved. The performance of the proposed model was evaluated using several induction machines with different horse power ratings ranging from 3 hp to 2400 hp. In all simulated cases, the proposed model was very accurate and duplicated faithfully the detailed d/q model results with a computation time as little as 25% of that required by the full-order model. Moreover, because of the numerical stability of the proposed model, computation time can be drastically reduced by using a larger time step in the simulation while maintaining reasonable accuracy.

List of Symbols

R_s, R_r = Stator and rotor phase resistances

X_{ls}, X_{lr} = Stator and rotor leakage phase reactances

X_m = Magnetizing reactance

Z_b = Base impedance

H = Inertia constant

ψ_{ds}, ψ_{qs} = d/q stator windings flux linkages

ψ_{dr}, ψ_{qr} = d/q rotor windings flux linkages

v_{ds}, v_{qs} = d/q stator windings terminal voltages
i_{ds}, i_{qs} = d/q stator windings currents
i_{dr}, i_{qr} = d/q rotor windings currents
$\omega_e, \omega_r, \omega_b$ = Synchronous (Reference Frame), rotor and base speeds
T_e, T_L = Electromagnetic and load torques
X_{ss} = $X_{ls} + X_m$
X_{rr} = $X_{lr} + X_m$
D = $X_{ss}X_{rr} - X_m^2$

Introduction

Accurate prediction of the dynamic behavior of three-phase induction machines is of prime importance in some power system studies such as power system stability studies, power system switching transients and relaying coordination. The well-established detailed fifth-order (d/q) model predicts quite accurately the dynamic response of induction machines [1-3]. This model, however, requires a large amount of computation time. Therefore, various reduced order models to predict the dynamic response of induction machines have been developed and investigated [3-15].

A traditional method of reducing the order of induction machine dynamic equations neglects the time rate of change of the stator flux linkages [4]. This approach is based on the fact that, changes in the stator flux linkages are much faster than those in the rotor flux linkages. The reduced order equations are then linearized using the small displacement approach. Such a model is investigated in [5] where some guidelines were suggested to determine when this model would be accurate. Various modifications of this method were developed in [3-7]. An investigation and comparison of three of these methods are given in [7].

A different approach that deals with load torque or voltage disturbance is reported in [9-12]. Such a model represents the dynamic behavior of an induction machine by a non-linear second order differential equation similar to the swing equation of a synchronous machine. The reduction in this model is based on the induction machine load angle and its simplified steady state equivalent circuit.

Two models have been reported in [13;14]. The model of [13] partially decoupled the detailed full-order model using linear transformation. Using the fact that mechanical transients are much slower than electrical transients, a piece-wiselinear model is derived in [14]. Both models, however, failed to accurately predict the machine dynamics right after start-up from stall. To circumvent this problem, the detailed model was initially used until the rotor speed reached a certain value, which depends on the machine ratings, after which the two models could then be used.

The machine equations can be expressed in either real or complex variable form. Despite the early introduction of complex variable analysis [16;17], the trend in the U.S.A. has been directed toward real variables except for a few isolated instances [18-20]. However, the complex variable method is extensively used outside the U.S.A. [21-23]. The complex variable analysis greatly simplified the mathematical representation of induction machines dynamics.

A fast and accurate dynamic model is reported in [15]. This model is based on the complex time variable analysis, recursive convolution and the fact that electrical transients are much faster than mechanical transients.

This paper presents an alternative simple model to predict the induction machine dynamics. Like the detailed representation, the proposed method considers electrical transients in both stator and rotor windings as well as mechanical transients. The new model was obtained using complex time variables [18], trapezoidal integration and the fact that mechanical transients are much slower than electrical transients. Trapezoidal integration is used widely in electromagnetic transients [24] and power system stability studies [25]. It is simple, self-starting, and numerically stable.

The proposed model is simple, numerically stable, very accurate and very fast as compared to the detailed model. Computation time was as little as 25% of that required by the detailed model. The proposed model and that of [15] have almost the same characteristics in terms of accuracy and computation time when using time steps smaller than or equal to 1 ms, however, the proposed model has better numeri cal stability and accuracy with time steps greater than 1 ms. Thus, the new model allows further reduction in computation time by selecting larger time steps in the simulations while maintaining reasonable accuracy. The numerical stability and the considerable saving in computation time of the new model are attributed to the trapezoidal integration which is employed in the proposed model.

