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Spatial-Domain Design of 2-D Quadrantal Symmetric Digital Filters using Singular-value Decomposition and Singular Perturbation Model Reduction

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Abstract. In this paper a new technique is presented to design low order 2-D quadrantally symmetric IIR digital filters. This technique is based on two steps: First, the 2-D impulse response specification (Hankel matrix) is decomposed into k parallel sections, each consists of two cascaded SISO 1-D filters, using the singular value decomposition. Second, a singular perturbational model reduction algorithm is applied to the 1-D filter to approximate the N-dimensional FIR into n-dimensional IIR filters, where $n < N$. The approximation step is based on computing the eigenspaces associated with the large eigenvalues of the cross-Gramian matrix W_{co}. Examples are given to illustrate the proposed technique.

Introduction

Two-dimensional (2-D) digital filters are used in many applications such as image processing, seismic or geophysical signal processing, ultrasonic data processing and biomedical tomography. The design of two-dimensional (2-D) digital filters has been an active area of research for many years (see [1-6] and the references listed therein). Many techniques have been proposed to design the 2-D digital filters. Some of these techniques are based on transformations of 1-D filters [1-4] and others are based on various methods of optimizations or optimization in conjunction with transformation [5-7]. The technique, that received a considerable attention in the past few years, is based on singular value decomposition (SVD) (see [8-13]). The reason for this interest is because, it offers, as pointed out in [11,12], the following advantages. First, the design can be accomplished by designing a set of 1-D subfilters and, therefore, the well-established algorithms for the design of 1-D filters can be employed. Second, the stability issues of the 2-D filter is guaranteed if the 1-D subfilters employed are stable, and third, the 1-D subfilters form a parallel structure that allows extensive parallel processing.

The SVD can be applied to impulse response matrix (input-output data) as in [8-10], or it can be applied to the sampled magnitude response as in [11-13]. In [8,9], a state space model of the separable-denominator transfer is obtained, while in [10], the state space representation is obtained by decomposing the 2-D impulse response matrix into two 1-D digital filters (single-input multi-output and multi-input single-output). Moreover, it was shown that an optimal decomposition could be obtained. In conjunction with the decomposition, in some of the previous work [9,10,12], the balanced model reduction is applied to the decomposed state space representations to obtain computationally efficient filters.

In this paper, we propose a new and computationally efficient algorithm to design 2-D digital filters using the SVD in conjunction with singular perturbation model reduction. This algorithm is different from the others algorithms in two aspects: First, the SVD is applied to the impulse response matrix (Hankel matrix) instead of the sampled amplitude response. This enables us to decompose the Hankel matrix into parallel sections of two 1-D filters in cascaded. Second, the real Schur form decomposition (RSFD) [14,15] is applied to these decomposed 1-D subfilters to convert them from high order FIR filters to reduced order IIR filters in the singular perturbed form. The conversion algorithm is based on finding the orthonormal eigenspaces that correspond to the large eigenvalues of the cross-Gramian matrix W_{co}. This algorithm avoids computing the balancing transformation, which tends to have numerical difficulties and ill-conditioning problem [16].

In Sections 2 and 3 we formalized the 2-D digital filters and we will show how the SVD is employed to decompose the 2-D filters into parallel branches of two 1-D filters in the z_1 and z_2 domains. Moreover, we see how we choose the decomposition index, which determines the number of parallel branches in our design. In Section 4 we describe how to use the real Schur form decomposition of W_{co} to obtain the reduced order singular perturbed IIR digital filters from the decomposed FIR filters obtained in Section 3. Section 5 gives two design examples to illustrate the significance of the proposed technique. Conclusion is given in Section 6.

Design Problem Formulation

A 2-D FIR SISO digital filter with a rectangular support in the region defined by $-N_i/2 \le n_i \le N_i/2,\ i = 1,2.$, (N_i is assumed to be even), can be characterized by the transfer function

$$H(z_1, z_2) = \sum_{n_1=-N_1/2}^{N_1/2} \sum_{n_2=-N_2/2}^{N_2/2} h(n_1, n_2)\, z_1^{-n_1} z_2^{-n_2} \qquad (1)$$

where $h(n_1, n_2)$ is the impulse response of the filter. For the quadrantally symmetric filter, the impulse response is real and

