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The Vibrational Energy States of a Trapped Atom between Conducting Plates

S. Al-Awfi
Department of Physics, Faculty of Science, King Abdulaziz University, Madinah, P. O. Box 344, Saudi Arabia

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Abstract. The vibrational energy states of the center-of-mass motion for an atom trapped by the optical dipole potential due to a cavity mode are calculated. In particular, a sodium atom between two conducting plates is considered with its dipole moment vector orientated parallel to the plates. The negative detuning situation and appropriate parameters are chosen in order to maintain a sufficient depth for the central well. Variations of the vibrational states with the mode intensity, longitudinal velocity, and plates separation are examined and discussed.

1. Introduction

It is well known that, in the absence of any external influence, an atom interacts with the vacuum electromagnetic fields that are constrained by the cavity in which an atom is placed leading to two types of physical effects [1-4]. First, the decay emission rate of the atom is modified, becoming position-dependent. Second, the atom experiences energy shifts to both levels [3-5].

On the other hand, in the presence of any excitation cavity modes, the state of the motion of the atom can be altered. With one of the modes excited with sufficient intensity, the atom experiences radiation pressure force, which is exploited in cooling (or heating) atomic motion [6-7]. The atom also experiences a dipole force (gradient of optical dipole potential), which causes trapping of the atom [7-8]. In this paper we evaluate the vibrational energy states of a trapped atom in optical dipole potential due to a cavity mode by using harmonic oscillator approximation. Such a study is useful in understanding the nature of the quantized motion of the trapped atom in a micro-cavity, in general [5].

This paper is organized as follows: in section 2, the outlines of the procedure

leading to the evaluation of the optical trapping potential in a cavity when a specific cavity mode is excited are given. In section 3, we describe our basic model and modify all relations in accordance with it. In section 4, we show the properties of atomic motion in the context of atom-guides. In section 5, we estimate the vibrational frequency of the trapped atom by using the harmonic oscillator approximation, and then discuss the effects of the mode intensity, longitudinal velocity, and plates separation on the vibrational frequency. Finally, the summary and conclusions are given in Section 6.

2. Trapping Potential in a Cavity

In the presence of any excitation cavity modes, the atom becomes subject to a light-induced force derivable from the dipole potential associated with the cavity mode. The steady state dipole force (the gradient of the potential) acting on the center of the mass of the atom can be written as [9]:

$$F_{dip.}(\mathbf{R},\mathbf{V}) = -\frac{1}{2}\hbar\Delta\frac{\nabla S(\mathbf{R},\mathbf{V})}{1+S(\mathbf{R},\mathbf{V})} \tag{1}$$

In the above equation $S(\mathbf{R},\mathbf{V})$ is the saturation parameter defined by:

$$\mathrm{S}(\mathbf{R},\mathbf{V}) = \frac{2\Omega^2(\mathbf{R})}{\Delta^2(\mathbf{R},\mathbf{V})+\Gamma^2(\mathbf{R})} \tag{2}$$

where $\Gamma(\mathbf{R})$ is a position-dependent decay rate for an electric dipole moment $\mathbf{d}$ situated at an arbitrary point $\mathbf{R}=(\mathrm{x},\mathrm{y},\mathrm{z})$ within a cavity, and $\Omega(\mathbf{R})$ is the Rabi frequency for an electric dipole $\mathbf{d}$ in the cavity mode, while $\Delta(\mathbf{R},\mathbf{V})$ is a dynamic detuning parameter, defined for the two-plate case by:

$$\Delta(\mathbf{R},\mathbf{V}) = \omega\ (\mathrm{k}_{\|},\mathrm{n}) - \omega_0 - \mathrm{k}_{\|}\mathrm{V}_{\|} = \Delta_0 - \mathrm{k}_{\|}\mathrm{V}_{\|} \tag{3}$$

where $k_{\|}$ and $V_{\|}$ are the magnitude of the wave vector and the velocity in the parallel direction of the cavity respectively.

