Design Criteria of Drain Tube Systems in the Central Region of Saudi Arabia

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Abstract. The unsteady movement of the water-table in equally spaced drain tubes was studied in this paper. A well known equation was used to determine the drain spacing by incorporating the elimatic conditions and soil types of the central region of Saudi Arabia. The evaporation from the water table was considered in the study. Non-dimensional charts based on the drawdown requirements are provided to be used as design criteria for tile drain spacing. A design procedure is provided and two numerical examples are presented for illustration purposes.

Nomenclature

Α	=	constant contained in Eqs. 16 and 18;
a	=	constant defined by Eq. 4
B	=	constant contained in Eqs. 16 and 18;
C		constant contained in Eqs. 16 and 18;
C	=	constant contained in Eq. 6;
C ₂	=	constant contained in Eq. 6;
C ₃	=	constant contained in Eq. 6;
D	=	depth of clay layer below drains;
Ď	=	constant contained in Eqs. 17 and 18;

- E = constant contained in Eqs. 17 and 18;
- \mathbf{F} = constant contained in Eq. 17;

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F	=	constant contained in Eq. 17;
G	=	constant contained in Eq. 17;
н	=	water-table height above drains midway between two drains;
Ho	=	depth of drains below ground surface;
K	=	hydraulic conductivity of soil;
L	=	spacing between drains;
Q _d	=	drain discharge per unit length of drain;
O_c	=	evaporation rate from water-table between two drains per unit length of drain;
q	=	evaporation rate from water-table at a point y above drains;
q	=	stcady state rainfall rate;
q _o	=	evaporation rate at ground surface;
S	=	length of the water-table;
Т	=	specified time period necessary to lower the water-table a vertical distance $(H_o - H)$;
t	=	time;
x	=	the horizontal Cartesian coordinate;
у	=	the vertical Cartersian coordinate; and
μ	÷	drainable porosity.

Introduction

Many investigators have attempted to solve the problem of draining a top clay cap underlain by an impervious substratum by a system of drain tubes [1,2]. Drainage equations are used to arrive at proper drain spacing [3,4,5], considering the maximum water-table height midway between drain tubes and the steady uniform rainfall rate [6]. For unsteady state conditions, these equations can also be used by assuming the unsteady state to consist of a succession of steady state increments [7,8,9]. Although modern numerical approaches have introduced the effect of considering the circular shape of drain tubes as well as the existence of the unsaturated zone, yet they disregarded the evaporation taking place from the subsoil water-tube [10,11]. In some hot countries it was found that introducing the effect of evaporation might result in increases in design spacings between drains in the order of 60% [8,12], which means more economical spacings. The first investigator who introduced the effect of evaporation on spacing between subsurface drains was Hammad [8]. In their treatments, Hammad [8] and Hathoot [12] regarded the water-tube as a horizontal line and considered a water-table evaporation formula [13] which was predicted in Egypt.

The objective of this paper is to present design criteria for drain spacing which accounts for the curved shape of the water-table and the subsoil evaporation measurements in the Central Region of Saudi Arabia.

The Unsteady Movement of the Water-table

The unsteady movement of the water-table has been conventionally assumed to be the same as a continuous succession of steady states with the flux through the water-table assumed to be uniform and given by the drain discharge rate divided by the surface area [3,4]. If we consider the combined effect of drain tubes and subsoil evaporation, the differential equation describing the water-table depression may be put in the form:

$$-\frac{dH}{dt}\mu L = Q_d + Q_e$$
(1)

in which H is the water-table height midway betwen drain tubes, t the time, μ the drainable porosity, L the spacing between drains, Q_d drain discharge per unit length of drain, and Q_e the subsoil evaporation rate taking place between two drains per unit drain length, Fig. 1.