$$h(n_1, n_2) = h(-n_1, -n_2) = h(-n_1, n_2) = h(n_1, -n_2) .$$

This type of filter has a separable denominator [9,10]. The transfer function $H(z_1, z_2)$ given in (1) can be rewritten as:

$$H(z_1, z_2) = \sum_{i=1}^{k} F_i(z_1) G_i(z_2) \tag{2}$$

where $F_i(z_1)$ and $G_i(z_2)$ are transfer functions of 1-D subfilters in z_1 and z_2 domains, respectively. k is called the decomposition index and it is determined by the allowable error, as we will see later in the next section. Moreover, k determines the number of parallel sections. Now, if these subfilters are FIR, then we have

$$F_i(z_1) = \sum_{n_1=-N/2}^{N/2} f_i(n_1) z_1^{-n_1} \tag{3}$$

and

$$G_i(z_2) = \sum_{n_2=-N/2}^{N/2} g_i(n_2) z_2^{-n_2} \tag{4}$$

where $f_i(n_1)$ and $g_i(n_2)$ are the impulse responses of the 1-D FIR filters in the directions n_1 and n_2 respectively.

Equations (1) to (4) show that a quadrantally symmetric filter can always be realized using a set of k parallel sections where the i^{th} section is characterized by the impulse response $h_i(n_1, n_2) = f_i(n_1) g_i(n_2)$.

Now, having the impulse response specifications $h(n_1, n_2)$ at the support $S_h = \{(n_1, n_2) : 0 \le n_i \le N,\ i = 1, 2\}$, or in a matrix form:

$$H_d = \begin{bmatrix} h(0,0) & h(0,1) & . & . & h(0,N) \\ h(1,0) & h(1,1) & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ h(N,0) & . & . & . & h(N,N) \end{bmatrix}, \quad \mathrm{rank}(H_d) = r. \tag{5}$$

The given 2-D specifications H_d can be decomposed into two 1-D specifications as $H_d = FG$ by the singular value decomposition, where F and G are two matrices of dimensions $(N+1)\times r$ and $r\times(N+1)$, respectively. Now, each column of F (row of G) represents the impulse responses of the SISO 1-D FIR filter in the z_1 and z_2, respectively. The decomposition and the approximation of 1-D FIR by IIR filters steps will be discussed in the following sections

The Singular Value Decomposition (SVD) of the 2-D Impulse Response

In this section, the SVD is applied to the impulse response of the 2-D quadrantally symmetric digital filter H_d as follows:

$$H_d = U\Sigma V^T = \sum_{i=1}^{r}\sigma_i u_i v_i^T \qquad (6)$$

where $U = [u_1 \quad u_2 \quad . \quad . \quad u_r]$, $V = [v_1 \quad v_2 \quad . \quad . \quad v_r]$ and

$\Sigma = [\sigma_1 \quad \sigma_2 \quad . \quad . \quad \sigma_r]$. Assume that the singular values σ_i 's are appeared in a descending order, i.e., $\sigma_1 \ge \sigma_2 \ge .... \ge \sigma_r$, then for $\sigma_k >> \sigma_{k+1}$, where $k < r$, the matrix H_d can be approximated as

$$H_d \approx \sum_{i=1}^{k}\sigma_i u_i v_i^T = \sum_{i=1}^{k} f_i g_i = \tilde{H}_d \qquad (7)$$

where $f_i \in R^{(N+1)\times 1} = \sigma_i^{1/2} u_i$ and $g_i \in R^{1\times(N+1)} = \sigma_i^{1/2} v_i^T$. The approximation error at this stage is [17]

$$\varepsilon_d = \left\| H_d - \tilde{H}_d \right\|_2 = \sigma_{k+1} \qquad (8)$$

Where $\|.\|_2$ denotes the Euclidian norm of the matrix involved. The approximated Hankel matrix $\tilde{H}_d$ can be rewritten as

$$\tilde{H}_d = \tilde{F}\tilde{G}, \qquad (9)$$

where $\tilde{F} \in R^{(N+1)\times k} = [f_1 \quad f_2 \quad . \quad . \quad f_k]$ and $\tilde{G} \in R^{k\times(N+1)} = [g_1^T \quad g_2^T \quad . \quad . \quad g_k^T]^T$.

