J. King Saud Univ., Vol. 5, *Eng. Sci.* (1), pp. 91 - 103 (A.H. 1413/1993)

MECHANICAL ENGINEERING

Empirical Correlations of Solar and Other Weather Parameters

Mahmoud S. Audi
Mechanical Engineering Department, University of Jordan, Amman, Jordan
(Received 29/1/1991; Accepted for Publication 20/7/1991)

Abstract. This paper presents the results of statistical analysis of extended data of nine solar and other weather parameters of the capital city area of Jordan. The regression models developed in this paper include the prediction of air temperatures, relative humidity, atmospheric pressures, daily sunshine hours, and daily solar energy incident on this area. It also includes four models developed in this research for the prediction of the clearness index of solar radiation in this area. This parameter besides its intrinsic use is needed in the prediction models of diffuse as well as direct solar radiation.

Introduction

Simulation as a design and research tool has advanced considerably during the past two decades. The aim of simulation is to predict the behavior of systems under varying inputs in a speedy and cost effective manner. The digital computer of all sizes has helped in achieving these two requirements. However, numerical simulation of systems requires mathematical models of these systems and the energy input and output parameters they interact with.

For example, in simulated studies of buildings for optimum energy consumption to maintain a desired indoors environment requires modelling of the heating and cooling loads of these buildings under varying weather and other conditions. This in turn requires mathematical representation of the weather parameters of the region where the buildings are located [1].

This paper presents prediction models for solar energy and other weather parameters that could be used in the design and simulation of engineering systems in the capital city region of Jordan. The data used in this analysis is extracted from a report published by the Meteorological Department of Jordan [2].

The weather parameters and the corresponding periods of data are given in Table 1.

Table 1. Parameters and Data Periods

Parameter	Description	Period in years
$\bar{T}$	Mean monthly air temperature, °C	27
$\bar{T}_{mx}$	Mean daily maximum air temperature, °C	
$\bar{T}_{mn}$	Mean daily minimum air temperature, °C	
$\bar{\phi}$	Mean monthly relative humidity, ϕ , %	
$\bar{P}$	Mean monthly atmospheric pressure, mbar	63
$\bar{P}_{mx}$	Monthly maximum atmospheric pressure, mbar	
$\bar{P}_{mn}$	Monthly minimum atmospheric pressure, mbar	
$\bar{n}$	Mean daily sunshine hours, hours	21
$\bar{G}$	Mean daily solar energy, MJ/m^2	5
$\bar{N}$	Mean daily maximum sunshine hours, hours	

Although the data considered here had been collected in a limited area, they represent a large terrain of Jordan. Furthermore, this particular area covers the major population center of Jordan. The data used in the derived correlations are averages of the periods listed in the above table.

Initially, several equations were attempted. In the final analysis, however, the shape of the graphical outlay of the data suggested a polynomial representation as a strong candidate.

Thus polynomials of degrees one (linear) through six were attempted and the statistical parameters of the results dictated the degree of the polynomial that was best for the particular parameter under consideration.

The statistical approach used in this study is outlined in the next section. This is followed by the section of the presentation and the discussion of the results.

Analysis and Procedure

The general form of the polynomial used to fit the data is given in the following relation.

$$f(\eta) = \sum_{i=0}^{n} \alpha_i \eta^i \qquad (1)$$

where η is the independent variable, α_0 is the constant of the polynomial, and the α_i s are the coefficients of the independent variable. $f(\eta)$ represents the various prediction parameters of this study, such as the $\bar{T}$, and $\bar{G}$. The highest degree of the polynomial is taken as 6 since no appreciable improvements were noticed in the values computed by the resulting equations when it increased beyond this value.

The procedure of determining the values of the constants and the coefficients is outlined as follows.

1. Each set of data was analyzed six times. This covers polynomials of orders one through six.

2. In each analysis the following computations were performed:

   - The multicorrelation coefficient, R^2
   - The constant of the polynomial α_0.
   - The coefficient or coefficients of the polynomial, α_i.
   - The standard errors of the constant and the coefficients, ε_i, and the ratios of the coefficients and the standard errors r_i.
   - The percentage error between the data and the calculated values, e_i. and,
   - The average values of r_i and e_i

3. A value of 0.9 or more for R^2 is desirable. This parameter, also called the determination coefficient, denotes the degree of agreement between the data and the calculated values. A value of 1 reflects complete agreement [3]. Also the higher the values of the ratios r_i the better would be the results. Values of 10 or more are desirable, see for example Draper *et al.* [4]. The best value of the percentage error is the one closest to zero. Thus a good R^2 value is not good enough unless r is large enough, and e is small enough.

