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Evaluation of Municipal Water Distribution System Reliability Using Minimum Cut-set Method

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Abstract. Reliability analysis of water distribution systems is a complex task, as it requires both definition and calculation of reliability measures. In this paper, a methodology for evaluating water distribution system reliability is developed and demonstrated on a real water distribution network. The methodology comprises of two steps: (1) nodal pressures are calculated using hydraulic simulation program (EPANET), and (2) the minimum cut-set method is applied to calculate nodal and system reliabilities of Al-Khobar water distribution network. The results show that the hydraulic reliability of the central part of Al-Khobar water distribution system is 69.73%.

Introduction

Water distribution system plays a vital role in preserving and providing a desirable life quality to the public, of which the reliability of supply is a major component. Reliability analysis of a water distribution system is concerned with its ability to deliver water to individual consumers in the required quantity and under a satisfactory pressure.

In general, reliability is defined as the probability that a system performs its mission within specified limits for a given period of time in a specified environment. Reliability of water distribution system is defined by Kaufmann *et al*. [1] as the probability that the system will perform its specified tasks under specified conditions and during a specified time. Goulter [2] and Cullinane *et al*. [3] defined reliability of water distribution system as the ability of the system to meet the demands that are placed on it. The demands are specified in terms of the flow to be supplied and the range of pressure at which these flow rates must be provided.

Literature Review

In order to analyze the reliability of water distribution system, different approaches are presently being employed by different researchers and analysts. Germanopoulos *et al.* [4] presented a methodology for assessing the security of supply from a water distribution system associated with different network failure events such as burst mains or source failures. They used a network simulation model to study the operation of the network both under normal operating conditions and conditions arising from crisis events. They identified the effect of different failure events on the supplies of the area and they used probability analysis of the occurrence of such events to provide an assessment of the security of water supply. They also identified the operational responses that should be triggered by crisis events. They concluded that the assessment of supply reliability obtained using the adopted methodology is considerably different from that suggested by the conventional approach, which simply relates supply reliability to the amount of emergency storage available in the network.

Lansey *et al.* [5] presented a constrained model for the minimum cost design of water distribution networks. Their methodology attempted to account for the uncertainties in required demands, required pressure heads, and pipe roughness coefficients. They formulated an optimization problem as a non-linear programming model which is solved using a generalized reduced gradient method. Their results show that uncertainties in future demands, pressure head requirements, and pipe roughness can have significant effects on the optimal design and cost. They observed that cost versus reliability relationship is convex, which means an incremental amount at a higher reliability level will result in a greater increase in the system cost than for an incremental change at a lower level.

Quimpo and Shamsi [6] developed a strategy for prioritizing decisions for the maintenance of a water distribution system. Using component and network reliability based on the time-varying connectivity concepts, the probabilities that the water will be available at demand points in the system are calculated to determine a reliability surface. At any time, this surface is used to locate low reliability areas, which identify parts of the system that need maintenance priority. The specific components that must be repaired or replaced are determined using a component importance criterion that measures the overall effect of component maintenance on the system reliability.

Mays [7] computed the reliability of water distribution system by treating the demand, pressure head, and pipe roughness as random variables. He assumed that water demand and pipe roughness coefficient follow a probability distribution, and then used a random number generator to generate the values of random variables for each node and pipe. Then, he performed hydraulic simulation and computed the pressure heads at the demand nodes, provided the demands are satisfied. Finally, he computed the nodal and system hydraulic reliabilities.

Calvin *et al.* [8] investigated the capacity reliability which is defined as the probability that the carrying capacity meets the flow demand. They described the use of capacity reliability for networks with more than one demand node through finding the probability of a feasible flow, given the probability distributions of flow capacities in pipes and fixed nodal demand. The solution procedure generates a set of inequalities that represents a necessary and sufficient condition for feasible flow. They proposed a solution procedure for evaluating the probability that all the inequalities are satisfied by eliminating redundant inequalities and by determining bounds for the probability of feasible flow. They developed a decision-making framework that applies both the capacity reliability measure and the solution approach for maintenance and rehabilitation decision making.

Ostfeld [9] developed a tailor-made reliability methodology for the reliability assessment of regional water distribution and applied it to the regional water distribution system. The methodology comprised of two interconnected stages: the analysis of storage/conveyance properties of the system and implementation of stochastic simulation through the use of the software "US Air Force Rapid Availability Prototyping for Testing Operational Readiness" (RAPTOR).