Notice here that, as we mentioned before, each column of $\tilde{F}$ (row of $\tilde{G}$) represents the impulse response of an FIR filter. In the next section, we use the singular perturbation technique to convert the high order FIR filters to low order IIR filters

Singular Perturbation Approximation of FIR by IIR Digital Filters

In this section, we use the singular perturbational model reduction [16] to convert the high order nonrecursive (FIR) into much lower order recursive (IIR) digital filters. The similarity transformation matrix T that, converts the FIR to IIR digital filters is computed by finding the eigenspaces that span the large and the small eigenvalues of the cross-Gramian matrix W_{co}. The eigenspaces are computed by using a very reliable and numerically stable algorithm known as real Schur form decomposition (RSFD).

From (9), the transfer functions of the 1-D FIR filters in the z_1 and z_2 domains are characterized by

$$\tilde{F}_i(z_1) = \sum_{j=0}^{N} f_{ji}\, z_1^{-j} \tag{10}$$

and

$$\tilde{G}_i(z_2) = \sum_{j=0}^{N} g_{ij}\, z_2^{-j} \tag{11}$$

respectively. Thus, the design of 2-D digital filter can be accomplished through the following steps:

1. From the transfer functions of the 1-D FIR filters, $\tilde{F}_i(z_1)$ and $\tilde{G}_i(z_2)$ for $i = 1, 2, ..., k$, find the state space representations (A, B, C, D).
2. Apply the singular perturbational model reduction to approximate the FIR filters, characterized by $\tilde{F}_i(z_1)$ and $\tilde{G}_i(z_2)$, by IIR digital filters, characterized by $H_{i1}(z_1)$ and $H_{i2}(z_2)$, respectively.
3. Connect $H_{i1}(z_1)$ and $H_{i2}(z_2)$ in cascaded as shown in Fig.1, the parallel realization of the 2-D digital filter.

Before we explain how the first two steps are done, let us introduce the transfer function $H_{FIR}(z)$ in place of $\tilde{F}_i(z_1)$ and $\tilde{G}_i(z_2)$ for each i to overcome the notational problem involved in the subscripts of $\tilde{F}, \tilde{G}$ and z. Thus $H_{FIR}(z)$ is written as:

$$H_{FIR}(z) = \sum_{j=0}^{N} h_j\, z^{-j} = D + C[zI_N - A]^{-1} B \tag{12}$$

where
$$h_j = \begin{cases} f_{ji} & \text{if } \tilde{F}_i(z_1) \text{ is selected} \\ g_{ij} & \text{if } \tilde{G}_i(z_2) \text{ is selected} \end{cases} \quad (13)$$

The state space (A, B, C, D) in the controllable canonical form is given by

$$A = \begin{bmatrix} 0 & 0 & . & . & 0 \\ 1 & 0 & . & . & 0 \\ 0 & 1 & 0 & . & . \\ . & . & . & . & . \\ 0 & . & . & 1 & 0 \end{bmatrix},\ B = \begin{bmatrix} 1 \\ 0 \\ . \\ . \\ 0 \end{bmatrix}, C = [h_1\ h_2\ ......\ h_N] \text{ and } D = h_0. \quad (14)$$

The purpose of the model reduction is to approximate the Nth order FIR by an n^{th} order IIR digital filter, where $n < N$ through a similarity transformation T.
Define the cross-Gramian matrix W_{co} [15,18] as

$$W_{co} = \sum_{k=0}^{\infty} A^k BCA^k \quad (15)$$

Equivalently, W_{co} can be computed by solving the Lyapunov equation

$$AW_{co}A - W_{co} + BC = 0 \quad (16)$$

Notice that W_{co} is invariant under the similarity transformation [15,18] and the singular values $\hat{\sigma}_i$, are given by

$$\hat{\sigma}_i(H_{FIR}(z)) = |\lambda_i(W_{co})|, \quad i = 1, 2, , ..., N \quad (17)$$

Notice also that the singular value $\hat{\sigma}_i$ is different from the singular value of the 2-D Hankel matrix H_d. The order n is chosen based on a specified error bound, ε which satisfies the inequality

$$\varepsilon = \|H_{FIR}(z) - H_r(z)\|_\infty \le 2 \sum_{i=n+1}^{N} \hat{\sigma}_i \quad (18)$$

where $\| \|_\infty$ denotes the maximum absolute value of its frequency response and $H_r(z)$ is the transfer function of the reduced order IIR filter and defined as

$$H_r(z) = D_r + C_r[zI_n - A_r]^{-1}B_r. \tag{19}$$

The state space representation of the singular perturbation reduced order IIR filter (A_r, B_r, C_r, D_r) will be defined later after we define the transformation T that transforms the state space representation, (14) into $(\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D})$. In the rest of this section, we will propose the algorithm that computes this transformation using the RSFD.

