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Studies on the Method of Orthogonal Collocation: I-A One-Point Collocation Method for the Transient Heat Conduction Problem

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Abstract. A one-point collocation scheme is developed for the transient heat conduction problem. The scheme makes use of the asymptotic solution of the problem for short time and the fact that initially the temperature profile can be divided into two zones, a fast temperature changing zone and inactive zone. For long time a Biot number dependent collocation point is used. The method provides approximate solutions which compares well with the numerical solution and other approximate solutions.

Notation

a	Shape factor in Eq. 1
A	Biot number
m	Defined in Eq. 56
Nu	Nusselt number
r	Dimensionless distance
r_1	r at the collocation point
t	Dimensionless time

t_o	Time at $\lambda = 0$
u	Dimenstionless temperature
u_s	Surface temperature
$\bar{u}$	Average temperature
u_1	u at the collocation point

Greek Letters

α	Defined in Eq. 26
β	Defined in Eq. 28
λ	Dimensionless distance of the inactive zone

Introduction

The transient heat conduction problem has been treated by several investigators [1-7] who were seeking approximate analytical solution for it. On the other hand, its numerical solution is well established through finite difference, finite element and orthogonal collocation methods [8].

$\bar{u}$

In this paper through the judicious choice of the collocation point and taking into consideration the steep profile at initial time, a one-point collocation method is used to obtain an approximate analytical solution which compares well with the numerical solution and other approximate solutions.

Although the problem has an analytical solution in the form of an infinite series, this solution requires a large number of terms for the short time and requires obtaining eigenvalues by the solution of transcendental equation. Another form for the solution is in terms of error function which is not convenient to use. In addition establishing an approximate solution for a linear problem could be the first step for doing the same for a non-linear problem.

Method Development

The dimensionless temperature profile u along the dimensionless distance r in the one dimensional transient heat conduction problem is represented by

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial r^2} + \frac{a}{r}\frac{\partial u}{\partial r} \tag{1}$$

with the initial condition at

$$t = 0,\ u = 0 \text{ for } r\ \varepsilon\ [0, 1] \tag{2}$$

and the boundary conditions;

$$\frac{\partial u}{\partial r} = 0 \text{ at } r = 0,\ t \geq 0 \tag{3}$$

$$\frac{\partial u}{\partial r} = A(1 - u_s) \text{ at } r = 1,\ t > 0 \tag{4}$$

where u_S is the surface temperature, i.e., $u|_{r=1}$, a = 0 for a slab, a = 1 for a cylinder, a = 2 for a sphere and A is the Biot number.

The average temperature $\bar{u}$ is defined by

$$\bar{u} = (a+1)\int_0^1 r^a\, u\, dr \tag{5}$$

We present here a one point collocation method that would lead to an approximate analytical solution. The method will be based on the dead zone concept [7, 9, 10] for the steep profile which occurs at the initial time.

For the one-point collocation, the temperature profile will be initially a sharp profile, thus we can divide the distance to an active zone and a dead zone. We will have

$$u = u_S \left(\frac{r-\lambda}{1-\lambda}\right)^2 \quad \text{for} \quad \lambda \leq r \leq 1 \tag{6}$$

and

$$u = 0 \quad \text{for} \quad 0 \leq r \leq \lambda \tag{7}$$

At t = 0, λ = 1 and as time progresses we reach a situation where λ = 0. At this instance, $u = u_S r^2$ for $r\ \varepsilon\ [0, 1]$. This will be the initial condition for a standard one point collocation as will be illustrated later. In the sequel, we will obtain expressions for determining λ, optimum collocation point, the temperature profile, the surface temperature and average temperature.

Substituting expression (6, 7) in the boundary condition, Eq. 4 and the average temperature Eq. 5, we will have

$$\frac{2u_s}{(1-\lambda)} = A\,(1 - u_s)$$

or

$$u_s = \frac{1}{\left(1+\frac{2}{A(1-\lambda)}\right)} \tag{8}$$

$$\overline{U} = (a+1)\int_\lambda^1 r^a u_s \left(\frac{r-\lambda}{1-\lambda}\right)^2 dr \tag{9}$$

For an integer value of a, this expression leads to

$$\overline{u} = u_s \frac{(1-\lambda)}{(a+2)(a+3)} \sum_{i=0}^{a} (i+2)(i+1)\lambda^{a-i} \tag{10}$$