Results, obtained by the proposed and the full-order models, are given for the simulations of voltage, frequency and load torque disturbances as well as acceleration from stall for small, medium and large three-phase induction machines.

Detailed Real Variables Model

Accurate prediction of induction machine transients can be obtained using the well-established detailed d/q formulation. The full order model of induction machines may be written in terms of either currents or flux linkages as state variables. In this study, flux linkages were selected as state variables because they tend to vary more slowly than currents providing more numerical stability. Using real variable analysis, the per unit equations of a symmetrical induction machine with no neutral connection may be expressed in a synchronously rotating frame of reference by [3]:

$$\frac{d\psi_{ds}}{dt} = \omega_b\left(-a_1\psi_{ds} + \frac{\omega_e}{\omega_b}\psi_{qs} + a_2\psi_{dr} + \upsilon_{ds}\right) \tag{1}$$

$$\frac{d\psi_{qs}}{dt} = \omega_b\left(-a_1\psi_{qs} - \frac{\omega_e}{\omega_b}\psi_{ds} + a_2\psi_{qr} + \upsilon_{qs}\right) \tag{2}$$

$$\frac{d\psi_{dr}}{dt} = \omega_b(a_3\psi_{ds} - a_4\psi_{dr} + s\ \psi_{qr}) \tag{3}$$

$$\frac{d\psi_{qr}}{dt} = \omega_b(a_3\psi_{qs} - a_4\psi_{qr} + s\ \psi_{dr}) \tag{4}$$

where

$$s = (\omega_e - \omega_r)/\omega_b, \tag{5}$$

$$\begin{aligned} a_1 &= R_s X_{rr}/D \ , \quad a_2 = R_s X_m / D \\ a_3 &= R_r X_m / D \ , \quad a_4 = R_r X_{ss} / D \end{aligned} \tag{6}$$

and

$$\psi_{ds} = X_{ss} i_{ds} + X_m i_{dr} \tag{7}$$

$$\psi_{qs} = X_{ss} i_{qs} + X_m i_{qr} \tag{8}$$

$$\psi_{dr} = X_m i_{ds} + X_{rr} i_{dr} \tag{9}$$

$$\psi_{qr} = X_m i_{qs} + X_{rr} i_{qr} \tag{10}$$

The mechanical equation is given by:

$$\frac{1}{\omega_b}\frac{d\omega_r}{dt} = \frac{1}{2H}\,(T_e - T_L) \tag{11}$$

with

$$T_e = \frac{X_m}{D}(\psi_{qs}\psi_{dr} - \psi_{ds}\psi_{qr}) \tag{12}$$

The d/q stator currents can be obtained from the flux linkages as follows:

$$i_{ds} = \frac{1}{D}(X_{rr}\psi_{ds} - X_m\psi_{dr}), \tag{13}$$

$$i_{qs} = \frac{1}{D}(X_{rr}\psi_{qs} - X_m\psi_{qr}), \tag{14}$$

Detailed Complex Variables Model

The machine equations can also be expressed in complex variables form. The complex variables analysis greatly simplifies the mathematical representation of induction machines dynamics. To obtain time domain complex equations for the induction machine, define the following variables [18]:

$$\begin{aligned} F_s &= F_{ds} + jF_{qs} \\ F_r &= F_{dr} + jF_{qr} \end{aligned} \tag{15}$$

where F denotes current, voltage or flux linkage. Applying these definitions to (1-4), the four real equations can be reduced to the following two complex equations:

$$\frac{d\psi_s}{dt} = \omega_b f_s \tag{16}$$

$$\frac{d\psi_r}{dt} = \omega_b f_r \tag{17}$$

where

$$f_s = -(a_1 + j\frac{\omega_e}{\omega_b})\psi_s + a_2\psi_r + v_s \tag{18}$$

$$f_r = a_3\psi_s - (a_4 + js)\psi_r \tag{19}$$

Applying relations of (15) to (7-10), the complex flux linkages can be related to the complex currents by:

$$\psi_s = X_{ss} i_s + X_m i_r \tag{20}$$

$$\psi_r = X_m i_s + X_{rr} i_r \tag{21}$$

The stator complex current and the developed electromagnetic torque are respectively related to the complex flux linkages by:

$$i_s = (X_{rr}\psi_s - X_m \psi_r)/D \tag{22}$$

$$T_e = \frac{X_m}{D} \Im\left(\psi_s \psi_r^*\right) \tag{23}$$

where * and $\Im$ denotes complex conjugate and imaginary part respectively. Moreover, (11) still represents the mechanical dynamics.