Notice that, because of the special structure of the state space of the FIR filter (14), equation (15) is simplified to:

$$W_{co} = \sum_{k=0}^{N-1} A^k BCA^k = \Omega_C \Omega_O, \tag{20}$$

where Ω_C and Ω_O are the controllability and the observability matrices, respectively, which are defined as:

$$\Omega_C = \begin{bmatrix} B & AB & .... & A^{N-1}B \end{bmatrix} \text{and } \Omega_O = \begin{bmatrix} C^T & A^T C^T & ...... & (A^T)^{N-1} C^T \end{bmatrix}^T \tag{21}$$

Notice also that A is nilpotent since $A^N = 0$. Therefore, equation (20) yields

$$W_{co} = \begin{bmatrix} h_1 & h_2 & . & . & h_N \\ h_2 & h_3 & . & . & 0 \\ . & . & . & . & . \\ . & . & . & . & . \\ h_N & 0 & . & . & 0 \end{bmatrix} \tag{22}$$

Again, notice that the matrix W_{co} is symmetric and is easily constructed. Therefore, there is no computation involved in finding W_{co}.

In the rest of this section, we summarize briefly, the singular perturbational model reduction algorithm in the following steps:

i. For each branch, design the FIR filter characterized by equation (12). Find the impulse response, $h_i, i = 0, 1, ..., N$ and from which construct the state-space representation (A, B, C, D) as in (14).
ii. Construct W_{co} as in equation (22)
iii. Compute the real Schur form decomposition (RSFD) of W_{co} [15],

$$T^T W_{co} T = \Lambda \tag{23}$$

where $\Lambda = \text{diag}(\lambda_1, \lambda_2, \ldots\ldots\ldots, \lambda_N)$ with the ordering

$$|\lambda_1| \geq |\lambda_2| \geq \ldots\ldots \geq |\lambda_n| \geq |\lambda_{n+1}| \geq \ldots\ldots \geq |\lambda_N|$$

The desired order of the IIR filter is determined based on the required error bound ε, which is defined by equation (18).

iv. Partition the matrices T and T^T such that

$$\begin{bmatrix} T_1^T \\ T_2^T \end{bmatrix} W_{co} [T_1 \quad T_2] = [\Lambda_1 \quad \Lambda_2], \tag{24}$$

where

$$\Lambda_1 = \text{diag}(\lambda_1, \lambda_2, \ldots\ldots\ldots, \lambda_n) \text{ and } \Lambda_2 = \text{diag}(\lambda_{n+1}, \lambda_{n+2}, \ldots\ldots\ldots, \lambda_N).$$

The matrices $T_1 \in R^{N\times n}$ and T_1^T span the right and the left eigenspaces associated with Λ_1 Similarly, $T_2 \in R^{N\times N\text{-}n}$ and T_2^T span the right and left eigenspaces associated with Λ_2.

v. Apply this transformation, T to (A, B, C, D) to obtain

$$\begin{bmatrix} T_1^T \\ T_2^T \end{bmatrix} A [T_1 \quad T_2] = \tilde{A} = \begin{bmatrix} \tilde{A}_{11} & \tilde{A}_{12} \\ \tilde{A}_{21} & \tilde{A}_{22} \end{bmatrix} \tag{25a}$$

$$\begin{bmatrix} T_1^T \\ T_2^T \end{bmatrix} B = \tilde{B} = \begin{bmatrix} \tilde{B}_1 \\ \tilde{B}_2 \end{bmatrix}, \quad C[T_1 \quad T_2] = \tilde{C} = [\tilde{C}_1 \quad \tilde{C}_2] \text{ and } \tilde{D} = D \tag{25b}$$

vi. The state space of the reduced model is defined as [16]:

$$A_r = \tilde{A}_{11} - \tilde{A}_{12}(\tilde{A}_{22} - I_{N-n})^{-1}\tilde{A}_{21} \tag{26a}$$

$$B_r = \tilde{B}_1 - \tilde{A}_{12}(\tilde{A}_{22} - I_{N-n})^{-1}\tilde{B}_2 \tag{26b}$$

$$C_r = \tilde{C}_1 - \tilde{C}_2(\tilde{A}_{22} - I_{N-n})^{-1}\tilde{A}_{21} \tag{26c}$$

$$D_r = \tilde{D} - \tilde{C}_2(\tilde{A}_{22} - I_{N-n})^{-1}\tilde{B}_2, \tag{26d}$$