Drain Discharge Equation

The steady state drainage equations based on theory of flow through porous media were compared by Lovell and Youngs [2]. It was found that the watertable height can be predicted accurately from the hodograph analysis and in close agreement with that obtained by seepage theory. For intermediate depths of the impervious layer the watertable height can be determined with reasonable accuracy by various assumptions and approximations [6]. The Houghudt's equation gave close results for optimum radius in the hodograph analysis for infinite soil depth. Lovell and Youngs [2] found out that the Houghoudt's equivalent depth drainage equation can be applied with reasonable accuracy. Youngs [6] transformed this equation to the following form:

$$\frac{H}{L/2} = \left(\frac{q}{K}\right)^{1/a}$$
(2)

in which H is the water-table height midway between two drains, L the spacing between drains, q the steady state rainfall rate and K the hydraulic conductivity of soil. The coefficient is given by:

$$a = 2 \left(\frac{2D}{L}\right)^{2D/L}, \quad 0 \le \frac{2D}{L} \le 0.35$$
 (3)

$$a = 1.36$$
, $\frac{2D}{L} > 0.35$ (4)

in which D is the depth of the impervious substratum below drains, Fig. 1. Equation 2 was found to provide results of fair accuracy in the range 0.01 < q/K < 0.1, which is a wide practical range. For convenience Eq. 2 may be put in the form:

$$Q_{d} = KL \left(\frac{2H}{L}\right)^{a}$$
(5)

in which Q_d is the discharge reaching each unit length of drain.

Evaporation Losses

A field experimental study on evaporation taking place from the subsoil watertable was recently made in Riyadh where evaporation rates are considerable throughout the year [14]. These experiments were conducted on three common soils: loamy sand, sandy loam and sandy clay loam. The study suggested that the conditions in winter season should be adopted as a basis for establishing a convenient drain spacing equation since the evaporation is minimum and the water-table is maximum and hence the evaporation from the subsoil water-table was found to be of the form:

$$\frac{q}{q_0} = (1 - C_1) + C_1 \exp\left[-C_2 \left(\frac{H_0}{y} - 1\right)^{C_3}\right]$$
(6)

in which q is the evaporation rate from the subsoil water-table at a point y above drains, q_0 evaporation rate at the soil surface and H_0 depth of drains below soil surface. C_1 , C_2 and C_3 are constants depending upon the type of soil as given in (Table 1). These constants were determined through regression for winter conditions [14].

Table 1. Constants C1, C2 and C3 for different soils.

Type of soil	C1	C ₂	C ₃
Loamy sand	0.925	1.324	1.118
Sandy loam	0.946	1.423	1.131
Sandy clay loam	0.957	2.400	1.002



Fig. 2. Experimental field data compared with theory (Kirkham, 1958) and the third degree parabola.

As subsurface evaporation depends upon the depth of the point under consideration it was necessary to consider the shape of the water-table. In Fig. 2 is shown the experimental data together with Kirkham's theoretical results [15]. Unfortunately Kirkham's equation contains an infinite series which is expected to complicate any proposed mathematical treatment. Alternatively the third degree parabolic equation is tried and proved fairly coincident with field data, Fig. 2. For the major part of the water-table, differences between field data and the parabolic water-table were less than \pm 5%. The differences are noticeable at small y values, where evaporation effect is small. Considering the origin of axes to be the drain center, Fig. 1, the equation of the water-table is given as:

$$y = H \left[1 - \left(1 - \frac{2x}{L} \right)^3 \right]$$
(7)

in which H is the height of the water-table midway between drains. The total evaporation rate taking place from the water-table can be written as:

$$\mathbf{Q}_{c} = \mathbf{0}^{\mathsf{S}} \mathbf{q} \, \mathrm{ds} \tag{8}$$

in which Q_e is the evaporation rate taking place between two drains per unit drain length and s the length of the water-table. We have

$$ds = \left[\frac{1}{dx}^{2} + \frac{1}{dy}^{2}\right]^{1/2}$$
(9)

Differentiating Eq. 7 with respect to x and rearranging:

$$\overline{dx}^{2} = \frac{\overline{dy}^{2}}{H^{2} \left[\frac{6}{L} \left(1 - \frac{2x}{L}\right)^{2}\right]^{2}}$$
(10)

substituting from Eq. 10 into Eq. 9:

$$ds = \left[\frac{1}{H^2 \left[\frac{6}{L} \left(1 - \frac{2x}{L} \right)^2 \right]^2} + 1 \right]^{1/2} dy$$
(11)

but from Eq. 7 we have

$$(1 - \frac{2x}{L}) = (1 - \frac{y}{H})^{1/3}$$
 (12)

substituting into Eq. 11 and simplifying:

ds =
$$\begin{bmatrix} \frac{L^2}{36H^2 (1 - \frac{y}{H})^{4/3}} + 1 \end{bmatrix}^{1/2} dy$$
 (13)

now substituting the value of ds as given by Eq. 13 and the value of q as given by Eq. 6 into Eq. 8 we get:

$$Q_{c} = 2 \int_{0}^{H} \left[\frac{L^{2}}{36H^{2} (1 - \frac{y}{H})^{4/3}} \right]^{1/2} q_{o} \{ (1 - C_{1}) + \frac{y}{H} \}^{1/2} = 0$$

$$C_1 \exp \left[-C_2 \left(\frac{H_0}{y} - 1\right)^{C_3}\right]$$
 dy (14)