For a slab, $a = 0$,

$$\overline{u} = u_s \frac{(1-\lambda)}{3} \tag{11}$$

For a cylinder, $a = 1$,

$$\overline{u} = u_s \frac{(1-\lambda)(\lambda+3)}{6} \tag{12}$$

For a sphere, $a = 2$,

$$\overline{u} = u_s \frac{(1-\lambda)(\lambda^2+3\lambda+6)}{10} \tag{13}$$

Substituting equations (6, 8) into the partial differential equation (1) we obtain

$$\frac{2A(r-\lambda)(1-\lambda)(2+A(1-\lambda)) - (r-\lambda)(1+A(1-\lambda))}{(1-\lambda)^2(2+A(1-\lambda))^2}\frac{d(1-\lambda)}{dt} = \frac{2A}{(1-\lambda)(2+A(1-\lambda))}\left[1+\frac{a(r-\lambda)}{r}\right] \tag{14}$$

This equation is to be satisfied at the collocation point $r=r_1(1-\lambda)+\lambda$. Thus we have

$$\frac{d(1-\lambda)}{dt}=\frac{[2+A(1-\lambda)][r_1(1-\lambda)(1+a)+\lambda]}{r_1(1-\lambda)[(2-r_1)+A(1-r_1)(1-\lambda)][r_1(1-\lambda)+\lambda]} \tag{15}$$

or written in another way as

$$\frac{d(1-\lambda)^2}{dt}=\frac{2[2+A(1-\lambda)][r_1(1-\lambda)(1+a)+\lambda]}{r_1[(2-r_1)+A(1-r_1)(1-\lambda)][(r_1(1-\lambda)+\lambda} \tag{16}$$

Although Eq. 16 can be integrated analytically, the solution will not be explicit in λ. So we either solve it numerically or try to obtain an approximate solution for λ. In addition we have to use an optimum value for r_1.

The first approximation is obtained as follows:

For the case of a slab ($a = 0$) and as $A \rightarrow \infty$

$$\frac{d[1-\lambda]^2}{dt}=\frac{2}{(1-r_1)r_1} \tag{17}$$

or

$$(1-\lambda)=\sqrt{\frac{2t}{r_1(1-r_1)}} \tag{18}$$

as $A \rightarrow 0$

$$\frac{d[1-\lambda]^2}{dt}=\frac{4}{(2-r_1)r_1} \tag{19}$$

Combining Eq. 17 and 19 in a form that depends on A and have the same solution as $A \rightarrow \infty$ and $A \rightarrow 0$ we obtain

$$\frac{d[1-\lambda]^2}{dt}=\frac{2}{\left(1-\frac{(A+1)}{(A+2)}r_1\right)r_1} \tag{20}$$

$$[1-\lambda]=\sqrt{\frac{2t}{r_1\left(1-\frac{(A+1)}{(A+2)}r_1\right)}} \tag{21}$$

This expression is extended through comparing numerical results with exact results for any shape to give

$$1-\lambda_1=\sqrt{\frac{2t}{r_1\left(1-r_1\left(\frac{A+1}{A+2}\right)\right)}}\left[1+0.5a\,r_1\sqrt{\frac{2t}{r_1\left(1-r_1\left(\frac{A+1}{A+2}\right)\right)}}\left(\frac{1+1.5A}{1+A}\right)\right] \tag{22}$$

The second approximation is obtained by substituting for (1 - λ) from Eq. 18 in the right hand side of Eq. 15 and then integrate to obtain

$$\begin{aligned}[1-\lambda]_2 &= \frac{(1-(a+1)r_1)}{(1-r_1)}d_1-\frac{r_1[(1-r_1)A+(2-r_1)(1-(a+1)r_1)]}{A(1-r_1)^2(A+2-r_1)} \\ &\ln(1+d_1d_2)-\frac{ar_1(2-2r_1+A)}{(1-r_1)^2(A+2-r_1)}\ln(1+d_1d_3)\end{aligned} \tag{23}$$

where

$$d_1=\sqrt{\frac{2t}{r_1(1-r_1)}}$$

$$d_2=\frac{A(1-r_1)}{(2-r_1)}$$

$$d_3=-(1-r_1)$$

The third approximation is obtained by substituting for (1 - λ) from Eq. 18 into Eq. 16 to obtain after integration

$$\begin{aligned}[1-\lambda]_3^2 &= \frac{(1-(a+1)r_1)}{(1-r_1)}d_1^2-\frac{2r_1[(1-r_1)A+(2-r_1)(1-(a+1)r_1)]}{A(1-r_1)^2(A+2-r_1)} \\ &\left[d_1-\frac{\ln(1+d_1d_2)}{d_2}\right]-\frac{2a\,r_1(2-2r_1+A)\left(d_1-\frac{\ln(1+d_1d_3)}{d_3}\right)}{(1-r_1)^2(A+2-r_1)}\end{aligned} \tag{24}$$

Comparing numerical results with exact results has shown that Eq. 23 is good for large A while Eq. 24 is good for small A.