Shinstine *et al.* [10] applied reliability models to large-scale municipal water distribution systems based on minimum cut-set method and examined the reliability levels that engineers implicitly design into their systems.

Reliability of water distribution system can be analyzed based on different criteria's. For example, it can be correlated to (1) inability of the system to supply proper water quality and quantity, (2) maintain reasonable heads throughout the distribution network, or (3) pumps failures. In this study, the focus is mainly towards the hydraulic failure of the water distribution system, which considers the system's failure due to demands and the inability of the pressure heads to satisfy or meet the requirements. Here, the hydraulic reliability associated with the central part of Al-Khobar city water distribution system is analyzed using the minimum cut-set approach. A method for calculating the complete pipe failure probability using Poisson method is used, which takes into account the number of breaks in the pipe. Finally, nodal and system reliability is calculated.

Development of Methodology

The proposed methodology for calculating nodal and system reliability is based on "minimum cut-sets" [10] in which nodal demand, pipe roughness, tank and reservoir water levels are considered as deterministic values. In order to explain this methodology, the following discussion is necessary.

"Hydraulic Availability" is defined as the ability of the water distribution system to provide service with an acceptable level of interruption in spite of abnormal conditions [3].

Availability is evaluated in terms of developing the required minimum pressure. Pressures between 20 psi and 80 psi [10] are considered to be desirable pressures under normal daily demands.

Goulter and Coals [11] proposed the use of discrete relationship between availability and pressure as shown in Fig. 1. The availability during a time period *t* can be expressed by the following mathematical relationship:

$$\left.\begin{aligned} HA_{jt} &= 1 \ldots\ldots\ldots\ldots \text{ for } \quad NP_{jt} \geq PR \\ HA_{jt} &= 0 \ldots\ldots\ldots\ldots \text{ for } \quad NP_{jt} < PR \end{aligned}\right\} \quad \text{at time } t \tag{1}$$

where

HA_j = hydraulic availability of node *j*,
NP_j = pressure at node *j*,
PR = required minimum pressure,
t = time during which hydraulic availability was evaluated.

Cullinane *et al.* [3] formulated an approach that describes availability index as a continuous “fuzzy” function. Using this concept, a significant index value may be assigned to pressure values slightly less than the arbitrary assigned required minimum pressure value, *PR*. Accordingly, a curve similar to Fig. 2 can be developed which resembles the curve of a normal distribution. Thus, the hydraulic availability function can be described mathematically as:

$$HA_j = (PR \leq NP_j)$$

$$HA_j = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{(H-\mu_H)}{\sigma_H}} e^{-\frac{t^2}{2}} dt = P\left[\frac{(H-\mu_H)}{\sigma_H}\right] \tag{2}$$

where

$H = NP_j$ = value of nodal pressure,

μ_H = mean nodal pressure,

σ_H = standard deviation of pressure,

$P[\bullet]$ = probability function.

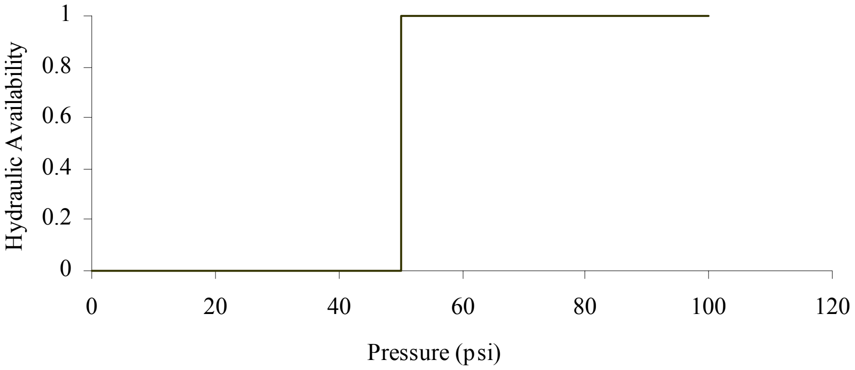


Fig. 1. Discrete hydraulic availability step function during time (*t*).