On the other hand, the dipole force $F_{dip.}$ can directly be written as the gradient of a dipole potential $\mathrm{U}_{\mathrm{dip.}}$ as follows [9]:

$$\mathrm{F}_{\mathrm{dip.}}(\mathbf{R},\mathbf{V}) = -\nabla\mathrm{U}_{\mathrm{dip.}}(\mathbf{R},\mathbf{V}) \tag{4}$$

Hence the dipole potential $U_{dip.}$ can be written as:

$$\mathrm{U}_{\mathrm{dip.}}(\mathbf{R},\mathbf{V}) = \left\{\left(\frac{\hbar\Delta(\mathbf{R},\mathbf{V})}{2}\right)\ln 1(+\mathrm{S}(\mathbf{R},\mathbf{V})\right\} \tag{5}$$

It is very clear that the ability of such potential to trap the atoms within the central region of the cavity depends on many factors, such as the cavity shape, the dipole

orientation, the type of detuning and the type of excited cavity mode [4].

3. Physical Model

The basic model considered here consists of a sample of a single atom which is trapped in the optical dipole potential due to an excitation of a cavity mode in vacuum between two conducting plates separated by a distance L.

The trapping potential of such model can be modified essentially when a cavity mode is excited at a frequency $\omega(k_{\|}, n)$, which is closely tuned to the dipole transition frequency ω_0. By setting $\mathbf{R} = (\mathbf{r}_{\|}, z)$ with $\mathbf{r}_{\|} = (x, y)$, we can write the optical dipole potential $U_{dip.}$ which traps the atom between the conducting plates in a static case (i.e. $V_{\|} = 0$) as:

$$U_{dip.}\left(k_{\|}, n, z\right) = \left\{\left(\frac{\hbar\Delta_0(k_{\|}, n)}{2}\right)\ln\left[1 + S(k_{\|}, n, z)\right]\right\} \tag{6}$$

where the saturation parameter $S(k_{\|}, n, z)$ for this case is given as:

$$S(k_{\|}, n, z) = \frac{2\Omega^2(k_{\|}, n, z)}{\Delta_0^2(k_{\|}, n) + \Gamma^2(z)} \tag{7}$$

where n refers to the order of the excited mode and $\Delta_0 = \omega\ (k_{\|}, n) - \omega_0$ is the static detuning of the cavity mode from the atomic resonance, and it is clear that the potential provides a minimum at the high intensity locations for red detuning while it provides a maximum at the high intensity locations for blue detuning.

In this paper we have chosen to work with sodium atom Na for many reasons: it is easy to produce a thermal sodium atomic beam, and sodium has an ideal level structure. Also, we can drive the cooling transition by a continuous wave dye laser employing one of the most efficient and reliable dyes, Rhodamine 6G [7]. In addition we have taken the decay rate $\Gamma(z)$, Rabi frequency $\Omega(k_{\|}, n, z)$ and thus the optical dipole potential $U_{dip.}\left(k_{\|}, n, z\right)$ for an atom trapped between two conducting plates for a specific case. With the dipole of the atom orientated parallel to the plates, and with red detuning $\Delta_0 < 0$, the parameters are chosen in order to maintain an enough central well depth. We have taken this specific case because the main objective of this work is to describe the quantum behavior for a confined atom. Therefore, Eq.(5) should be rewritten as :

$$U_{dip.}\left(k_{\|}, n, z\right) = \left\{\left(\frac{\hbar\Delta_0(k_{\|}, n)}{2}\right)\ln\left[1 + \frac{2\Omega_{\|}^2(k_{\|}, n, z)}{\left[\Delta_0^2(k_{\|}, n) + \Gamma_{\|}^2(z)\right]}\right]\right\} \tag{8}$$

where $\Gamma_{\|}(z)$ is given by [1]:

$$\Gamma_{\|}(z) = \Gamma_o \sum_{n=0}^{[2L/\lambda]} \frac{3\lambda}{4L}\left[1+\left(\frac{n\lambda}{2L}\right)^2\right] \sin^2\left(\frac{n\pi}{L}z\right) \tag{9}$$

and Γ_0 is the corresponding spontaneous decay rate in free space, is given by:

$$\Gamma_0 = \frac{8\pi^2\mu^2}{3\hbar\varepsilon_0\lambda^3} \tag{10}$$

Beside $n=1$, $\Omega_{\|}(z)$ is given by [4]:

$$\Omega_{\|}(z) = \sqrt{2}\ \Omega_0\left(\frac{\lambda}{2L}\right)\sin\left(\frac{\pi\ z}{L}\right) \tag{11}$$

where Ω_0 is the free space Rabi frequency, which is given as:

$$\Omega_0 = \left(\frac{\mu^2 I_0}{2\hbar^2\varepsilon_0 c}\right)^{1/2} \tag{12}$$

There are in fact two distinct theoretical limits of the trapping forces, depending on the saturation parameter. Firstly, the low intensity limit, when $S<<1$. Secondly, the high intensity limit, when $S>>1$. In the latter case we need to use the dressed atom approach [10], while in the first case we can use the above equations as basis of the dynamic motion. Here we consider the limit of large detuning in which $|\Delta_0|$ is assumed to be much greater than the peak Rabi frequency Ω and greater than the atomic recoil frequency ($v_{rec} = \hbar k^2/2M = 1.58\times10^5 s^{-1}$).

Fig. 1 shows the variation of the optical dipole potential $U_{dip.}(k_{\|}, n, z)$ between the two plates corresponding to the parameters given in Table 1. Consequently, the red detuned atom provides a quasi-harmonic potential that traps the atom in the region of the minimum of $U_{dip.}(k_{\|}, n, z)$ which occurs at $z_{min} = L/2$.

4. Atomic Motion

In the context of cooling or trapping atoms, we have to distinguish between two atomic motions; firstly, the longitudinal motion due to the spontaneous forces, which may be slowed down to cool the atom; secondly, the transverse motion due to the optical potential, which leads to trap the atoms. The first motion can be treated classically because its velocity is very large (about $10\ km/s$) [4,11-12], while the second motion has a very low velocity. The maximum transverse velocity corresponding to the central

potential depth can be given as [13]**:**

$$V_{max} = \left[2U_{min} / M\right]^{1/2} \tag{13}$$

where M is the mass of the atom. From Fig. 1, we can directly obtain V_{max} which is about $2m/s$. This means that the momentum of the photon ($P_p = \hbar k = 1{\cdot}13\times10^{-27}\,N\cdot s$) is comparable to the momentum of the atom ($P_a = MV_{max} = 7.682\times10^{-26}\,N\cdot s$), or, in other words, the de Broglie atomic wavelength ($\lambda_{dB} = h/MV_{max} = 8{\cdot}67\times10^{-9}\,m$) is comparable to the radiation wavelength ($\lambda = 589.0\times10^{-9}\,m$). Consequently, we have to consider the limit in which the atom obeys quantum-mechanical laws. Also, the atoms move in the quantum regime since the separation between the plates is $L = 500nm$.

Table 1. The parameters corresponding to Fig.1

Parameters	Symbol	Value	Unit
Wavelength of a resonant beam	λ	589×10^{-9}	m
Transition frequency	ω_0	$3{\cdot}2\times10^{15}$	s^{-1}
Plates separation	L	500×10^{-9}	m
Atomic mass	M	$3{\cdot}82\times10^{-26}$	kg
Free space decay rate	Γ_0	$61{\cdot}3\times10^{6}$	s^{-1}
Free space Rabi frequency	Ω_0	9.8×10^{8}	s^{-1}
Static detuning	Δ_0	-37×10^{-9}	Hz
Scaling velocity	V_0	1.0×10^{4}	ms^{-1}
Scaling intensity	I_0	$1{\cdot}0\times10^{7}$	Wm^{-2}
Scaling potential	U_0	$3{\cdot}2\times10^{-27}$	J

It can also be seen from Fig.1 that the central well depth is approximately $23U_0$ (where $U_0 = \hbar\Gamma_0/2 = 3.23\times10^{-27}\,J$). This is deep enough to permit many quasi-harmonics trapping (vibrational) states. The vibrational frequency of trapped atoms can be estimated simply by using the harmonic oscillator approximation [8,14].