Proposed Model

Equations (16 and 17) can be integrated over one time step from $t - \Delta t$ to t as follows:

$$\int_{\psi_s(t-\Delta t)}^{\psi_s(t)} d\psi_s(\tau) = \omega_b \int_{t-\Delta t}^{t} f_s(\tau)\, d\tau \tag{24}$$

$$\int_{\psi_r(t-\Delta t)}^{\psi_r(t)} d\psi_r(\tau) = \omega_b \int_{t-\Delta t}^{t} f_r(\tau)\, d\tau \tag{25}$$

Applying the trapezoidal rule of integration to the right hand side of (24) and (25) yields:

$$\psi_s(t) - \psi_s(t-\Delta t) = \omega_b \frac{\Delta t}{2}\{f_s(t) + f_s(t-\Delta t)\} \tag{26}$$

$$\psi_r(t) - \psi_r(t-\Delta t) = \omega_b \frac{\Delta t}{2}\{f_r(t) + f_r(t-\Delta t)\} \tag{27}$$

Substituting respectively, for f_s and f_r from (18) and (19) for t and $t - \Delta t$, collecting terms and rearranging give:

$$(a_1 + \frac{2}{\omega_b \Delta t} + j\frac{\omega_e}{\omega_b})\psi_s(t) - a_2\psi_r(t) = \psi_{sh} \tag{28}$$

$$-a_3\psi_s(t) + \{a_4 + \frac{2}{\omega_b \Delta t} + js(t)\}\psi_r(t) = \psi_{rh} \tag{29}$$

where

$$\psi_{sh} = -(a_1 - \frac{2}{\omega_b \Delta t} + j\frac{\omega_e}{\omega_b})\psi_s(t-\Delta t) + a_2\psi_r(t-\Delta t) + \upsilon_s(t+\Delta t) + \upsilon_s(t) \tag{30}$$

$$\psi_{rh} = a_3\psi_s(t-\Delta t) - \{a_4 - \frac{2}{\omega_b \Delta t} + js(t-\Delta t)\}\psi_r(t-\Delta t) \tag{31}$$

It is important to note that ψ_{sh} and ψ_{rh} are known quantities at time t from past history terms at $(t - \Delta t)$ and a known term at t $(\upsilon_s(t))$.

The two differential equations, (16) and (17), have been transformed into two simple algebraic equations, (28) and (29). Equation (28) is linear while (29) is non-linear because of $s(t) = \{\omega_e - \omega_r(t)\}/\omega_b$. Since mechanical transients are slower than electrical transients, $\omega_r(t - \Delta t)$ can be substituted for $\omega_r(t)$ into the expression of s(t). With this substitution, (28) and (29) can be easily solved for $\psi_s(t)$ and $\psi_r(t)$ *i.e.*

$$\psi_s(t) = \left[\left\{a_4 + \frac{2}{\omega_b \Delta t} + js(t)\right\}\psi_{sh} + a_2\psi_{rh}\right] / D_c \tag{32}$$

$$\psi_r(t) = \left\{a_3\psi_{sh} + \left(a_1 + \frac{2}{\omega_b \Delta t} + j\frac{\omega_e}{\omega_b}\right)\psi_{rh}\right\} / D_c \tag{33}$$

where ψ_{sh} and ψ_{rh} are respectively given by (30) and (31) and,

$$s(t) = \{\omega_e - \omega_r(t-\Delta t)\} / \omega_b \tag{34}$$

$$D_c = \left(a_1 + \frac{2}{\omega_b \Delta t} + j\frac{\omega_e}{\omega_b}\right).\left\{a_4 + \frac{2}{\omega_b \Delta t} + js(t)\right\} - a_2 a_3 \tag{35}$$

Moreover, the rotor speed can be updated using the following equation which was obtained by applying the trapezoidal rule of integration to (11)

$$\frac{1}{\omega_b}\omega_r(t)=\frac{1}{\omega_b}\omega_r(t-\Delta t)+\frac{1}{4H}\left\{T_e(t)-T_L(t)+T_e(t-\Delta t)-T_L(t-\Delta t)\right\} \quad (36)$$

with $T_e(t)$ evaluated from (23).