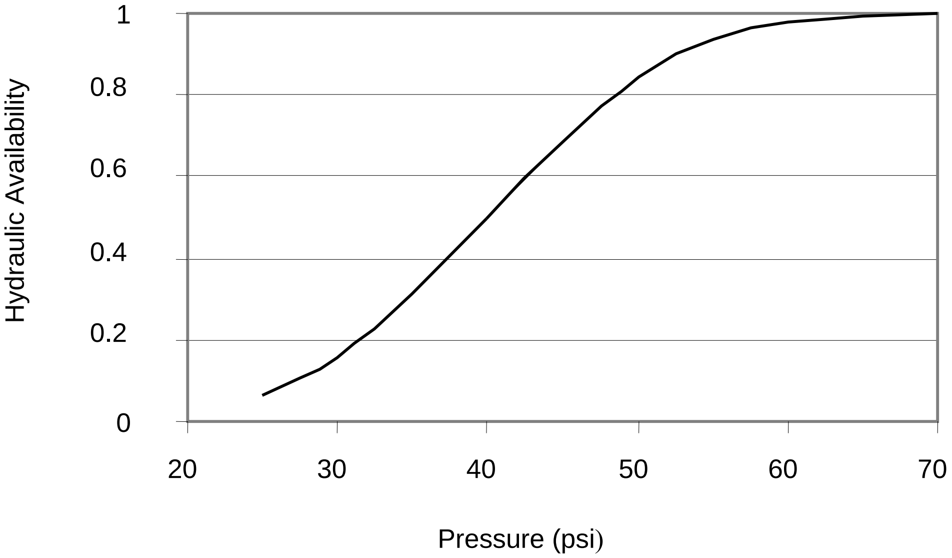


Fig. 2. Continuous hydraulic availability function.

Calculation of nodal and system reliability

The minimum cut-set approach is adopted to calculate the nodal and system reliability, R_{node} and R_s, respectively. According to Su *et al.* [12], the minimum cut-set can be defined as "a set of system components (e.g., pipes) which, when failed, causes failure of the system". However, system failure will not occur if any component of the set does not fail [13].

Assuming that a pipe break can be isolated from the rest of the system, the minimum cut-sets are determined by closing a pipe or combination of pipes in the water distribution system and using a hydraulic simulation model to determine the values of pressure head at each demand node of the system. In this study, EPANET was used [14]. By comparing these pressure heads with the minimum pressure head requirements, the reliability model can determine whether or not this pipe or combination of pipes is a minimum cut-set of the system or an individual demand node. A minimum cut-set for a node is the one that causes reduced hydraulic availability at that node, while a minimum cut-set for the system is a cut-set that reduces the hydraulic availability for any node in the system. To calculate the number of combinations for pipe closure for the cut-set determination, it is observed that the failure of two or three pipes is purely a "random" phenomenon. Therefore, in order to determine the pipe combinations for the cut-set determination, subsets of pipe combinations should be determined by applying a random approach. For instance, if there are *K* numbers of pipes in the water distribution system, then out of those *K* pipes, *T* subsets should be randomly generated and each sub-set could have only one pipe or a combination of two or three pipes. A flow chart of the procedure is shown in Fig. 3.

According to Shinstine et al. [10], for *n* components (pipes) in the i^{th} minimum cut-set of a water distribution system, the failure probability of the i^{th} minimum cut-set (MC_i) is:

$$P(MC_i) = \prod_{i=1}^{n} P_i = P_1 \bullet P_2 \bullet P_3 \bullet .... \bullet P_n \tag{3}$$

Using the step function for hydraulic availability and assuming that the occurrence of the failure of the components within a minimum cut-set is statistically independent, for a water distribution network with four minimum cut-sets (MC_i) with the system reliability, R_s, the failure probability of the system P_s is then defined [13] as:

$$P_s = P(MC_1 \cup MC_2 \cup MC_3 \cup MC_4) \tag{4}$$

$$P_s = P(MC_1) + P(MC_2) + P(MC_3) + P(MC_4) \tag{5}$$

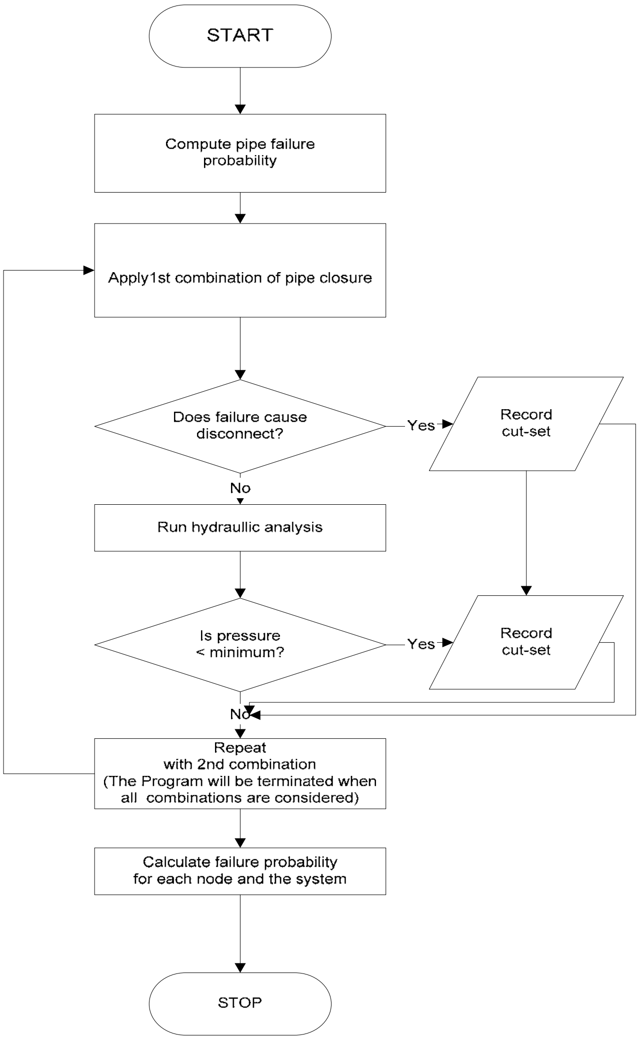


Fig. 3. Minimum cut-set flow chart.

$$P_s = \sum_{i=1}^{4} P(MC_i) \tag{6}$$

In general form,

$$P_s = \sum_{i=1}^{M} P(MC_i) \tag{7}$$

The system reliability, R_s, is expressed as:

$$R_s = 1 - P_s = 1 - \sum_{i=1}^{M} P(MC_i) \tag{8}$$

where M = number of minimum cut-sets in the system.

It is possible to weigh the nodal terms as the function of the nodal demand. Nodal reliabilities can be computed with the same relationship including only failures that affect the individual node.

Using the continuous hydraulic availability concept, a true minimum cut-set does not exist. The probability of a cut-set occurring is consistent; however, reliability is defined as the nodal hydraulic unavailability (1–*HA*). The system reliability is then expressed as [15]:

$$R_s = 1 - P_s = 1 - \sum_{i=1}^{M} (1 - HA_{net}^{i}) P(MC_i) \tag{9}$$

where HA_{net}^{i} is the network hydraulic availability and expressed as:

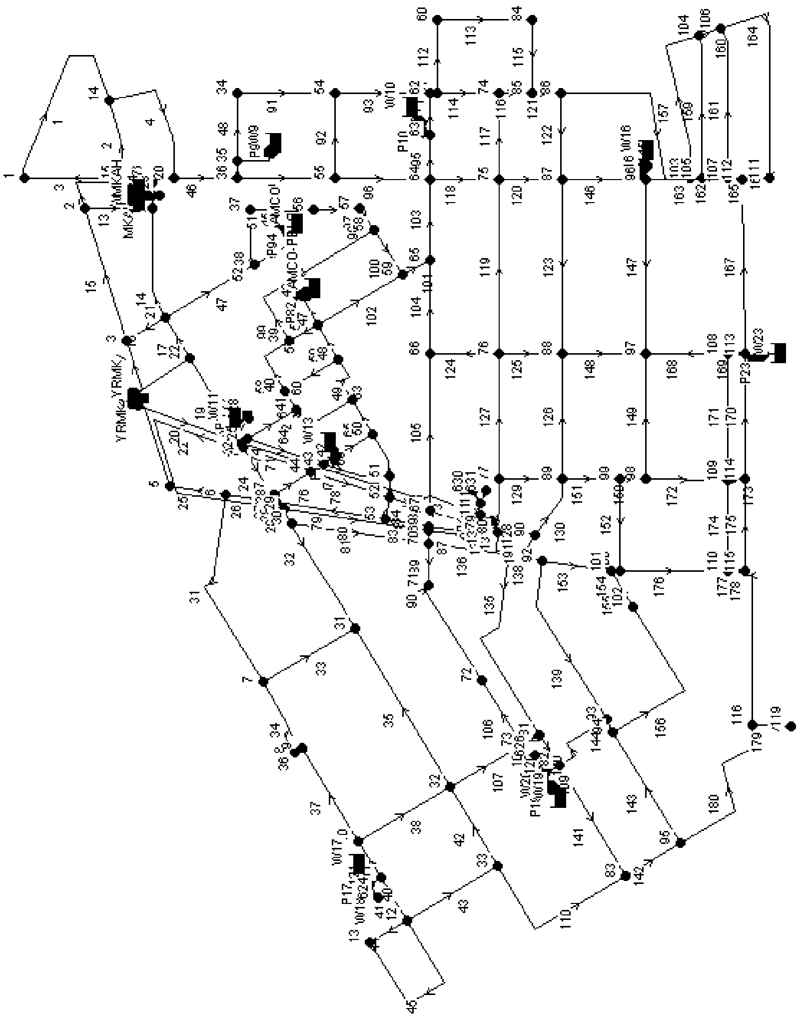
$$HA_{net} = \prod_{j=1}^{J} HA_j \tag{10}$$