So, the reduced order IIR filter characterized by equation (19) is an approximate to the full order FIR filter characterized by equation (12). Now, $H_r(z) = H_{i1}(z_1)$ if the first row of equation (13) is chosen and $H_r(z) = H_{i2}(z_2)$ if the second row of equation (13) is chosen. Repeat steps (i)-(vi) for $i = 1, 2, ..., k$. See the parallel realization of the 2-D recursive digital filter in Fig. 1.

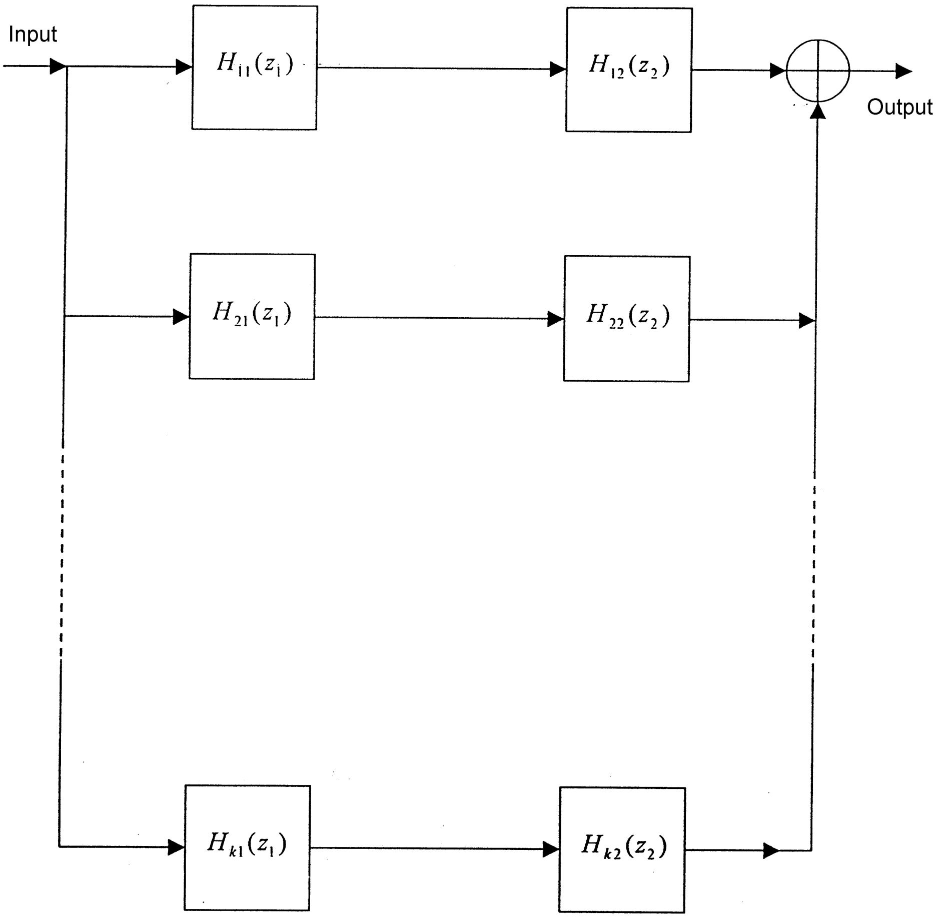

Fig. 1. Parallel realization of 2-D IIR (Recursive) digital filter.

In this section, a 2-D circularly symmetric low pass and fan digital filters are designed to illustrate the effectiveness of the proposed technique in the decomposition and in the model reduction steps.

