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An Improved Algorithm for Efficient Computation of Current Distribution in Power Cables

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Abstract. Unequal current distribution inside underground cables in close proximity causes unfavorable induced voltages and losses in their sheaths. Because of the problem complexity, most previous researchers in the field have not modeled the skin and proximity effects with sufficient accuracy. In this paper, a new technique is presented for tackling the eddy current problem of non-magnetic multi-conductor systems. The technique utilizes an efficient impedance matrix formulation, which yields accurate current distribution assessment. Applications to a set of test cable systems are also presented in the paper to demonstrate the usefulness of the proposed technique.

Introduction

Single conductor sheathed underground cables of different arrangements for three-phase voltages up to 132 kV are being widely used. The current distribution inside cable strands are usually non-uniform due to both skin effect and its proximity to the rest of the cables in the arrangement. Further, sheath-induced voltages cause eddy-currents and losses. These voltages, in the case of open-circuited sheaths, could become hazardous to personnel operating in the vicinity of the cable. In addition, excessive sheath losses may be produced in case of bonded sheaths.

Since the early decades of this century several contributions in this area of interest have been reported. H. Halperin and K.W. Miller have suggested different methods for bonding cable sheaths to reduce sheath losses without causing excessive induced voltages in the sheath [1]. Useful expressions for sheath voltages, currents and losses were given for unbonded and solidly-bonded sheaths in [1].

However, the proximity effect has been ignored in deriving these expressions. H.B. Dwight has introduced simplified analysis, which did not consider the proximity effect properly, to calculate the current distribution in cables (or bus bars) as well as the eddy-current losses in their sheaths (or enclosures) for specific arrangement [2]. Toshio Imai has extended the formulation of H.B. Dwight and applied it to randomly spaced multiple conductors [3]. The effect of the internal cable current on the sheath losses was included through subsidiary calculation [3-5]. Cable sheaths and busbar enclosures were considered thin throughout these approaches. Correction factors were introduced to consider the effect of sheath thickness on its eddy-current losses [3-5]. Reference [6] included an iterative solution to determine the current sharing for different cable arrangements, based on many simplifying assumptions and formulas. The results reported are not reliable as they deduced from the first iteration due to convergence problems.

None of the previous approaches has taken the skin and proximity effects properly in terms of accuracy achieved in the calculations. In calculating the losses in certain sheath the outer cables were replaced with filamentary conductors and the effect of their sheath currents was ignored [2-5]. In calculating cable parameters to determine the current sharing, the cable of interest is replaced with solid conductor of equivalent radius while the remaining cables are represented by filamentary conductors [2,6].

Indirect approach, based on optimization techniques, was introduced to improve the ampacity of parallel single-core cables by uninsulated additional conductors [7]. To avoid unnecessary approximations, an efficient and direct formulation for the eddy current problem of a nonmagnetic multi-conductor system with translational symmetry was developed and used to compute the skin and proximity effects on the impedance matrix of transmission networks [8]. The approach presented in reference [8] is modified and applied to some arbitrary power cable installations with open circuited (unbonded) sheaths as described in the following sections.

Formulation

For n straight parallel nonmagnetic conductors, the current density distribution inside conductor k can be described by the following Fredholm integral equation [8]

$$J_k(\rho) = J_{sk} + j\omega\mu_o \sigma_k \sum_{\ell=1}^{n} \frac{1}{2\pi} \int_{s_\ell} J_\ell(\rho') \, \ell n \left|\rho - \rho'\right| ds'$$

$$k = 1, \ldots, n \qquad (1)$$

where ρ and ρ' are two dimensional position vectors corresponding to the observation and integration points respectively, ω is the angular supply frequency, μ_o is the permeability of free space, and σ_k is the conductivity of conductor k.