For convenience Eq. 14 is rearranged and put in the form:

$$\frac{Q_e}{q_0 L} = 2 \frac{H_0}{H} \int^{H/H_0} \left[\frac{1}{36 \left(1 - \frac{y}{H_0} \frac{H_0}{H}\right)^{4/3}} + \left(\frac{H}{L}\right)^2 \right]^{1/2}$$

$$\left\{ \left(1 - C_1\right) + C_1 \exp\left[-C_2\left(\frac{H_0}{y} - 1\right)^{C_3}\right] \right\} d\left(\frac{y}{H_0}\right)$$
(15)

Equation 15 has been numerically integrated for H/L between 0.01 and 0.22, and H/H₀ between 0.2 and 1.0. The results are presented for loamy sand, sandy loam and sandy clay loam in Figs. 3,4 and 5, respectively. It is evident from the above figures that curves are straight lines with very small slopes. If we nelegect the effect of H/L on the evaporation ratio Q_e/Lq_0 and consider intermediate values of the



Fig. 3. Evaporation ratio versus water table height ratio for loamy sand.



Fig. 4. Evaporation ratio versus water table height ratio for sandy loam.



Fig. 5. Evaporation ratio versus water table height ratio for sandy clay loam.



Fig. 6. Evaporation ratio versus intermediate head ratio.

evaporation ratio, it is found that the error is generally less than $\pm 3\%$ for the extreme ends of each curve. For interior values, which are more common in practice, the error is much less than $\pm 3\%$. In Fig. 6 is plotted the evaporation ratio Q_e/Lq_0 versus intermediate values of H/H_0 for the three types of soil under consideration. Prior to study the unsteady movement of the water-table by applying Eq. 1, curves of Fig. 6 should be conveniently represented by mathematical equations. For loamy sand it is found that the following polynomials fit the corresponding curve of Fig. 6:

$$\frac{Q_e}{Lq_0} = A + B \left(\frac{H}{H_0}\right) + C \left(\frac{H}{H_0}\right)^2, \quad 0.2 < \frac{H}{H_0} \le 0.4 \quad (16)$$

$$\frac{Q_e}{Lq_0} = D + E \left(\frac{H}{H_0}\right) + F \left(\frac{H}{H_0}\right)^2 + G \left(\frac{H}{H_0}\right)^3 , \quad \left(\frac{H}{H_0}\right) > 0.4$$
(17)

On the other hand curves corresponding to sandy loam and sandy clay loam are fitted to the following polynomial;

$$\frac{Q_e}{Lq_0} = A + B \left(\frac{H}{H_0}\right) + C \left(\frac{H}{H_0}\right)^2 + D \left(\frac{H}{H_0}\right)^3 + E \left(\frac{H}{H_0}\right)^4$$
(18)

the values of the constants are given in (Table 2).

Type of sail	A	В	С	D
Loamy sand	0.1716	-0.843165	1,87055	0.1084679
Sandy loam	0.2081648	-1.4075392	3.6206104	-2.0022708
Sandy clay knam	0.0305129	0.2601291	-1.5220624	3.0696458
Type of soil		E	F	G
Loamy sand	· · · ·	-0.717247	2.3777	-1.06852
Sandy loam		0.2659895		
Sandy clay loam		-1.2446875		

Table 2. Constants contained in Eqs. 16 through 18

Differences between curves of Fig. 6 and the corresponding polynomials are found, in general, to be less than $\pm 1\%$.