Example 1: Consider the 2-D circularly symmetric low pass digital filter which satisfies the following specifications:

$$|H(\omega_1, \omega_2)| = \begin{cases} 1 & \text{for } 0 \le \Omega \le \omega_p \\ 0 & \text{for } \omega_s \le \Omega \le \pi. \end{cases}$$

Where $\Omega = \sqrt{\omega_1^2 + \omega_2^2}$, $\omega_p = 0.4\pi$ and $\omega_s = 0.45\pi$. Thus, $\omega_c = 0.425\pi$, and the corresponding impulse response smeared by Hamming window is given by

$$h_d(n_1, n_2) = \frac{0.2125\, J_1(0.425\pi\sqrt{(n_1 - 24)^2 + (n_2 - 24)^2})}{\sqrt{(n_1 - 24)^2 + (n_2 - 24)^2}} \times W(n_1 - 24, n_2 - 24), 0 \le n_1, n_2 \le 48$$

where $W(i, j) = 0.54 + 0.46\cos(2\pi\sqrt{i^2 + j^2}/48)$ and $J_1(x)$ is the first order Bessel function of the first kind. The desired magnitude response, which will be considered here as an ideal magnitude response is shown in Fig. 2.

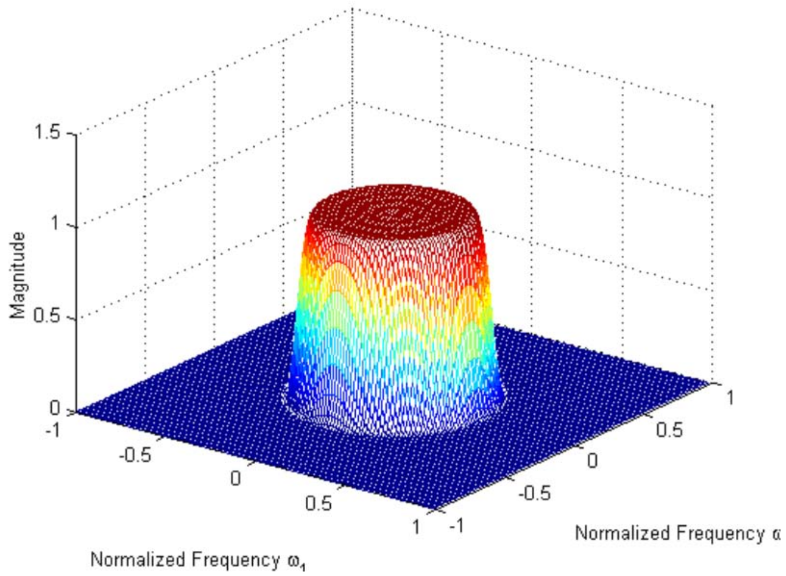

Fig. 2. Ideal magnitude response of circularly symmetric 2-D low pass digital filter.

Following the procedure of Section 3, we apply the SVD to the resulted 49 x 49 impulse response matrix. The singular values of H_d are displayed. It is found that only 25 of these singular values are nonzero, which means that the actual number of parallel branches is 25, but because of the smallness of some of these singular values, we can only retain 4 of them, i.e., choosing the number of branches k to be 4, gives a satisfactory result. So, this is the first approximation, reducing the number of parallel branches from 25 to only 4. The second approximation is to reduce the order of individual 1-D filters. This is achieved by the singular perturbation model reduction. So, if we follow the procedure of Section 4, the filters order is reduced from 48 to the values given in Table 1, where the first element in the bracket is the order of $H_{i1}(z_1)$, and the second element is the order of $H_{i2}(z_2)$ in Fig.1. This choice, of number of parallel branches and the filters orders, gives a maximum passband error of 3.29% and maximum stopband error of 2.95%. The resulted magnitude response of the 2-D IIR filter (reduced) is shown in Fig. 3.

Table 1. Passband and stopband errors for the reduced order 2-D IIR digital filter

No of Parallel Branches k	**Orders of the Subfilters** (n_1, n_2)	**Passband Error** E_P	**Passband Error** E_S
4	[illegible] (16,16)	0.0329	0.0295

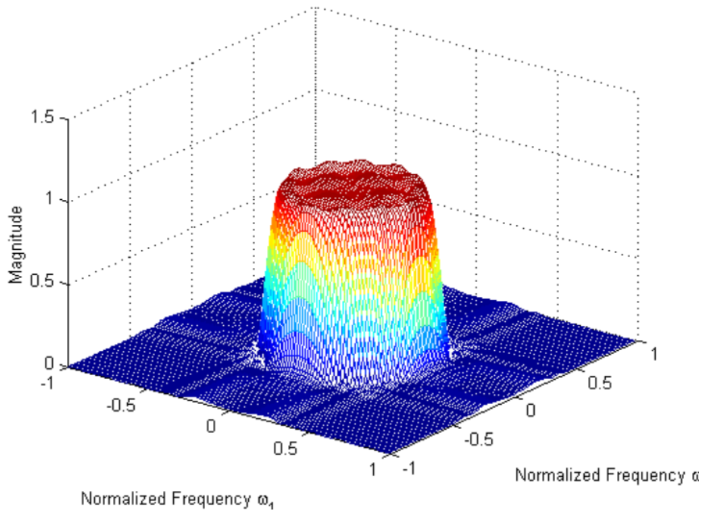

Fig. 3. Magnitude response of circularly symmetric 2-D IIR low pass filter.