Thus we combine both expressions into the following:

$$[1-\lambda]_4 = \frac{\left[(1-\lambda)_3 + A(1-\lambda)_2\right]}{(1+A)} \tag{25}$$

Now the optimum value for r_1 is obtained as follows:

For short time and for the case of a slab, the following asymptotic solution for $\bar{u}$ can be obtained from the asymptotic results of Martin and Saberian [1] as,

$$\bar{u} = \alpha = \frac{1}{\frac{1}{A} + \frac{\sqrt{t}(1+5A\sqrt{\pi t})}{(\sqrt{\pi}+10A\sqrt{t})}} \tag{26}$$

Our collocation analysis gives the following expression for $\bar{u}$;

$$\bar{u} = \frac{u_s(1-\lambda)}{3} = \frac{(1-\lambda)^2}{3(1-\lambda)+6/A} \tag{27}$$

Substitution from Eq. 26 into Eq. 27 we obtain a quadratic equation in (1-λ) which gives the solution

$$(1-\lambda) = \frac{1}{\beta} = \frac{3\alpha + \sqrt{9\alpha^2 + \frac{24\alpha}{A}}}{2} \tag{28}$$

substituting from Eq. 28 into Eq. 21, we obtain

$$r_1 = \frac{\left(1-\sqrt{1-8\beta^2 t\frac{(1+A)}{(2+A)}}\right)(2+A)}{2(1+A)} \tag{29}$$

$$\text{As } A \to \infty,\ \alpha \to \sqrt{\frac{4t}{\pi}},\ \ \beta \to \frac{1}{3\alpha} \to \sqrt{\frac{\pi}{36t}} \ \text{ and } \ r_1 \to \left(1-\sqrt{1-\frac{2\pi}{9}}\right)/2 \tag{30}$$

$$\text{As } A \to 0, \quad \alpha \to At, \quad \beta \to \frac{1}{\sqrt{6t}} \quad r_1 \to 1 - \frac{\sqrt{3}}{3} \tag{31}$$

For small time the expression inside the square root could become negative. In this case, r_1 is given by

$$r_1 = \frac{2+A}{2(1+A)} \tag{32}$$

As λ becomes zero the inactive zone disappears and we will have just one zone for which standard collocation can be applied. The collocation point is taken as suggested by Villadsen and Michelsen [7] to be A-dependent as

$$r_1^2 = \frac{(a+1)(A+a+5)}{(a+5)(A+a+3)} \tag{33}$$

At the instant of switching from the two zones into one zone, we will have

$$u_s = \frac{1}{1+\frac{2}{A}} \tag{34}$$

$$u = \frac{r^2}{1+\frac{2}{A}} \tag{35}$$

and

$$\bar{u} = \frac{(a+1)A}{(a+3)(A+2)} \tag{36}$$

As time increases, define

$$u = \frac{(r^2 - r_1^2)}{(1-r_1^2)} u_s + \left(\frac{r^2-1}{r_1^2-1}\right) u_1 \tag{37}$$

where u_1 is the temperature at the collocation point r_1, $\bar{u}$ is then given by

$$\bar{u} = (a+1)\int r^a \, u \, dr = \frac{1}{(a+3)}\left[\frac{(a+1)-(a+3)r_1^2}{(1-r_1^2)} u_s + \frac{2u_1}{(1-r_1^2)}\right] \tag{38}$$

At the collocation point, Eq. 1 becomes

$$\frac{du_1}{dt}=\frac{2(a+1)(u_s-u_1)}{(1-r_1^2)} \tag{39}$$

and Eq. 4 becomes

$$\left.\frac{du_1}{dr}\right|_{r=1}=\frac{2}{(1-r_1^2)}(u_s-u_1)=A(1-u_s)=\frac{1}{(a+1)}\frac{du_1}{dt} \tag{40}$$

Therefore

$$\frac{du_1}{dt}=(a+1)A(1-u_s) \tag{41}$$

$$\frac{du_1}{dt}=\frac{(a+1)(1-u_1)}{\frac{1-r_1^2}{2}+\frac{1}{A}}=(a+1)A(1-u_s) \tag{42}$$

Thus

$$u_s=1-\frac{(1-u_1)}{1+\frac{(1-r_1^2)}{2}A} \tag{43}$$

$$u_1=\frac{Ar_1^2}{A+2}+\frac{(2+A(1-r_1^2))}{A+2}\left[1-e^{\frac{-(t-t_o)}{\frac{1-r_1^2}{2(a+1)}+\frac{1}{A(a+1)}}}\right] \tag{44}$$

$$u_s=\frac{A}{(A+2)}+\frac{2}{(A+2)}\left[1-e^{\frac{-(t-t_o)}{\frac{1-r_1^2}{2(a+1)}+\frac{1}{A(a+1)}}}\right] \tag{45}$$

where t_o is the time at which $\lambda=0$.