Design of Drain Spacing

As the expressions for both the discharge taken by drain tubes and by evaporation are available, Eq. 1 may be put in the following form:

$$-\frac{dH}{dt} \mu L = KL F(\frac{H}{H_0}, \frac{H_0}{L}, \frac{D}{L}) + q_0 L f(\frac{H}{H_0})$$
(19)

in which

$$F\left(\frac{H}{H_0}, \frac{H_0}{L}, \frac{D}{L}\right) = \left(2\frac{H}{H_0}\frac{H_0}{L}\right)^a$$
(20)

and

$$f\left(\frac{H}{H_0}\right) = \frac{Q_e}{L_{q_0}}$$
(21)

Rearranging Eq. 19 we get

$$\frac{-d(H/H_0)}{dt/\mu H_0/K} = F(\frac{H}{H_0}, \frac{H_0}{L}, \frac{D}{L}) + \frac{q_0}{K}f(\frac{H}{H_0})$$
(22)

Separating variables and setting integration limits:

$$\int_{0}^{t} \frac{dt}{(\mu H_{0}/K)} = \frac{(H/H_{0})_{2}}{(H/H_{0})_{1}} \frac{d(H/H_{0})}{F(\frac{H}{H_{0}}, \frac{H_{0}}{L}, \frac{D}{L}) + \frac{q_{0}}{K} f(\frac{H}{H_{0}})}$$
(23)

In the drawdown requirements of plant, the root zone should be cleared within a specified time period, T, which depend on the kind of plant, otherwise the plant would die [8]. If we consider the worst condition in which the soil is completely water logged after a heavy rainfall or just after excessive irrigation, we have $(H/H_0)_1 = 1.0$, and the lower limit $(H/H_0)_2$ will depend upon the depth of the root zone of plant, $(H_0 - H)$. Considering the above statements in Eq. 23 we get:

$$\frac{T}{(\mu H_0/K)} = \frac{1.0}{(H/H_0)_2} \frac{d(H/H_0)}{F(\frac{H}{H_0}, \frac{H_0}{L}, \frac{D}{L}) + \frac{q_0}{K} f(\frac{H}{H_0})}$$
(24)



Fig. 7. Drain spacing design chart for loamy sand.



Fig. 7. Continued.



Fig. 8. Drain spacing design chart for sand clay loam.



Fig. 8. Continued.



Fig. 9. Drain spacing design chart for sandy clay loam



Fig. 9, Continued.

Since Eq. 24 is non-integrable the time ratio $T/(\mu H_o/K)$ may be evaluated numerically. It is of practical importance to provide graphs for the time ratio as a function of the variables included in Eq. 24 [16]. Since, μ ,K and the constants contained in $f(H/H_o)$ are characteristics of the soil, it is convenient to provide graphs for each individual soil. As design should, in general, be based on the worst expected conditions, the minimum value of q_o is considered. Graphs of Figs. 7, 8 and 9 are devoted to loamy sand, sandy loam and sandy clay loam, respectively.

In each figure, the time ratio $T/(\mu H_0/K)$ is plotted versus the drain depth ratio H_0/L . Groups of curves are presented such that each group corresponds to a certain lower limit $(H/H_0)_2$, which in turn ranges between 0.9 and 0.2. Within each group curves are plotted for $D/L \approx 0.005$, 0.02, 0.04, 0.08 and 0.175 or more. The above mentioned ranges of variable are chosen to cover the conditions that might be found in practice. Each group is presented in a separate graph since there is interference between some successive groups which may cause confusion and perhaps incorrect predicted values. Figures 7.8 and 9 can be used as design charts for drain spacing and the following procedure is recommended:

- 1. the following data should be available in advance: type of soil (μ, K) , depth of drains, H_{α} , the new depth H, or $(H/H_{\alpha})_2$, and the time T.
- 2. evaluate the time ratio $T/(\mu H_0/K)$.
- 3. assume D/L, and hence L.
- 4. for the figure corresponding to the type of soil and the $(H/H_0)_2$ group locate the value of the time ratio on the vertical axis and draw a horizontal line.
- 5. the point of intersection of the horizontal line with the curve corresponding to the assumed D/L value will correspond in turn to an H_0/L value.
- 6. as H_0 is known the value of L may be evaluated.
- 7. if the assumed and predicted values of L are the same, they will represent the required spacing, otherwise more trials should be made.

Numerical example 1

In a loamy sand soil where $\mu = 0.036$ and K = 0.3 m/day drains are installed 2.0 m below ground surface and are 2.0 m above an impervious substratum. It is required to design the drain spacing to satisfy lowering the water-table midway between drains from 2.0 m to 1.8 m within 4.0 days.

Solution

Since the soil is loamy sand, we have to use Fig. 7.