Example 2. As a second example, consider the design of 63×63 fan filter with $\tan(\phi) = 0.35$, using Kaiser window [7].

The ideal magnitude response of this filter is depicted in Fig. 4. Again the SVD of Section 3 is applied and k is chosen to be 8. The exact number of the parallel branches is 22. Applying the singular perturbation model reduction algorithm to the FIR filters of order 62, giving us IIR filters of orders listed in Table 2. The maximum passband and stopband errors are 3.04% and 3.13% respectively. The magnitude response of the reduced order 2-D IIR filter is shown in Fig. 5.

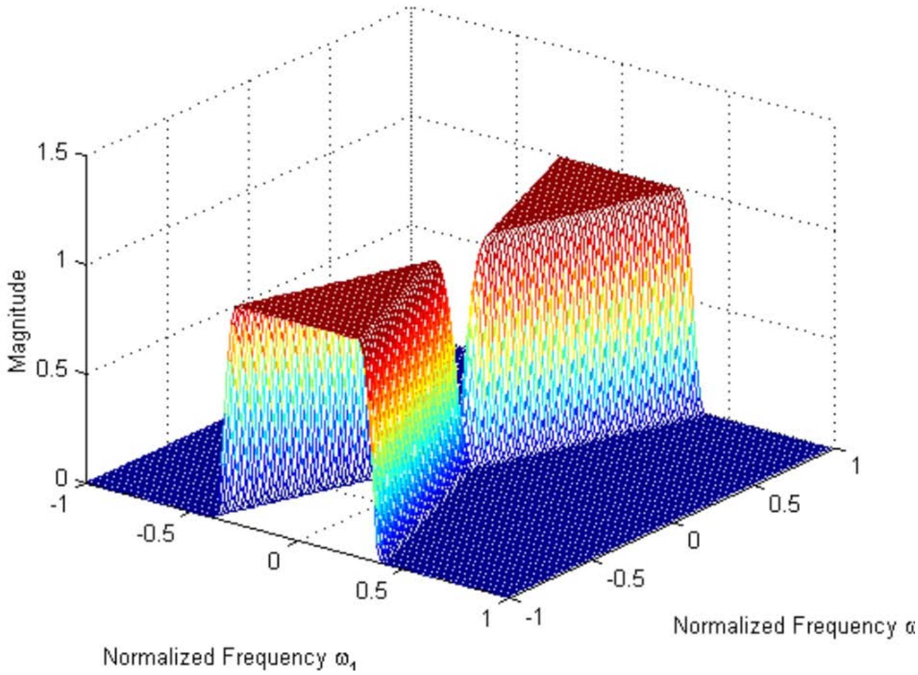

Fig. 4. Ideal magnitude response of a fan filter of order 63 x 63.

Table 2. Passband and stopband errors for the reduced order 2-D IIR fanfilter

No of Parallel Branches k	Orders of the Subfilters (n_2, n_1)	Passband Error E_P	Passband Error E_S
8	(31,16), (32,18), (33,17), (34,18) (35,18), (36, 20), (36,18), (36, 20)	0.0304	0.0313