Simulation Results

To test the accuracy of the proposed model, several induction machines ranging in size from 3 to 2400 hp were used in the simulation. The results obtained by the proposed model were compared to those obtained by the detailed d/q model. In all cases, the trapezoidal model was very accurate and it was almost impossible to distinguish between the curves obtained by the proposed model and those obtained by the full-order model. Results are presented for voltage, frequency and load-torque disturbances as well as free acceleration from stall for three induction machines (3, 820 and 2400 hp). The simulations were carried out using a Δt of 1 ms. The three machines, the parameters of which are given in Table 1, have their stator windings connected in star. It is important to note that the voltages which are given in the Table 1 are line voltages.

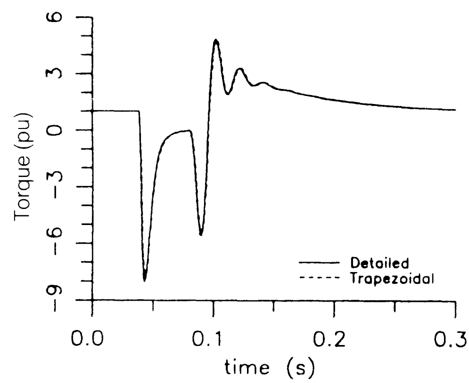
Table 1. Machines parameters

Parameters	Small m/c	Medium m/c	Large m/c
hp	3	820	2400
f (Hz)	60	50	50
# of poles	4	10	2
rpm	1725	597	2990
Voltage (V)	220	3300	11000
Z_b (Ω)	21.627	17.802	67.583
R_s (Ω)	0.435	0.0900	0.2479
R_r (Ω)	0.816	0.0893	0.2088
X_{ls} (Ω)	0.750	1.4572	5.4548
X_{lr} (Ω)	0.750	0.9526	2.7491
X_m (Ω)	26.130	25.857	164.4
J (kg-m²)	0.089	250.63	62.53

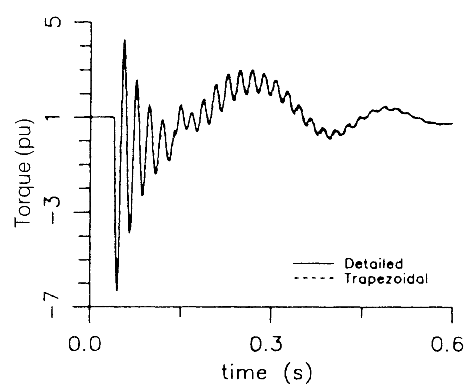
Figures 1-3 respectively show the torques, stator current magnitudes and rotor speed responses of the three machines during and after a short circuit at the stator terminals. The fault occurred at t = 0.04 s for the three machines and cleared at t = 0.08 s for the small machine, t = 0.14 s for the medium machine and t = 0.15 s for the large machine. Figures 1-3, show the responses of the small, medium and large machines respectively. In order to further test the accuracy of the proposed model, a 5% drop in the supply frequency for the small and medium machines and a 5% increase in the frequency for the large machine were simulated. Figure 4 (a-c) respectively show the torques and rotor speeds responses of the small, medium and large machines for the frequency disturbance. The disturbance was at t = 0. Moreover, load torque disturbance was also investigated for the three machines. The load torque disturbance was simulated by removing the full load torque from the machines at t = 0.04 s and returning them back to their respective machines at t = 0.08 s for the small machine and t = 0.24 s for both the medium and large machines. Figure 5 (a-c) respectively show the torque and rotor speeds responses of the small, medium and large machines for the load torque disturbance. The torques and rotor speeds responses of the small, medium and large machines during free acceleration from stall are shown in Fig. 6(a-c) respectively.

It is important to note that, although the stator current magnitudes are not shown for the frequency and load-torque disturbances as well as free acceleration, their predictions by the proposed model are also in excellent agreement with those obtained by the full-order model.