where HA_j = hydraulic availability of node j.

If HA_{net} equals one, the failure probability of the cut-set is not included in Eq. (9); thus, it is identical to Eq. (8) for the step function hydraulic availability case. To compute the system reliability with continuous hydraulic availability, all cut-sets are included.

Application of the Methodology

The developed methodology was applied on Al-Khobar water distribution system to evaluate its hydraulic reliability. The water distribution system, shown in Fig. 4, consists of 191 pipes, 131 junctions, 12 reservoirs, 2 tanks and 12 pumps. In the analysis, nodal demand, Chezy's roughness coefficient for pipe, tank and reservoir water level are assumed and considered as deterministic values. A sample data are summarizes as shown in Table 1.



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The pipe failure combinations required for the cut-set calculations are determined by assuming randomness in the simultaneous failure of two or three pipes. Then, a steady-state hydraulic analysis is performed using the hydraulic simulation software EPANET [14], and nodal pressures are calculated for the pipe closure combinations shown in Table 2.

Table 2. Pipe closure combinations for Al-Khobar sub-network

Closure Comb No.	Pipe ID	Pipe ID	Pipe ID
1	160	111	20
2	83	-	-
3	146	-	-
4	150	-	-
5	112	119	-
6	164	21	-
7	37	-	-
8	175	44	-
9	12	-	-
10	150	-	-
11	114	27	-
12	158	142	-
13	17	96	-
14	141	73	-
15	7	-	-
16	7	YRMK-900[*]	-
17	166	158	-
18	50	132	-
19	10	142	-
20	21	-	-
21	41	26	-
22	39	116	163
23	170	-	-
24	24	25	-
25	37	127	-

[*] Al-Yarmouk reservoir tank.

Table 2. *Contd.*

Closure Comb No.	**Pipe ID**	**Pipe ID**	**Pipe ID**
26	170	26	55
27	7	-	-
28	127	-	-
29	158	150	128
30	10	26	-
31	10	-	-
32	178	115	-
33	27	163	-
34	50	-	-
35	170	20	-
36	162	-	-
37	110	128	-
38	96	132	-
39	164	172	-
40	180	146	-
41	83	150	-
42	119	-	-
43	114	86	-
44	6	43	-
45	41	172	-
46	83	-	-
47	178	50	-
48	83	-	-
49	160	-	-
50	19	21	-

The results of this study show that the system reliability of Al-Khobar network is 69.73% and the nodal reliabilities of the system are ranging from a minimum value of 74.32% to a maximum value of 99.99%. This means that the probability of Al-Khobar water distribution system to have a required minimum pressure of 33 psi at all junctions is 69.73%, and the probability that each junction will have a required minimum pressure of 33 psi varies from 74.32% to 99.99% depending upon the individual junction.

Table 3 summarizes the calculated reliability at selected nodes. It reveals that among all the 131 junctions of Al-Khobar network, junction J-57 has the lowest reliability value equal to 74.32% while junction J-520 has the highest reliability value of 99.99%. The reason for lower reliability value at junction J-57 is due to the larger number of cut-sets resulting in higher cut-set failure probability. On the other hand, the high reliability value observed at junction J-520 is due to its location, which is near to YRMK (Al-Yarmouk water reservoir tank) where high pressure is expected during the day.