From Eqs. 38, 43-45, we obtain:

$$\bar{u} = 1 - \frac{(1-u_1)(1+\frac{A}{(a+3)})}{(1+\frac{(1-r_1^2)A}{2})} \tag{46}$$

and

$$\bar{u} = \frac{(a+1)A}{(a+3)(A+2)} + \frac{2(A+a+3)}{(a+3)(A+2)}\left(1 - e^{\frac{-(t-t_0)}{\frac{1-r_1^2}{2(a+1)}+\frac{1}{A(a+1)}}}\right) \tag{47}$$

Substituting Eq. 46 into Eq. 42, we obtain

$$\frac{d\bar{u}}{dt} = \frac{(a+1)(1-\bar{u})}{\frac{1-r_1^2}{2}+\frac{1}{A}} \tag{48}$$

We notice that as $A \to 0$, $\bar{u} \to u_1 \to 0$, $u_s \to u_1 \to 0$

Comparison with previous work

The application of the method of moments [7] to Eq. 1-4 would lead to the following expression for the average temperature

$$\bar{u} = 1 - \exp\left(\frac{(a+1)t}{\frac{1}{A}+\frac{1}{(a+3)}}\right) \tag{49}$$

For short time the true average is lower than that calculated from Eq. 49 whereas for long time it is higher. Thus expression (49) is generally not accurate enough.

Higher accuracy for long time is obtained through Eq. 47 derived in this paper.

On the other hand Dixon [6] developed an expression similar to Eq. 42 for the case of a cylinder and instead of using Eq. 38 for calculating the average temperature, he used the approximation

$$\bar{u} \cong u_1 \tag{50}$$

This makes his results less accurate specially for large A.

Harriott [2] obtained the following expression for the case of a sphere and $A \to \infty$

$$\bar{u} = \sqrt{1 - e^{-10t}} \tag{51}$$

This expression has an overall accuracy better than Eq. 49 but still less accurate than Eq. 47 for long time.

Martin and Saberian [1] obtained an expression of high accuracy given by

$$\bar{u} = 1 - \exp\left(\frac{-(a+1)t}{\frac{1}{A} + \frac{1}{Nu(t)}} \right) \tag{52}$$

where

$$Nu(t) = \sqrt{Nu_\infty^2 - b^2 + (Nu_o + b)^2} \quad , b = 0.2 \tag{53}$$

$$Nu_o = \frac{\sqrt{\pi} + 10A\sqrt{t}}{\sqrt{t}(1 + 5A\sqrt{\pi t})} \tag{54}$$

$$Nu_\infty = \frac{a + 3 + A}{1 + \frac{(a+1)A}{m^2}} \tag{55}$$

$$m = \begin{cases} \frac{\pi}{2} & \text{slab} \\ 2.4048 & \text{cylinder} \\ \pi & \text{sphere} \end{cases} \tag{56}$$

This expression satisfies the short time and long time asymptotes. Better values for m^2 are the collocation weights 2.5 for a slab, 6 for a cylinder and 10.5 for a sphere.

In this paper we developed short time and long time expressions which will be compared with Martin and Saberian [1] expressions in the next section.

Numerical Results

The numerically exact solution for equations (1-4) is obtained through the application of standard orthogonal collocation method [7] using 16 collocation points. Four approximate solutions are compared with the exact solutions; for short time, together with Eq. 6-13 the approximate solutions for λ, the dimensionless distance of the inactive zone, given by the (i) numerical solution of the differential Eq. 16, (ii) equation (22), (iii) Eq. 25 with the long time solution given by Eq. 43, 44, 46. The switching time t_0 from the short time to the long time solution occurs when $\lambda = 0$. The fourth approximate solution for comparison is the one derived by Martin and Saberian [1] (Eq. 52-55). In Tables [1-3], we compare the integral of the square error I given by

$$I = \int_0^1 \left[\bar{u}_{\text{approximate}} - \bar{u}_{\text{exact}}\right]^2 dt \tag{57}$$

for the four approximations.