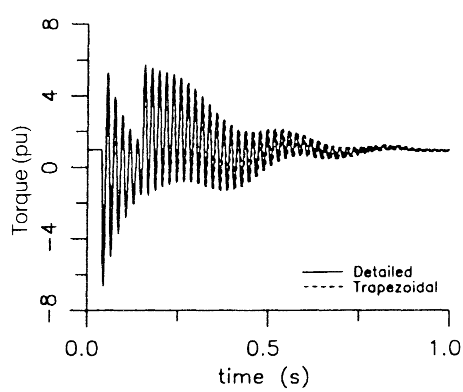
It is very clear that the curves obtained by the trapezoidal integration model follow very closely those of the detailed model. In fact they are hardly distinguishable. The proposed model faithfully duplicates the detailed model results regardless of the horse power rating of the induction machine and the type of disturbance. It was also observed that the computation time required by the trapezoidal model is as little as 25% of that required by the full-order model. The computation time of the proposed model can be reduced further by selecting a larger time step for the simulations. It is worth mentioning that the proposed model gave fairly good results with a time step of 10 ms for the simulation of voltage, frequency and load torque disturbances as well as free acceleration as clearly indicated in Figs 7 and 8. The results of Figs 7 and 8 were carried out with Δt = 1 ms and 10 ms for the detailed and proposed models respectively. In this case, the computation time required by the trapezoidal model was reduced to less than 3% of that required by the detailed model.



(a). Small machine

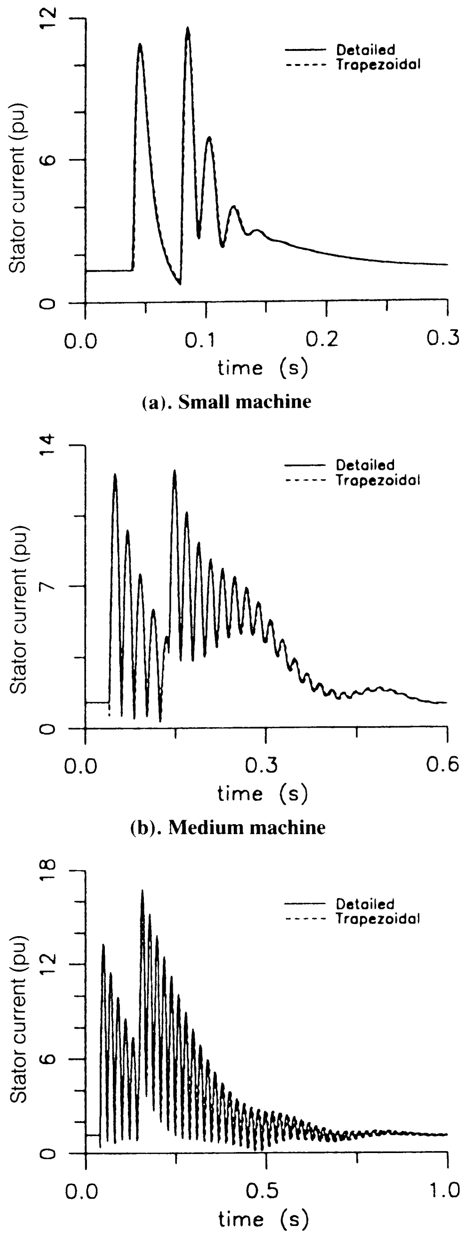


(b). Medium machine



(a). Large machine

Fig. 1. Torques responses during and after a short circuit at the stator terminals