4. In applying the step above one of the models stands out as best representative of the data. Such model was selected for inclusion in this paper.

The multicorrelation coefficient, R^2 is given by the following equation.

$$R^2 = 1 - \frac{\sum_{i=1}^{n} (\omega_i - f(\eta_i))^2}{\sum_{i=1}^{n} (\omega_i - \bar{\omega})^2} \tag{2}$$

where ω_i are the data values, $\bar{\omega}$ is the average of these values, and $f(\eta_i)$ is the predicted value obtained by using the derived correlation equation.

The standard error of the constant of the polynomial is given by the following equation.

$$\varepsilon_{\upsilon} = \frac{1}{2} \sqrt{\sum_{i=1}^{n} (\omega_i - f(\eta_i))^2} \quad (3)$$

where the terms in this equation are as defined above.

The standard errors of the coefficients of the polynomial are given by the following equation.

$$\varepsilon_i = \sqrt{\frac{1}{n} + \frac{\overline{\eta^2}}{\sum_{i=1}^{n} (\eta_i - \bar{\eta})^2}\, \varepsilon_{\upsilon}} \quad (4)$$

where $\bar{\eta}$ is the average value of the independent variable. The other terms in this equation are as defined above.

The ratios of the constants of the polynomials or the coefficients to the standard error and the percentage errors of the predicted values are given by the following relations, respectively.

$$r_i = \frac{\alpha_i}{\varepsilon_i} \quad (5)$$

$$e_i = \frac{Data_i - Result_i}{Data_i} 100 \quad (6)$$

Results and Discussion

Approach

In correlating the data by following the procedure outlined above and calculating the average of the standard error ratios and the averages of the percentage errors, thirteen statistical results similar to those given in Table 2 for the mean monthly air temperature are obtained in this study - one for each of the parameters listed in Table 1 and four for the clearness index.

Table 2. Typical Results

n	R^2	$\bar{r}$	$\bar{e}$
1	0.117	1.48	42.79
2	0.903	6.50	15.05
3	0.979	3.11	6.91
4	0.998	11.69	1.83
5	0.998	4.57	2.23
6	0.998	3.55	2.55

In considering the R^2 values, shown in the Table above, five of the six polynomials are satisfactory. All values of the five values R^2 are greater than 0.9. The linear option (n=1) is obviously unacceptable, its R^2 value is 0.117. Moreover, the average percentage error of the prediction by the linear equation is also far too high. It is 42.79%.

By further inspection of the rest of the results in this Table, it can be seen that the fourth degree polynomial provides the best result. The R^2 value is one of the highest three, its average percentage error of 1.83% is the lowest, and its average standard error ratio of 11.69 is the highest.

Thus, the choice of the best polynomial in the case of the mean air temperature is straight forward, since it satisfies the best of the three statistical indicators. In selecting the best polynomial for some of the other parameters the R^2 value and one of the other two values were accepted when there was no absolute choice.

In a further study of the results, predicted values along with data were plotted to consider a visual comparison of the data and the predicted values. An example of these plots is given in Fig. 1. The linear and the second degree equations were not plotted to avoid crowding the Figure. It is also seen that they are the least accurate of the results.

It is visually noticeable that the polynomial of the fourth degree is the best throughout the whole range. This confirms the result obtained by the statistical analysis. However, for the period from January to July the two highest order polynomials seem to be satisfactory, but when the other part of the year is considered great deviation from the data can be seen. Obviously, these polynomials cannot satisfactorily represent the data of that part of the year.