Table 3. Nodal and system reliability values of Al-Khobar water distribution network

Junction ID	Reliability (%)
J-14	84.47
J-16	84.45
J-31	84.47
J-116	86.85
J-57 (Min)	74.32
J-520 (Max)	99.99
System	*69.73*

In the evaluation process of water distribution system reliability, the mean and standard deviations at the nodes are required to estimate hydraulic availability needed to calculate nodal and system reliabilities. The results indicate that higher values of mean and standard deviations of nodal pressures will increase the variation of the generated values of pressure at the nodes, which will consequently result in low values of nodal and system reliabilities.

The number of pipe closure combinations also affects the nodal and system reliabilities. A high number of pipe closure combinations will result in a large number of cut-sets that will result in low values of nodal and system reliabilities. Therefore, appropriate number of pipe closure combinations should be selected based on field experience to get realistic values of nodal and system reliabilities.

Finally, it is recommended to use the developed methodology for large water distribution networks, such as the entire Al-Khobar water distribution system. This is because the possibilities of pipe closure combinations are high compared to the case dealing with small water distribution network.

Conclusion

Reliability analysis of a water distribution system is concerned with measuring its ability to meet consumers' demands in terms of quantity and quality, under normal and emergency conditions. This paper developed a methodology based on minimum cut-set method, which can be applied to evaluate the hydraulic reliability of water distribution systems. The methodology applied on Al-Khobar water distribution system shows that the system reliability is 69.73%, which means that the probability of Al-Khobar water distribution system to have a required minimum pressure of 33 psi at all the junctions is 69.73%. The reason for low reliability of Al-Khobar network, compared to similar distribution networks, is due to the high annual failure rates of the pipes resulting in higher cut-set failure probabilities. If this reliability value is compared with the reliability value of Tucson, Arizona, U.S.A. which is 96.0 %, it is observed that the Al-Khobar network is less reliable. The reason of high reliability value of Tucson water distribution network is the proper maintenance of the network by the private water business companies, as in U.S.A. most of the municipal water distribution networks are managed and maintained by private water business companies. The system reliability value of Al-Khobar network could be improved by reducing the annual failures of the pipes through proper maintenance and replacement of older pipes in the network. The improvement in system reliability could be easily achieved by simply improving the reliability of individual junctions. Therefore, as a preliminary step in improving system reliability values, critical junctions of the network should be identified and the pipes affecting the reliability values of these junctions should be properly maintained and replaced, if necessary.

Depending upon the residential and commercial consumers' requirements of Al-Khobar network, "levels of reliability" should be established and water agencies should be made responsible for its proper implementation and monitoring.

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تقييم موثوقية أداء نظام شبكة توزيع المياه باستخدام طريقة المجموعات الصغرى المتقطعة

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(قدِّم للنشر في ٢٠/٠٤/٢٠٠٤م ؛ وقبل للنشر في ٢٥/٠٤/٢٠٠٥م)

ملخص البحث. يعتبر تحليل الموثوقية الهيدروليكية لشبكات توزيع المياه من الأمور المعقدة لما ينطوي على ذلك من تعريف دقيق لمعنى الموثوقية بالإضافة إلى طريقة حساب الموثوقية. في هذه الدراسة تم تطوير طريقة يمكن من خلالها تقييم الموثوقية الهيدروليكية لنظم توزيع المياه، وتعتمد الطريقة على الخطوتين التاليتين: (١) استخدام برنامج (EPANET) لمحاكاة حركة المياه في شبكة التوزيع من أجل حساب الضغوط عند النقاط المكونه لشبكة التوزيع ، (٢) استخدام طريقة المجموعات الصغرى المتقطعه لحساب الموثوقية الهيدروليكية لنظام شبكة التوزيع. وقد تم تطبيق الطريقة المقترحة على شبكة مياه مدينة الخبر وأوضحت النتائج بأن الموثوقية الهيدروليكية لشبكة توزيع مياه الخبر غير عالية مقارنةً بإحدى شبكات التوزيع. وأوصت الورقة بتبني بعض التدابير التي من شأنها تحسين الموثوقية الهيدروليكية للشبكة.