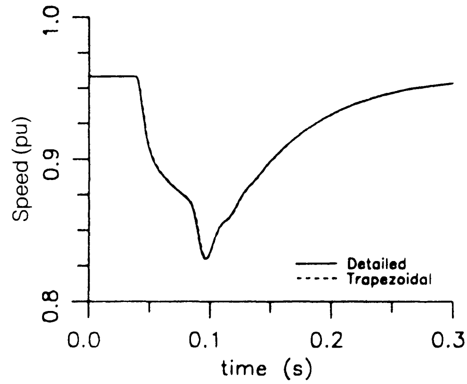


(a). Small machine

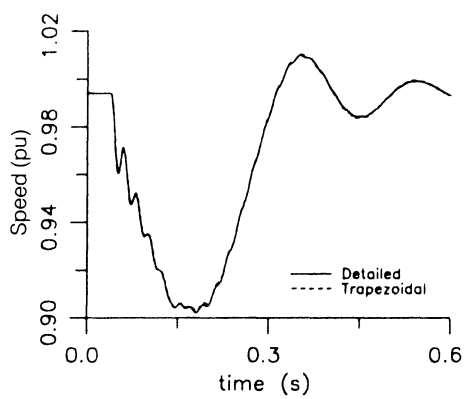
(b). Medium machine

(c). Large machine

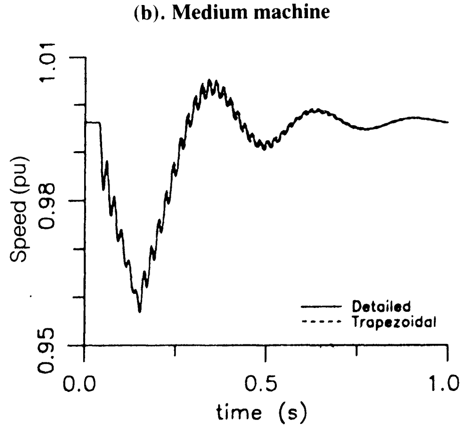
Fig. 2. Stators currents magnitudes responses (referred to the synchronously rotating reference frame) during and after a short circuit at the stator terminals



(a). Small machine

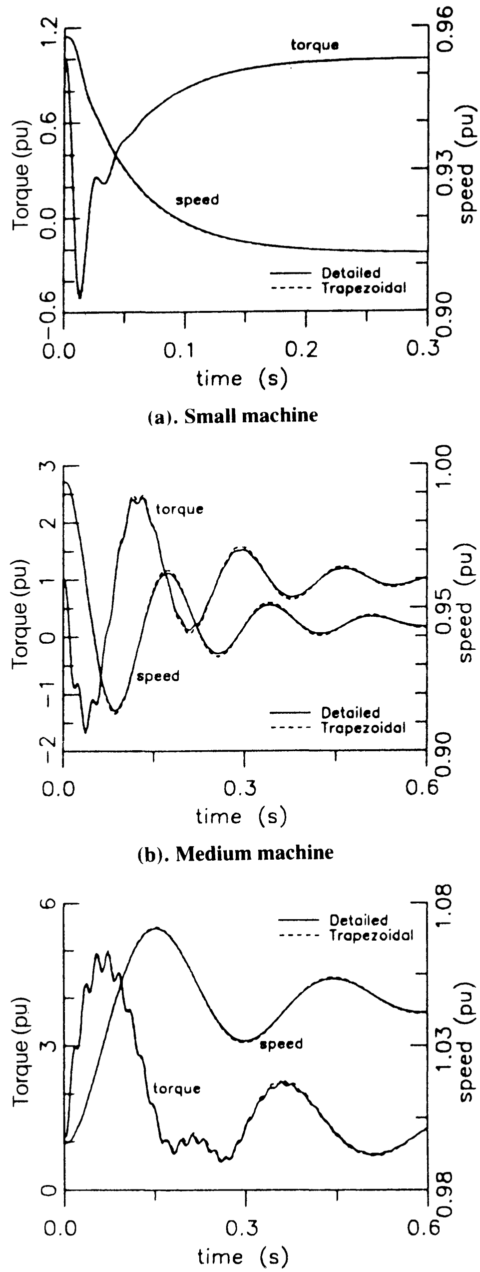


(b). Medium machine



(c). Large machine

Fig. 3. Rotors speeds responses during and after a short circuit at the stator terminals