5. Harmonic Oscillator Approximation

We have shown that there is an adequate dipole potential depth (where the highest of vibrational frequency of trapped atoms is about $v = 23U_0/h \approx 112MHz$), which allows many quasi-harmonic trapping (vibrational) states to exist. We approximate the dipole potential in Fig.1 by the harmonic oscillator approximation (the parabolic approximation) about the dipole potential minimum for mode n as [14]:

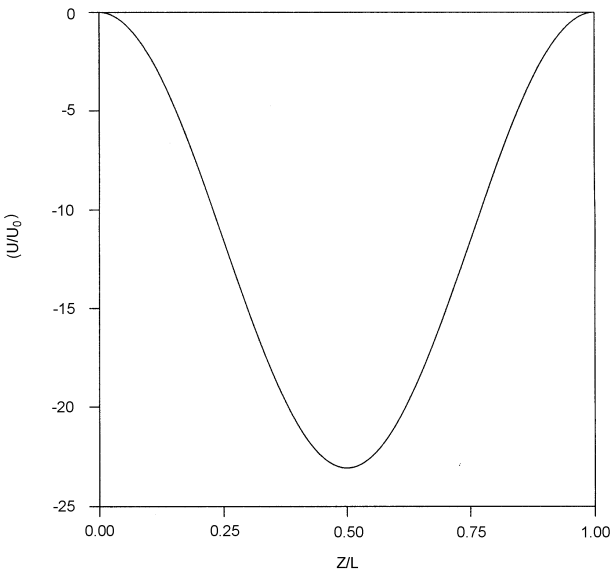

Fig. 1. The dipole potential of the sodium atom between conducting plates when the dipole is parallel to the plates. See Table.1 for values of parameters.

$$U\left(k_{\|}, n, Z\right) \approx U_{min.} + \frac{1}{2} M\omega_n{}^2 \left(z - z_{min}\right)^2 + ... \tag{14}$$

where ω_n is the vibrational frequency of the atom in the dipole potential and the sub-number refers to the order of the excited mode. The quantity $M\omega_n$ is the stifness constant and $U_{min.}$ is the potential minimum which is given by:

$$U_{min} = U_{dip.}\left(k_{\|}, n, z_{min}\right) = \left\{\left(\frac{\hbar\Delta_0}{2}\right)\ln\left[1 + \frac{2\Omega_{\|}^2\left(k_{\|}, n, z_{min}\right)}{\left[\Delta_0^2 + \Gamma_{\|}^2\left(z_{min}\right)\right]}\right]\right\} \tag{15}$$

The harmonic oscillator approximation and the parameters assumed above will give a central well depth approximately like Fig.1. It is shown clearly by a dotted line in Fig.2. The harmonic vibrational frequency ω_n can be estimated simply by using the harmonic oscillator approximation as follows:

$$\omega_n = \left\{\frac{2}{M}\frac{d^2U_{dip}}{dz^2}\right\}_{z_{min}}^{\frac{1}{2}} \tag{16}$$

We have explicitly:

$$\omega_n = \left\{ \frac{4\hbar\pi^2\Omega_{\|}^2\Delta}{(\Delta^2 + 2\Omega_{\|}^2)L^2M} \right\}_{z_{min}}^{\frac{1}{2}} \tag{17}$$

While the corresponding harmonic oscillator ground state width is obtained by:

$$\delta z = \left(\hbar / M\omega_n\right)^{1/2} \tag{18}$$

The harmonic oscillator approximation is valid if $\delta z << z_{min}$ so that the atomic wave function is well localized around the potential minimum.