Discretizing the cross-sectional area of each conductor into small elements and assuming that the current density is constant over each element, the above equation reduces to a system of linear algebraic equations

$$[B][J] = [J_s] \tag{2}$$

Rearranging equation (2) we obtain

$$[J] = [b][J_s] \tag{3}$$

The total current in each current-carrying conductor is given by

$$I_k = \sum_{i}^{N_k} J_i \delta_i \tag{4}$$

where N_k is the number of elements of conductor k and δ_i is the cross-sectional area of element i. The summation of equation (4) reduces to zero if the conductor is open circuited (e.g. unbonded sheath). Performing the summation for the different conductors and knowing that $J_s = \sigma E_s$, we obtain the following equation

$$\begin{bmatrix} I \\ 0 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} E_s \\ E_{ss} \end{bmatrix} \tag{5}$$

where E_s is the matrix of the impressed source term of current carrying conductors, and E_{ss} is the matrix of the induced voltages per unit length in the open-circuited conductors.

Equation (5) can be rearranged in the following form

$$\begin{aligned}[I] &= [Y_{11}][E_s] + [Y_{12}][E_{ss}] \\ &= \left[[Y_{11}] - [Y_{12}][Y_{22}]^{-1}[Y_{21}]\right][E_s] \\ &= [Y_t][E_s]\end{aligned} \tag{6}$$

Including equation (6) into the transmission line equation

$$[V_s] = [Z_{T.L}][I] + [Z_{load}][I] \tag{7}$$

where $[Z_{T.L}]$ refers to transmission network impedance matrix, $[Z_{T.L}] = \ell.[Y_t]^{-1}$, ℓ is length of the transmission line, we can calculate the current in the different three phases.

$[E_s]$ can be obtained from

$$[E_s] = [Y_b]^{-1}[I] \tag{8}$$

Substituting back in (5) we get

$$[E_{ss}] = [Y_{22}]^{-1}[Y_{21}][E_s] \tag{9}$$

To calculate the eddy-current losses in cable sheaths (3) is used to calculate the current distribution in the different elements of each sheath.

It is worthy to mention that this formulation is also applicable to busbars (with open circuit enclosures) feeding large industrial loads. When applied to power cables, each strand is considered as a separate conductor. In this case (5) can be used to calculate the current in each strand and to proceed in the calculations another summation similar to that in (4) is done to calculate the current in each cable and finally the process is repeated to get an equation describing the current in each phase in terms of $[E_s]$ and $[E_{ss}]$.

Results

The algorithm described in the above section is implemented in computer program and applied to different cable configurations. The specifications of the cable used are summarized in Table 1. Tables 2-5 summarize the results of the proposed technique for the four different configurations listed in Table 1. The tables include the results of the induced sheath voltages as computed by the formulas listed in [1], based on the nominal cable current as well as on the actual values calculated by the proposed approach. The actual values are less than the nominal values due to the voltage drop on the cables. Such voltage drop would cause the voltage at various points along the cable length to be less than the rated value, which is being taken at the sending end of the cable.

It is clearly seen that there is a good agreement between the two sets of results for triangular arrangement (configuration # 1), and the minor difference between the results is due to proximity effect, which is the same for the three cables. The difference between the two sets of results for the other three configurations is higher due to the unequal effect of proximity, which depends on both spacing and current phase angle of the outer phases. The power loss in the sheaths of phase C (leading the middle phase) is higher than those of phase A (lagging the middle phase).

Table 1 Cable data

Specifications

Cables:	13.8 kV, 3x single–core, copper XLPE 300 mm^2
Standard:	IEC
Conductor:	18/1 (spec. AW131-2)
	Diam. 23 mm
	Screen thick. 0.8 mm
Insulation:	XLPE
	Thick. 4.5 mm
	Screen thick 1 mm
Sheath:	Metallic screen (spec. AW423-2) Thick: 0.4 m

Operating conditions

Nominal current:	900 A
Sheath grounding:	At one point
Conductor temperature:	90 °C
Frequency:	60 Hz
Conductivity:	4.47×10^7 S m^{-1}

Cable arrangement

Spacing:	0.07 m
Length:	600 m
Phase sequence:	A, B, C
Reference voltage:	V_B

Configurations

#1 single circuit, triangular	A B C
#2 single circuit, flat	ABC
#3 double circuit, flat	$A_1B_1C_1$ $A_2B_2C_2$
#4 double circuit, flat	$A_1B_1C_1$ $C_2B_2A_2$

Table 2. Configuration # 1

Quantity	Phase	Present work	Reference [1]
Current (A)	A	895.27	900
	B	895.27	900
	C	895.27	900
Sheath voltage (V)	A	54.82	55.3 (55.1)*
	B	54.82	55.3 (55.1)*
	C	54.82	55.3 (55.1)*
Sheath power (W)	A	2610	
	B	2610	
	C	2610	

Note: Total power per circuit = 7830 W

*Values based on present calculated currents.