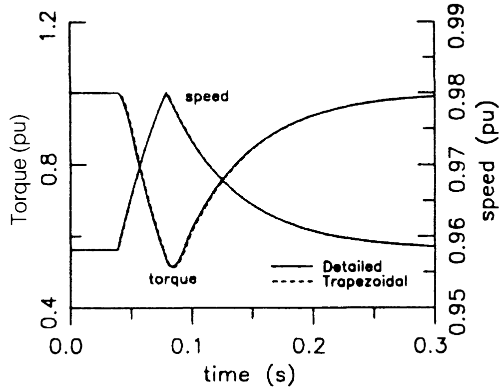


(a). Small machine

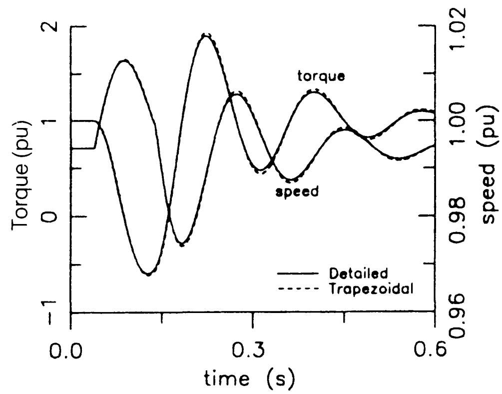
(b). Medium machine

(c). Large machine

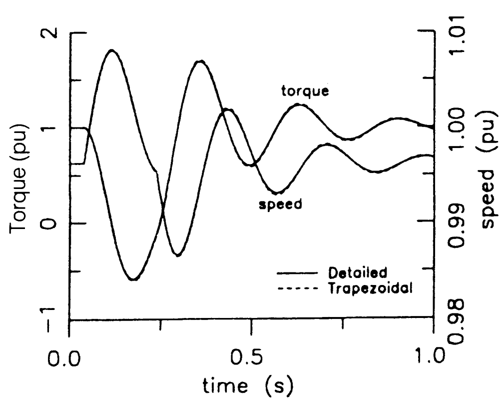
Fig. 4. Torques and Rotors speeds reswponses to frequency disturbance



(a). Small machine

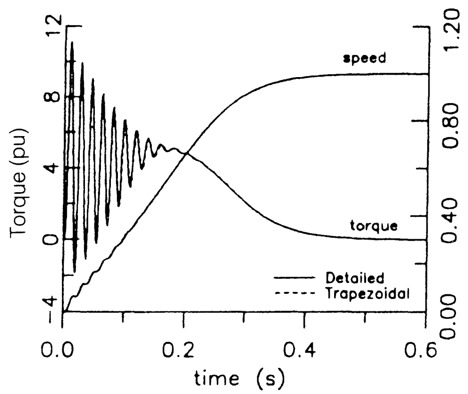


(b). Medium machine

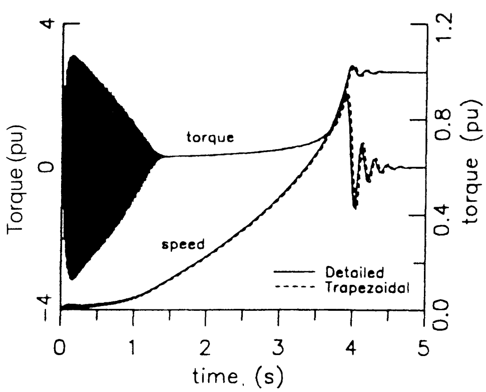


(c). Large machine

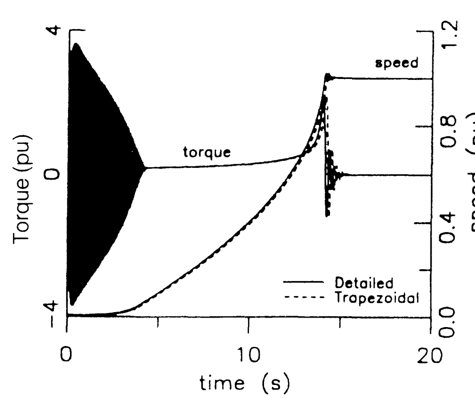
Fig. 5. Torques and Rotors speeds responses t during and after load torque disturbance