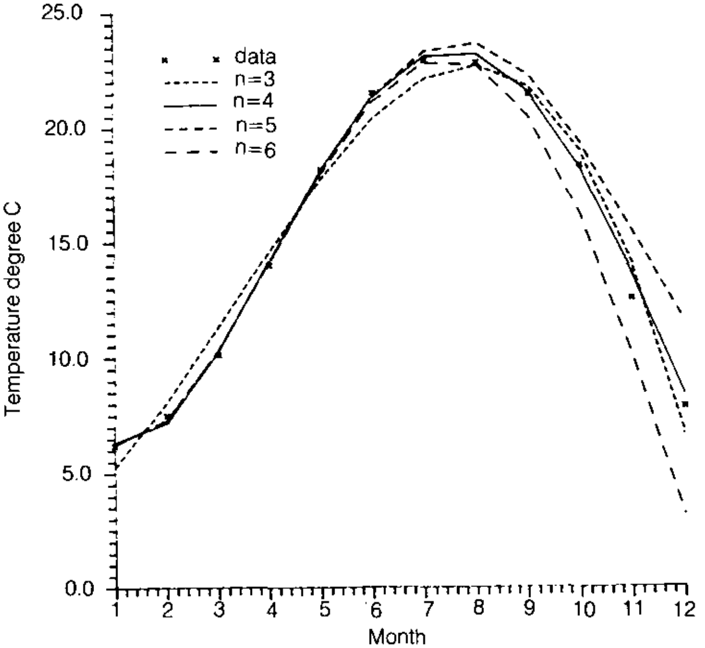

Fig. 1. **Comparison of prediction models**

This detailed analysis was applied to the statistical results of the rest of the parameters under study. The prediction equations are given in the following subsections.

Air Temperature

Three monthly mean daily air temperatures are considered in this analysis. These are the mean, the maximum, and the minimum daily temperatures. These values are averaged for each of the months.

The following equation is for the calculation of the mean temperature, $\bar{T}$.

$$\bar{T}. = 9.1187 - 5.1584m + 2.6565m^2 - 0.3000m^3 + 0.0095m^4 \qquad (7)$$

where m is the month of the year. The R^2 value for this regression is 0.998, and the value of the average percentage error, $\bar{e}$, is 1.831%, and the value of the average standard error ratio, $\bar{r}$ is 11.687. Take the month of January as an example to compare the recorded value and the computed value of the mean temperature: The recorded value is 6.2 °C compared to 6.3 °C as computed from the above equation. Thus a negligible error of 1.6% is expected in the predicted values.

The maximum temperature can be predicted by the following equation.

$$\bar{T}_{mx} = 28.1136 - 7.2958m + 3.5892m^2 - 0.4438m^3 + 0.0178m^4 - 0.0178m^5 \quad (8)$$

where the R^2 value is 0.996, the value of $\bar{e}$ is 4.2%, and the value of $\bar{r}$ is 4.53. The recorded maximum temperature for January is 24 °C, which is very close to the value computed from the above equation, namely, 23.98 °C.

The average minimum air temperature can be predicted by using the following regression equation.

$$\bar{T}_{mn} = 6.56 - 6.4430m + 2.8390m^2 - 0.3169m^3 + 0.0103m^4 \quad (9)$$

where the R^2 value is 0.997, the value of $\bar{e}$ is 4.34%, and the value of $\bar{r}$ is 10.43. The recorded temperature is 2.4 °C compared with the value of 2.65 computed from the above equation. The error in prediction of this value is expected to be about 10%.

In addition, the meteorological records show that the absolute maximum temperature in the Amman area is 41.5 °C recorded in August, and the absolute minimum temperature of –8.3 °C recorded in January.

Atmospheric pressure

Regression equations for the calculation of the monthly mean atmospheric pressure, along with the monthly maximum and minimum pressures are presented here. The pressures are in mbars.

The monthly atmospheric pressure is predictable by the following equation.

$$\bar{P} = 935.810 - 13.8133m + 7.3494m^2 - 1.831m^3 + 0.2168m^4 - 0.0117m^5 + 0.0002m^6 \quad ....(10)$$

where the R^2 value is 0.960, the value of $\bar{e}$ is 0.05%, and the value of $\bar{r}$ is 184.08. The value recorded for January is 927.6 mb which is negligibly different from the computed value of 927.7 mb.

The mean maximum atmospheric pressure can be predicted by the following equation.

$$\bar{P}_{mx} = 938.542 + 4.2865m - 1.2877m^2 - 0.0392m^3 + 0.0211m^4 - 0.0006m^5 - 0.00002m^6 \quad ....(11)$$

where the R^2 value is 0.970, the value of $\bar{e}$ is 0.50%, and the value of $\bar{r}$ is 106.00. The recorded value is 941.3 which is almost equal to the computed value of 941.5 for the month of January using the above equation.

The mean minimum atmospheric pressure can be predicted by the following equation.

$$\bar{P}_{mn} = 918.061 - 8.9995m + 2.2429m^2 - 0.1737m^3 + 0.0037m^4 \qquad (12)$$

where the R^2 value is 0.902, the value of $\bar{e}$ is 0.12%, and the value of $\bar{r}$ is 127.57. The value in the records of this quantity for the month of January is 912 mb compared favorably with the value 911.5 computed using the above equation.

The records show that the highest atmospheric pressure recorded in the Amman area is 1035.5 mbar in February and the lowest value of 993.5 mbar in March. The altitude of Amman is 980 meters above sea level.