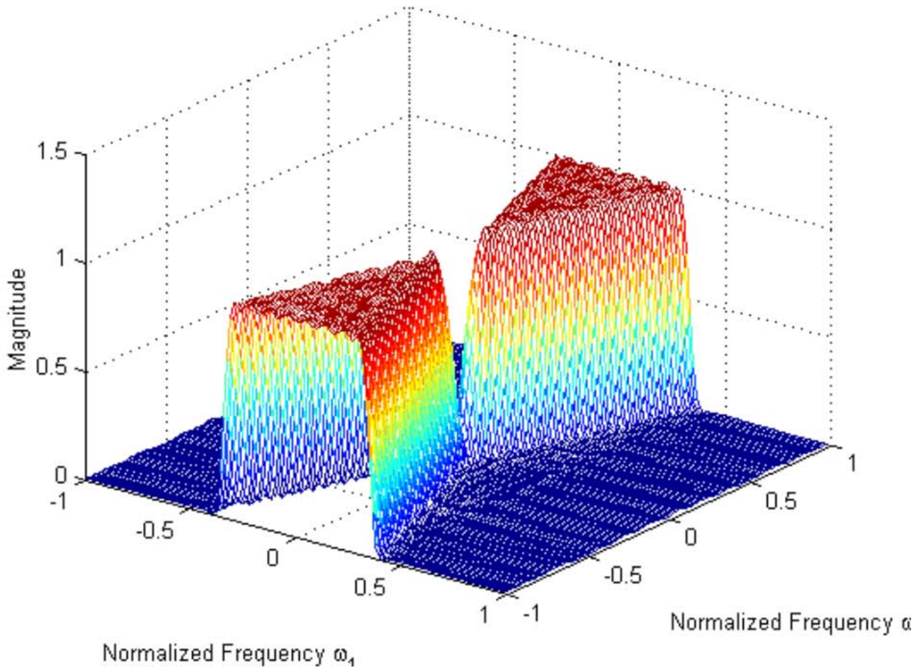

Fig. 5. Magnitude response of reduced order IIR fan filter.

Notice here in this example, that the reduction in the n_1 direction (ω_1 band) is less than one third, while the reduction in the n_2 direction (ω_2 band) is about one half. This is expected because the filter band ω_2 is extended along the whole band, while the filter band ω_1 is extended to less than $\frac{\pi}{2}$, see Figs 4 and 5. It was noticed in [19] that for the two reduction algorithms, balanced model truncation (BMT) and Hankel-norm optimal approximation (HOA), the wider band we have, the less reduction we obtain. In [20], it was shown that BMT gives relatively smaller error at high frequency compared with the singular perturbation approximation. A comparison of error performance for the truncated model reduction and singular perturbation reduction for different types of filters was given in [16], Moreover, it was shown that, as in the BMT and HOA, the singular perturbation model reduction for the narrow passband filters can be reduced more effectively than the wider passband filters. This explains why the reduction in n_2 is less than the reduction in n_1 directions in this example.

Conclusion

In this paper, a new design technique for quadrantally symmetric 2-D recursive digital filters using SVD and real Schur form decomposition is presented. This technique is based on two steps: decomposing the 2-D impulse matrix, H_d into k parallel sections, each comprising two 1-D FIR digital filters connected in cascaded. The FIR subfilters of order N are converted to lower order IIR filters of order n_i, where $i = 1, 2, ..., k$ using the singular perturbational model reduction algorithm.

The given examples had shown that the recursive magnitude response is very close to the ideal and the maximum errors in the passband and the stopband are relatively small.

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تصميم مرشحات رقمية ثنائية الأبعاد ومتماثلة ربعياً باستخدام تحليل القيمة المفردة واختزال النماذج ذات الاضطراب الشاذ

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(استلم في ٢٠٠١/٠٧/٠٧م،وقبل للنشر في ٢٠٠١/٠٩/٢٢م)

ملخص البحث. قدمت في هذه الورقة طريقة مبتكرة لتصميم مرشحات رقمية ثنائية الأبعاد ومتماثلة رُبعياً كما أنها ذات رتبة صغيرة وذات استجابة نبضية لانهائية. تعتمد هذه الطريقة على مرحلتين: في المرحلة الاولى تحلل مصفوفة الاستجابة النبضية للمرشح الأصلي إلى أعداد صغيرة من الفروع المتوازية ويتكون كل فرع من مرشحين رقميين متعاقبين ذ وي بعد واحد والمرحلة الثانية تتمثل في تبسيط واختزال المرشحات الرقمية ذات البعد الواحد والتي عادة ما تكون ذات رتبة عالية وذات استجابة نبضية نهائية إلى مرشحات ذات رتبة صغيرة وذات استجابة نبضية لا نهائية والغرض من هذه الخطوة هو تبسيط وتقليل العمليات الحسابية وكذلك تقليل الزمن اللازم للإشارات المارة في هذه المرشحات. في نهاية البحث قدمت أمثلة لشرح وبيان مزايا الطريقة المقترحة في هذا الورقة .