(a). Small machine

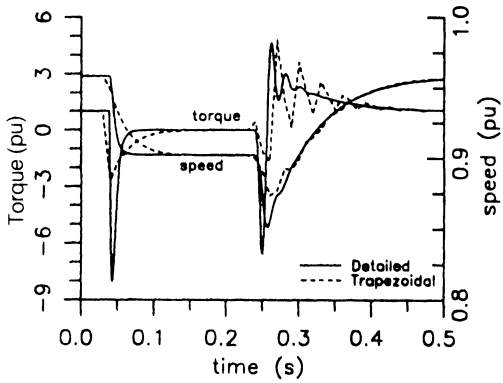


(b). Medium machine

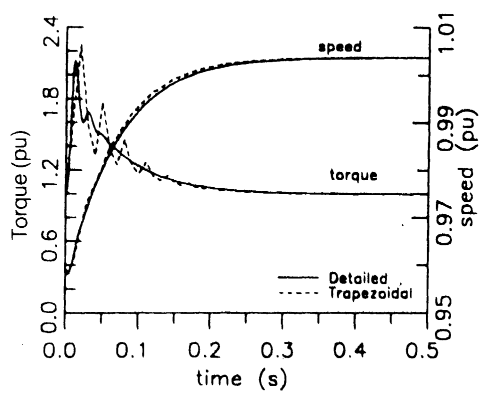


(c). Large machine

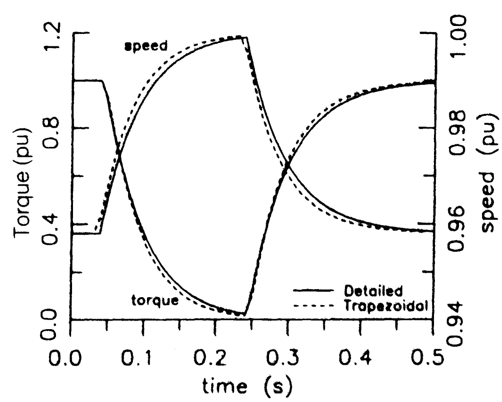
Fig. 6. Torques and Rotors speeds responses to free acceleration from stall



(a). Voltage disturbance



(b). Frequency disturbance



(c). Load-torque disturbance

Fig. 7. Small machine responses with Δt = 1 ms for the detailed model and Δt = 10 ms for the trapezoidal model

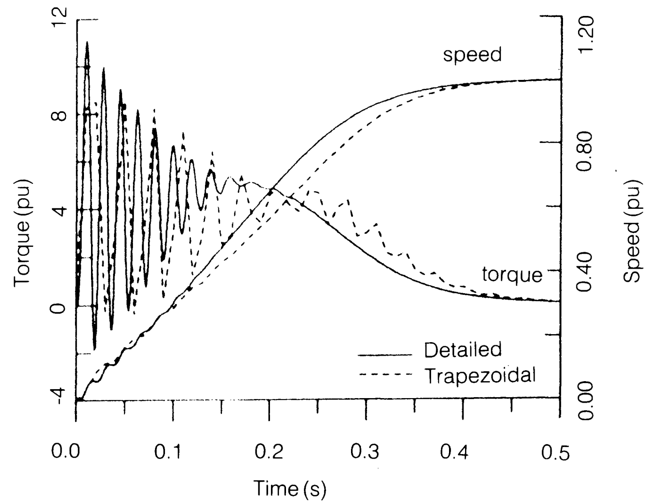


Fig. 8. Small machine free acceleration response with $\Delta t = 1$ ms for the detailed model and $\Delta t = 10$ ms for the trapezoidal model

Conclusions

A simple, fast, accurate and numerically stable dynamic model has been presented in this paper. The model is based on complex variable analysis, trapezoidal integration and the fact that mechanical transients are slower than electrical transients. The proposed model predicts the dynamic response of a three-phase induction machine with excellent accuracy regardless of the machine horse power rating and type of disturbance.

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نموذج تمثيل الحالات العابرة للمكائن الحثّية بإستخدام المتغيّرات المركّبة وقاعدة شبه المنحرف

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ملخص البحث. يقدم هذا البحث نموذجًا لحساب الاستجابة الديناميكيّة للمكائن الحثّية بدقة متناهية، وهذا النموذج سهل وسريع ودقيق ومستقر عدديًا. لقد تمّ تحويل المعادلات التفاضلية للمكائن الحثّية إلى معادلات جبريّة يمكن حلّها بسهولة وذلك باستخدام المعادلات المركّبة للمكائن الحثّية وقاعدة شبه المنحرف بالإضافة إلى حقيقة أن الحالات العابرة الميكانيكيّة أقل سرعة من مثيلاتها الكهربائيّة، وقد تمّ تقويم أداء النموذج المقترح باستخدام عدة مكائن حثّية مختلفة القدرة الحصانيّة المقدرة والتي تتراوح من ٣ إلى ٢٤٠٠ حصانًا، وكان النموذج المقترح دقيقًا جدًا في جميع الحالات وتمكن بصدق من استنساخ نتائج النموذج المحوري المفصل وفي مدة حسابيّة تقدّر بـ ٢٥٪ من المدة الحسابيّة للنموذج الكامل، وبسبب الاستقرارية العدديّة للنموذج المقترح فإنّه يمكن تقليل مدته الحسابية بصورة كبيرة باستخدام درجة زمنيّة أكبر في عمليّة التمثيل وفي الوقت نفسه الحفاظ على دقة معقولة.