Table 1. Comparison between integeral square error for the approximate solutions for the case of a slab (a=0)

A	1st Approximation Eq. 16	2nd Approximation Eq. 22	3rd Approximation Eq. 25	Martin and Saberian Approximation
0.1	1.66×10^{-9}	3.512×10^{-8}	4.745×10^{-8}	6.944×10^{-8}
1	2.5546×10^{-5}	5.0905×10^{-7}	9.5528×10^{-5}	$7.8169\ 10^{-6}$
10	1.8298×10^{-4}	3.0487×10^{-5}	1.8089×10^{-4}	1.1642×10^{-4}
100	1.9827×10^{-5}	9.0135×10^{-6}	1.7292×10^{-5}	2.3113×10^{-4}
1000	7.2232×10^{-6}	6.7316×10^{-6}	7.2778×10^{-4}	2.5184×10^{-4}
10000	6.6079×10^{-6}	6.5659×10^{-4}	6.6287×10^{-4}	2.5404×10^{-4}

Table 2. Comparison between integeral square error for the approximate solutions for the case of a cylinder (a=1)

A	1st Approximation Eq. 16	2nd Approximation Eq. 22	3rd Approximation Eq. 25	Martin and Saberian Approximation
0.1	4.3404×10^{-6}	2.1001×10^{-6}	4.4994×10^{-6}	2.9022×10^{-7}
1	4.6451×10^{-6}	4.1509×10^{-5}	4.5035×10^{-6}	9.2794×10^{-6}
10	1.3496×10^{-4}	2.3763×10^{-5}	6.3526×10^{-5}	4.6063×10^{-5}
100	3.0465×10^{-5}	2.5136×10^{-5}	1.8476×10^{-5}	2.3113×10^{-5}
1000	1.8984×10^{-5}	2.3338×10^{-5}	1.9325×10^{-5}	9.0169×10^{-5}
10000	1.8509×10^{-6}	2.3174×10^{-5}	2.005×10^{-5}	9.1186×10^{-5}

Table 3. Comparison between integeral square error for the approximate solutions for the case of a sphere (a=2)

A	1st Approximation Eq. 16	2nd Approximation Eq. 22	3rd Approximation Eq. 25	Martin and Saberian Approximation
0.1	1.0856×10^{-5}	4.5867×10^{-6}	5.6463×10^{-6}	8.5852×10^{-7}
1	2.2866×10^{-4}	8.4376×10^{-5}	1.2100×10^{-5}	1.4700×10^{-5}
10	1.6387×10^{-4}	4.2543×10^{-5}	9.5589×10^{-5}	3.7592×10^{-5}
100	6.5906×10^{-5}	5.0170×10^{-5}	3.9699×100^{-5}	5.0728×10^{-5}
1000	4.2956×10^{-5}	4.7643×10^{-5}	3.2992×10^{-5}	5.3951×10^{-5}
10000	4.1643×10^{-5}	4.7368×10^{-5}	3.2551×10^{-5}	5.4374×10^{-5}