It is not difficult to check that for the parameter values for sodium, and the $n = 1$ p-polarized mode in the cavity between the conducting plates, the vibrational frequency and hence the vibrational energy can be estimated as:

$$\omega_1 \approx 2\times10^7\,s^{-1} \quad \& \qquad E_1 \approx 2\times10^{-27}\,J \tag{19}$$

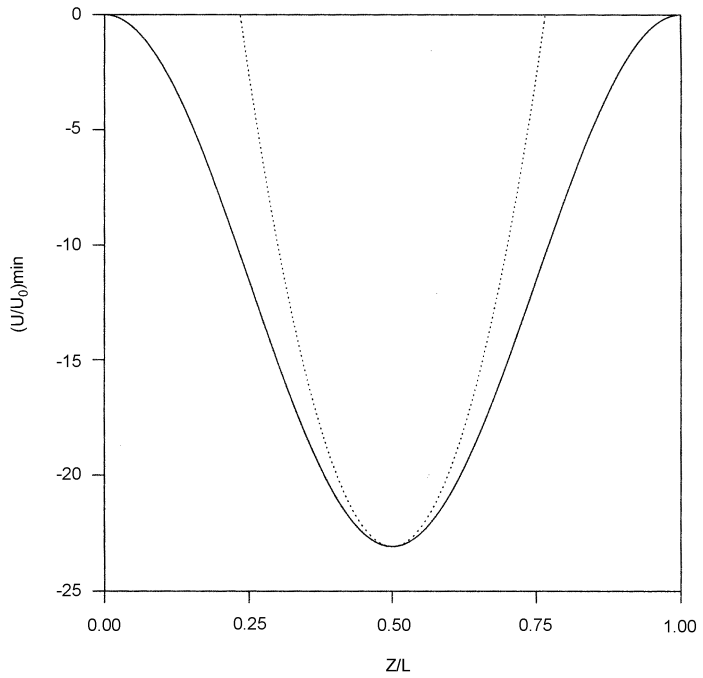

Fig. 2. The dipole potential of the sodium atom between conducting plates when the dipole is parallel to the plates for negative detuning showing the potential minimum at the center. The dotted line shows the parabolic approximation to the dipole potential. See Table.1 for values of parameters.

It is important now to examine how the vibrational frequency ω_1 varies with changing of the mean factors of the model: mode intensity, longitudinal velocity and plates separation. Fig.3 displays the variations in the vibrational frequency ω_1 (in units of Γ_0) with the mode intensity, assuming different values of the longitudinal velocity. It can be seen that the value of vibrational frequency decreases with the increasing velocity, because the depth of the well decreases with increasing velocity [4]. Fig. 4 shows the variations in the vibrational frequency with longitudinal velocity taking different values of the mode intensity. We note that the value of the vibrational frequency increases with increasing intensity because the depth of the well increases with increasing intensity [4]. In Fig. 5, we show the variations in the vibrational frequency with plates separation; it is worth noting that the value of the vibrational frequency decreases with increasing plates separation because, it is well-known in the context of the Cavity-QED, the dipole well becomes shallow with increasing the cavity dimension.