Design Criteria of Drain Tube Systems ...

$$\frac{\text{TK}}{\mu\text{H}_0} = \frac{4.0(0 \cdot 3)}{0.036 \cdot (2.0)} = 16.67$$
$$\left(\frac{\text{H}}{\text{H}_0}\right)_2 = \frac{1.8}{2.0} = 0.9$$

Now assume D/L = 0.01, i.e. L = 200.0 m from Fig. 7(a) for the above values we get $H_0/L = 0.015$ from which L = 133.3 m. As a second trial consider D/L = 0.02, i.e. L = 100 m, for the same time ratio we get: $H_0/L = 0.013$, from which L = 153.8 m. Considering the above trails if we assume D/L = 0.167 from which L = 120.0 m, we have $H_0/D = 0.016$ from which L = 125.0 m. Therefore, the design spacing may be taken as L = 120 m. If we want to see to what extent evaporation affects the spacing let us apply the following equation in which evaporation is neglected, [6]:

$$\frac{\mathrm{KT}}{\mu \,(\,\mathrm{L}\,/\,2\,)^{\mathrm{a}}} = \frac{1}{\mathrm{a}-1} \,(\,\mathrm{H}^{1\cdot\mathrm{a}} - \,\mathrm{H}^{1\cdot\mathrm{a}}_{\mathrm{0}}\,) \tag{25}$$

where a is given by Eq. 4.

Solving Eqs. 25 and 2 by trial and error we get the spacing L = 75.0 m. It is evident that a saving of about 60% is achieved when considering the effect of evaporation.

Numerical example 2

It is required to design the spacing between drain tubes in a system serving sandy-loam soil ($\mu = 0.038$ and K = 0.27 m/day). Drains are to be installed 1.8 m below ground surface and they are 10.0 m above an impervious substratum. To satisfy the drawdown requirements 60.0 cm of the topsoil should be drained within 5.0 days.

Solution

$$H = 1.8 - 0.6 = 1.2 m$$

$$H/H_0 = \frac{1.2}{1.8} = 0.667$$

Therefore we have to deal with Fig. 8(c) in which $H/H_0 = 0.7$ and Fig. 8(d) in which $H/H_0 = 0.6$.

The time ratio $= \frac{TK}{\mu H_0} = \frac{5.0(0.27)}{0.038(1.8)} = 19.74$

For Fig 8(c) successive trials yield L = 71.7 m, (D/L = 0.139 and H₀/L = 0.0251).

From Fig. 8(d) for D/L = 0.183 and $H_0/L = 0.033$ we get L = 45.5 m.

By interpolation, considering the two predicted values of L, we get the required spacing as

L = 71.7 -
$$\frac{0.7 - 0.667}{0.7 - 0.6}$$
 (71.7 - 54.5)

 $= 66.0 \,\mathrm{m}.$

It is worthy to note that if the effect of evaporation is neglected the spacing = 56.5 m, the percentage saving being about 17%.

Conclusion

Through the solution of two numerical examples it is shown that the spacing design charts provided in this paper are practical and cover a wide range of the variables. Consideing the evaporation, that takes place from the subsoil water-table, is of special importance in hot and dry climatic regions since it results in more economical drain spacings with corresponding savings that may exceed 60%. Special considerations must be given to evaporation measurements at sites prior to apply the equation in design.

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ملخص البحث. تم في هذا البحث دراسة الحركة غير المستقرة لسطح الماء الحُرُّ في حالة تسرب المياه تجاه أنابيب صرف ذات مسافات بينية متساوية ، تركزت الدراسة حول الظروف المناخية وأنواع التربة الزراعية الشائعة في المنطقة الوسطى من المملكة.

استخدمت معادلة دولية معروفة لحساب التصرف لكل وحدة طول من أنابيب الصرف . لقد تم إدخال تأثير التبخر الحادث من سطح الماء الحُرَّ الذي أُخِذَ شكله المنحني في الاعتبار.

تم رسم مجمـوعة من المنحنيات اللابعدية التي يمكن استخدامها في تصميم المسافة بين أنابيب الصرف وقد تمت حسابات المنحنيات على أساس احتياجات النبات في صرف الماء من منطقة الجذور في زمن مناسب.

قدم البـاحثـون الخطوات الواجب اتباعها عند استخدام المنحنيات التصميمية كما قدموا مثالين عدديين لتوضيح ذلك.