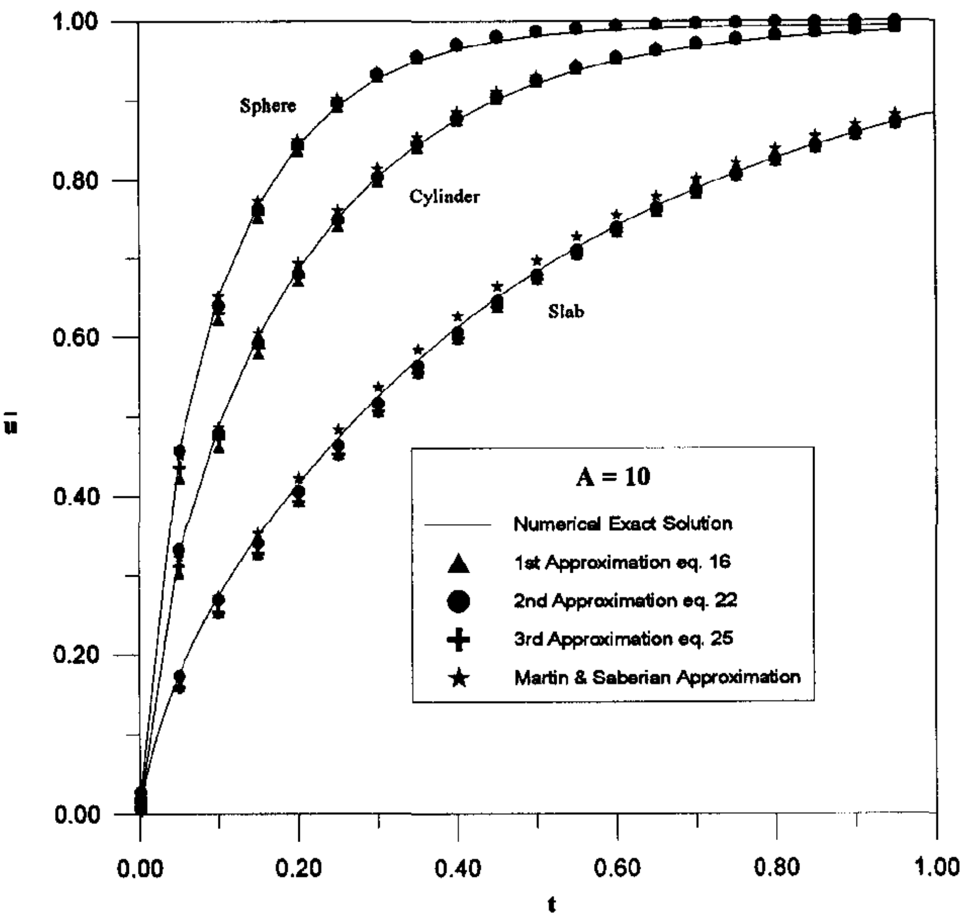

Fig. 1. Comparison of average temperatures for different shapes.

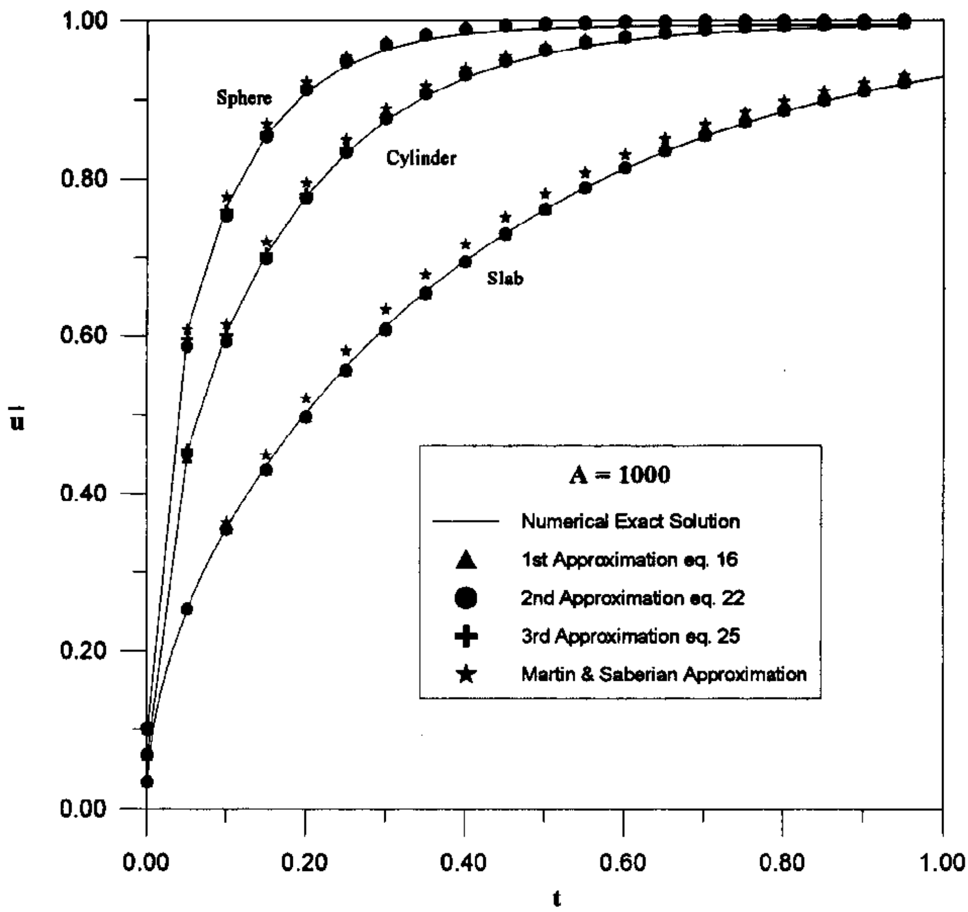

Fig. 2. Comparison of average temperatures for different shapes.