Relative humidity

The mean monthly relative humidity, $\bar{\phi}$, can be predicted by the following equation.

$$\bar{\phi} = 67.318 + 16.865m - 11.4413m^2 + 2.2565m^3 - 0.1864m^4 + 0.0057m^5 \qquad (13)$$

where the R^2 value is 0.985, the value of $\bar{e}$ is 1.98%, and the value of $\bar{r}$ is 8.51. By comparing the value of 75% in the records for the month of January with the computed value of 74.8% it is seen that the error in the computed values is negligible.

The meteorological records for the Amman area show that the maximum number of days with relative humidity greater than 80% is 13.5 and that happens in January. The number of days with relative humidity less than 30% is 7.1 which happens in May.

Solar energy

In this section, the prediction equations of the mean daily sunshine hours, the mean daily incident solar energy, and four equations for the prediction of the mean daily clearness index of the area of Amman, Jordan, are presented. They are presented along with their corresponding values of R^2, the average percentage error, $\bar{e}$, and the average standard error ratio, $\bar{r}$.

Sunshine hours

The mean daily sunshine hours, $\bar{n}$, are given by the following equation.

$$\bar{n} = 6.2545 - 1.6718m + 0.8047m^2 - 0.0414m^3 - 0.0060m^4 + 0.0004m^5 \qquad (14)$$

for which R^2 is 0.9896, e is 2.4%, and r is 3.4598.

The total annual sunshine hours for most of Jordan given by the World Maps of Climatology [5] is 3600 hours, the computed value of this quantity as given by the above equation is about 3200 hours, with a deviation of about 10%. However, the January and the July values as computed using the above equation fall within the band of values given in the above reference [5]. For example, for January, Amman falls in the band of 150 to 200 hours, the computed value is 165.6 hours.

Solar energy

The mean daily incident solar energy is given by the following equation.

$$\bar{G} = 9.8343 - 3.0087m + 2.3863m^2 - 0.3311m^3 + 0.0127m^4 \tag{15}$$

for which R^2 is 0.9996, e is 1.2%, and r is 24.1563.

Clearness index

The monthly mean daily clearness index of a location of interest, $\bar{K}_T$ is defined as the ratio of the monthly mean solar energy, $\bar{G}$, at a given location over the monthly mean extraterrestrial solar energy, $\bar{G}_0$ at the same latitude as the location of interest. The value of $\bar{G}_0$ was computed by taking the actual average of the daily values for each of the months.

The regression equations of $\bar{K}_T$ are given in terms of the sunshine hours ratio which is defined as the actual sunshine hours, $\bar{n}$, over the monthly mean daily maximum sunshine hours, $\bar{N}$. This value is computed for each day of the month and averaged over the number of days in the month.

The regression equations are given as follows.

$$\bar{K}_T = \frac{\bar{G}}{\bar{G}_0} = 0.5277 + 0.3422 \quad \frac{\bar{n}}{\bar{N}} \tag{16}$$

for which R^2 is 0.8002, e is 2.462%, and r is 13.62.

$$\frac{\bar{G}}{\bar{G}_0} = 0.0781 + 1.6857\left(\frac{\bar{n}}{\bar{N}}\right) - 0.966\left(\frac{\bar{n}}{\bar{N}}\right)^2 \tag{17}$$

for which R^2 is 0.8582, e is 2.089%, and r is 2.60.

$$\frac{\bar{G}}{\bar{G}_0} = 0.8727 - 1.916\left(\frac{\bar{n}}{\bar{N}}\right) + 4.368\left(\frac{\bar{n}}{\bar{N}}\right)^2 - 2.582\left(\frac{\bar{n}}{\bar{N}}\right)^3 \quad (18)$$

for which R^2 is 0.8620, e is 1.943%, and r is 9.59.

$$\frac{\bar{G}}{\bar{G}_0} = 14.134 - 82.139\left(\frac{\bar{n}}{\bar{N}}\right) + 183.857\left(\frac{\bar{n}}{\bar{N}}\right)^2 - 178.656\left(\frac{\bar{n}}{\bar{N}}\right)^3 + 63.929\left(\frac{\bar{n}}{\bar{N}}\right)^4 \quad (19)$$

for which R^2 is 0.8770, e is 1.645%, and r is 120.46.

As can be seen from the three statistical parameters the fourth degree polynomial is the best of the other regression correlations. Nevertheless the linear version is the most common.

The constant and the coefficient of the linear regression equation are usually denoted by "a" and "b". These values depict the location and the turbidity of the atmosphere in the area where they apply. The values derived here of 0.5277 and 0.3422, respectively, are very close to values derived by the author [6] more than a decade ago. However, they are different from the values of 0.174 and 0.615, respectively, derived by

The source of data used by Alsaad [7] is limited and different from the one used by the author.