Table 3. Configuration # 2

Quantity	Phase	Present work	Reference [1]
	A	898.00	900
Current (A)	B	895.27	900
	C	892.55	900
	A	73.74	73.59 (73.43)*
Sheath voltage (V)	B	54.8	55.3 (55.01)*
	C	72.24	73.59 (72.89)*
	A	1250	
Sheath power (W)	B	5115	
	C	1348	

Note: Total power per circuit = 7713 W
*Values based on present calculated currents.

Comparing the results of Table 4 and Table 5, configuration # 4 is recommended as the voltages induced and power losses in the sheaths are lower than that of configuration # 3.

Table 4. Configuration # 3

Quantity	Phase	Present work	Reference [1]
	A_1	899.72	900
Current (A)	A_2	899.71	900
	B_1	895.1	900
	B_2	895.1	900
	C_1	890.74	900
	C_2	890.74	900
	A_1	102.12	101.34 (101.31)*
Sheath voltage (V)	A_2	102.12	101.34 (101.31)*
	B_1	68.5	69.4 (69.02)*
	B_2	68.5	69.4 (69.02)*
	C_1	98.3	101.34 (100.3)*
	C_2	98.3	101.34 (100.3)*
	A_1	3722	
Sheath power (W)	A_2	3722	
	B_1	11908	
	B_2	11908	
	C_1	3878	
	C_2	3878	

Note: Total power per circuit = 19508 W
*Values based on present calculated currents.

Table 5. Configuration # 4

Quantity	Phase	Present work	Reference [1]
	A_1	896.7	900
Current (A)	A_2	896.7	900
	B_1	895.03	900
	B_2	895.03	900
	C_1	894.07	900
	C_2	894.07	900
	A_1	46.75	45.38 (45.21)*
Sheath voltage (V)	A_2	46.75	45.38 (45.21)*
	B_1	68.5	69.4 (69.02)*
	B_2	68.5	69.4 (69.02)*
	C_1	43.45	45.38 (45.08)*
	C_2	43.45	45.38 (45.08)*
	A_1	2707	
Sheath power (W)	A_2	2707	
	B_1	1831	
	B_2	1831	
	C_1	2819	
	C_2	2819	

Note: Total power per circuit = 7357 W
*Values based on present calculated currents.

Conclusion

The new technique presented in this paper, for evaluating current distribution in closely-spaced cables, provides more accurate results as compared to other existing techniques. The accuracy gain was achieved by modeling both skin and proximity effects properly. The use of efficient impedance matrix formulation has achieved the required accuracy with, however, a modest additional computation burden. Results of the applications presented in the paper for four different cable configurations confirmed that non-uniformity in current distribution could, in some cases, account for significant differences in phase sheath voltages and power losses.

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طريقة محسنة لحسابات عالية الكفاءة لتوزيع التيار في كابلات القدرة

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ملخص البحث. بسبب توزيع التيار غير المتساوي في الكابلات الأرضية جهودا حثية غير مرغوب فيها. وبسبب هذه المشكلة المعقدة فإن معظم الباحثين السابقين في هذا المجال لم يتمكنوا من نمذجة تأثير الظاهرة السطحية والظاهرة التقاربية لتوزيع التيار بالدقة المطلوبة. وفي هذه الورقة تم تقديم طريقة جديدة لمتابعة ظاهرة التيار الدوامي في أنظمة الموصلات المتعددة غير الممغنطة ، وهذه الطريقة تستخدم صياغة مصفوفة المعاوقة والتي تعطي تقويما سليما لعملية توزيع التيار. وتم أيضا اختبار الطريقة المقترحة وتطبيقها على مجموعة من أنظمة الكابلات لإبراز مدى مرونة تطبيقها وجدواها.