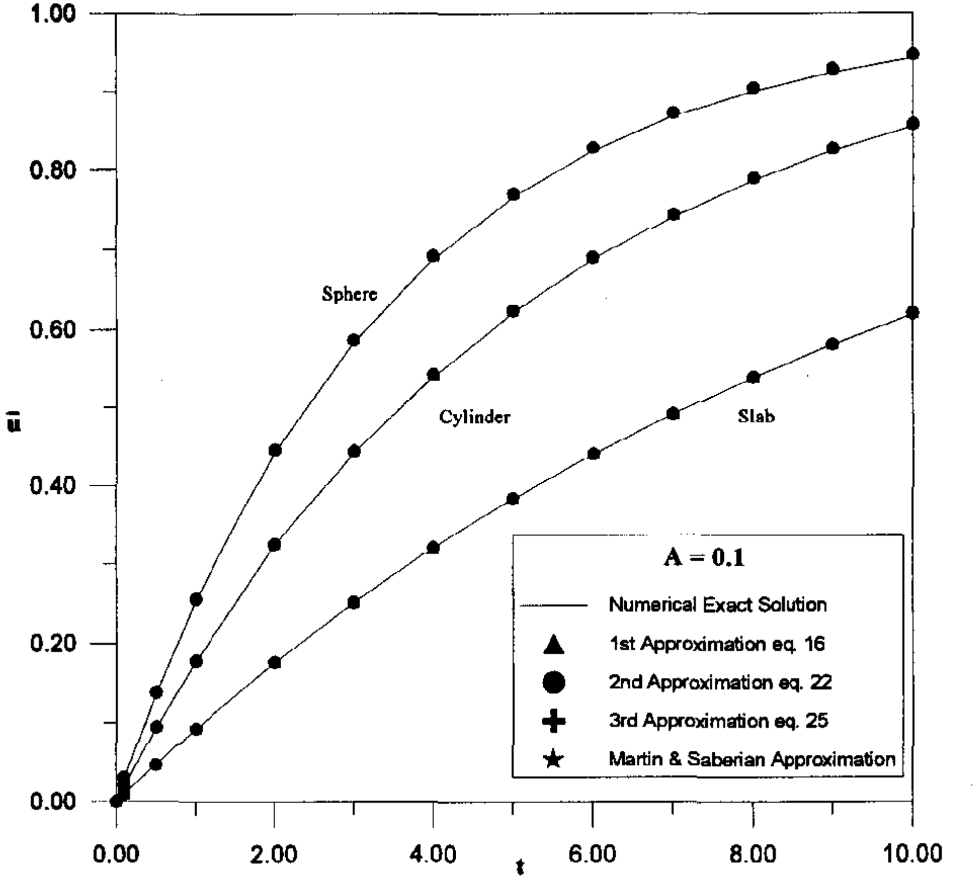

Fig. 3. Comparison of average temperatures for different shapes.