The clearness index is a parameter used in predicting the fraction of the beam or the diffuse radiation energy over the total energy.

An equation for the calculation of diffuse radiation using the clearness index was developed by Klein [8]. This equation is given as follows.

$$\frac{\bar{G}_d}{\bar{G}} = 1.390 - 4.027\,\bar{K}_T + 5.531\,\bar{K}_T^2 - 3.108\,\bar{K}_T^3 \quad (20)$$

where $\bar{G}_d$ is the daily diffuse solar energy, (MJ/m^2), received on a horizontal surface.

A linear form of this equation was developed for the Amman area by Alsaad [9] is as follows.

$$\frac{\bar{G}_d}{\bar{G}} = 0.675 - 0.747\,\bar{K}_T \tag{21}$$

For the beam radiation fraction, a linear regression equation was derived by Becker [10].

$$\frac{\bar{G}_b}{\bar{G}} = -0.14 + 1.26\,\bar{K}_T \tag{22}$$

Therefore the total, the diffuse and the beam mean daily solar energy flux in the Amman area can be predicted with high degree of accuracy for the design of solar energy systems.

Conclusion

The advent of the digital computer and the more recent proliferation of high power PCs have helped in the advancement of the science of system simulation. The numerical simulation of buildings for residence or for any other applications calls for the input of local weather parameters.

In this paper, regression equations for thirteen solar energy related and other weather parameters were developed based on long time records of data for the Amman area of Jordan. The four models for the prediction of the clearness index are included with the corresponding statistical information to help the user select the equation with the desired level of accuracy. Moreover, regression equations for the diffuse and the direct solar radiation in terms of the clearness index are also included.

The approach used in the analysis is outlined. The choice of the best model was based on the three statistical parameters: The coefficient of determination R^2, the average percentage error between the data and the prediction values, and the standard error ratios of the constants and the coefficients of the derived equations.

References

[1] Eto, J. H. "On Using Degree-days to Account for the Effect of Weather on Annual Energy Use in Office Buildings," *Energy and Buildings.* 12 (1988), 113-127.

[2] Meteorological Department. *Jordan Climatological Data Handbook,* Jordan: Meteorological Department, 1988.

[3] Audi, M. S. "Empirical Equations of Rectifiable Data Using an Electronic Spreadsheet." *Journal of Engineering Technology,* American Society of Engineering Education, Spring (1988), 32-35.

[4] Draper, N. and Smith, II. *Applied Regression Analysis.* New York: Wiley-Interscience, 1966.

[5] Landsberg, H. E.; Lippmann, H.; Paffen, KH. and Troll, C. *World Maps of Climatology,* 3rd ed. New York: Springer-Verlag, 1966.

[6] Audi, M. S. "Estimation of Solar Energy Flux in Jordan." *Dirasat,* 6 (1979), 121-134.

[7] Alsaad, M. A. "Solar Radiation Map for Jordan." *Solar and Wind Technology,* 7 (1990), 267-275.

[8] Klein, S. A. "Calculations of Monthly Average Insulation on Tilted Surfaces." *Solar Energy,* 19 (1977), 325-329.

[9] Alsaad, M. A. "Various Diffuse Solar Correlations for Amman, Jordan." *Research Journal of the University of Allepo,* 10 (1989).

[10] Becker, P. "Monthly Average Solar Radiation in Panama – Daily and Hourly Relations Between Direct and Global Insulation." *Solar Energy,* 39, No. 5 (1987), 445-453.

معادلات حساب الإشعاع الشمسي وغيره من محدِّدات الطقس

محمود س. عودي

أستاذ مشارك، قسم الهندسة الميكانيكية، الجامعة الأردنية، عمان، الأردن

(استُلم في ٢٩/١/١٩٩١م؛ قُبل للنشر في ٢٠/٧/١٩٩١م)

ملخص البحث. تحتوي هذه الورقة على نتائج دراسة إحصائية للإشعاع الشمسي وغيره من محدِّدات الطقس لمدينة عمّان بالأردن. إن المعادلات التي طُورت نتيجة هذه الدراسة تضم حساب درجة حرارة الهواء، ونسبة الرطوبة، والضغط الجوي، وساعات الإشعاع اليومية، والطاقة الشمسية. وأضيف إلى ذلك معادلات حساب معامل الصفاء للإشعاع الشمسي. وبذلك فإن هذه النتائج تضم أيضًا معادلات لحساب الإشعاع الشمسي المُبعثر والمباشر.