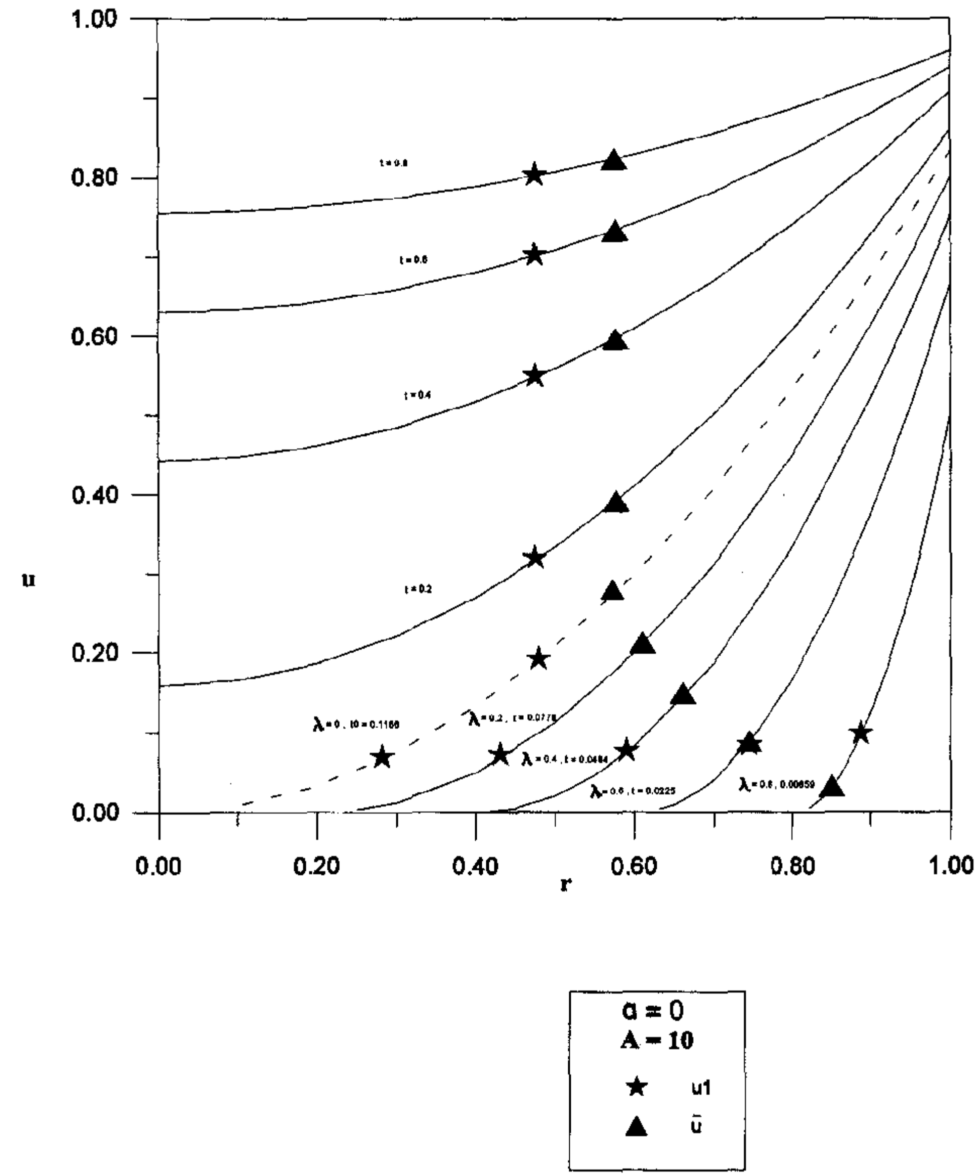

Fig. 4. Temperature profile for transient heat conduction.

It is noticed that the second approximation has the best overall performance whereas that of Martin and Saberian is not as accurate for large A. Figure 1 compares the average temperature change for the case of A = 10 and different shapes. Again the second approximation shows the best performance. Figure 2 shows the same trend for the case A = 1000. For small A (A = 0.1). Fig. 3 shows that it needs longer time to reach steady state since the heat flux at the surface is smaller. Table 4 gives the value of the collocation points for different time and λ whereas Fig. 4 gives the change in u with time and distance during the two zones period and the one zone period. It is indicated on the same figure the value of u at the collocation point (u_1) and the average u ($\bar{u}$). When $\lambda = 0$, the collocation point and hence u_1 changes.

Table 4. Values of collocation points for the case of a slab (a = 0) and A = 10 for different time and λ

λ	t	r_1 two-zone	r_1 one-zone
0.8	0.0066	0.4429	
0.6	0.0225	0.3562	
0.4	0.0464	0.3215	
0.2	0.0778	0.3015	
0.0	0.1166	0.2882	0.4804

Conclusions

Using a one point collocation method, we are able to obtain accurate approximations for the average temperature. The second approximation represented by Eq. 22 for short time and Eqs. 43, 44, 46 for long time is particularly simple and more accurate than other approximations. The success of the method is mainly due to the proper selection of the collocation point which is chosen using the asymptotic solution for short time and using an A dependent collocation point for long time.

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دراسات على طريقة التنظيم المتعامد:
(١) طريقة التنظيم باستخدام نقطة واحدة لحل مسألة التوصيل الحراري الانتقالي

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ملخص البحث. طُورت طريقة للتنظيم باستخدام نقطة واحدة لحل مسألة التوصيل الحراري الانتقالي وهذه الطريقة تستخدم الحلول التقاربية للمسألة لوقت قصير وكذلك تستفيد من الحقيقة التي تقول إنه في باديء الأمر يمكن تقسيم التغير في درجة الحرارة إلى منطقتين: منطقة بها تغيير سريع في درجة الحرارة ومنطقة لا يحدث فيها تغير في درجة الحرارة وكذلك تستفيد هذه الطريقة من أنه عند وقت طويل فإن نقطة التنظيم تعتمد على رقم بيوت وإن هذه الطريقة تعطي حلولا تقريبية بدقة قريبة من الحلول التي تعطيها الطرق العددية والطرق التقريبية الأخرى.