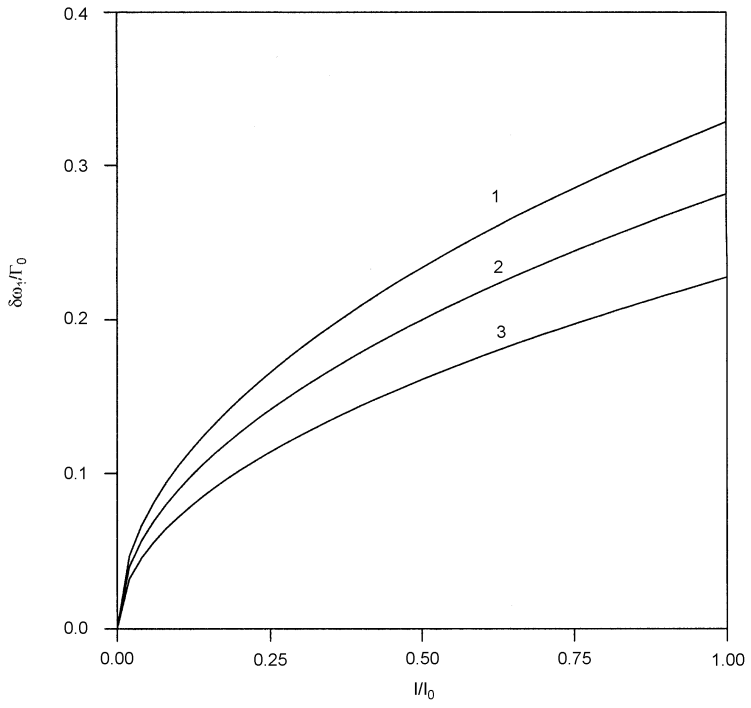

Fig. 3. Variation of the vibrational frequency with the excitation intensity of the n=1 p-polarized cavity mode for a sodium atom between the plates. Here, the electric dipole matrix element is oriented parallel to the plate. The labels 1-3 stand for $V = 0.25,\ 0.5$ and 1×10^4 m/s.

6. Comments and Conclusions

In conclusion, we have investigated in detail the motion of an atom trapped in spatially varying electromagnetic fields between conducting plates. Firstly, we evaluate the optical dipole potential that acts on the atom confining within the central region with negative detuning. Appropriate parameters were chosen in order to maintain a sufficient central well depth.

We have also studied the transverse atomic motion of the atom in the trapping potential and described it in terms of vibrational states. In addition, we have evaluated the vibrational states of a trapped atom between conducting plates and have shown that a sufficient depth for the potential is needed to allow several quasi-harmonic trapping states. The vibrational frequency of the trapped atoms was estimated by using the harmonic oscillator approximation. The effect of the mode intensity, longitudinal velocity and plates separation on the vibrational frequency was also discussed.

In fact, from a quantum-mechanical point of view, the precise details of the vibrational energy levels can be obtained straightforwardly by the numerical solution of the one-dimensional Schrodinger equation involving the potential. However, the details along this line have not yet been reported. They are currently under investigation and the results will be reported in due course.

References

[1] Hinds, E. "Perturbative Cavity Quantum Electrodynamics." *Adv. Atom. Mol. and Opt. Phys*., 2 (1993), 1-56.

[2] Sukenik, C. I., Boshier, M.G., Cho, D., Sandoghdar, V. and Hinds, E. A. "Measurement of the Casimir-Polder Force." *Phys. Rev. Lett*., 70 (1993), 560-563.

[3] Sandoghdar, V., Sukenik, C.I., Haroche, S. and Hinds, E.A. "Spectroscopy of Atoms Confined to the Single Node of a Standing Wave in a Parrallel-plate Cavity." *Phys. Rev*., A53 (1996), 1919-1922.

[4] Al-Awfi, S. and Babiker, M. "Atom Dynamics between Conducting Plates." *Phys Rev*., A58 (1998), 2274-2282.

[5] Marksteiner, S., Savage, C. M., Zoller, P. and Rolston, S. L. "Coherent Atomic Waveguides from Hollow Optical Fibers: Quantized Atomic Motion." *Phys Rev*., A50 (1994), 2680-2690.

[6] Babiker, M., Power, W. and Allen, L. "Light-induced Torque on Moving Atoms." *Phys Rev. Lett*., 73 (1994), 1239-1242.

[7] Adams, C. and Riss, E. "Laser Cooling and Trapping of Neutral Atoms." *Prog. Quant. Electron*., 21 (1997), 1-73.

[8] Allen, L., Babiker, M., Lai, W. and Lembessis, V. "Atom Dynamics in Multiple Laguerre-Gaussian Beam." *Phys Rev*., A54 (1996), 4259-4270.

[9] Power, W., Allen, L., Babiker, M. and Lembessis, V. "Atomic Motion in Light Beams Possessing Orbital Angular Momentum." *Phys Rev*., A52 (1995), 479-488.

[10] Al-Awfi, S. and Babiker, M. "Field-dipole Orientation Mechanism and Higher Order Modes in Atom Guide." *Modern Optics*., 48 (2001), 847-855.

[11] Savage, C.M., Marksteiner, S. and Zoller, P. "Atomic Waveguides and Cavities from Hollow Optical Fibers." In: *Fundamentals of Quantum Optics III*, Ehlotsky, F. (Ed.). Berlin: Springer-Verlag (1993), 61-74.

[12] Renn, M. J., Montgomary, D., Vdovin, O., Anderson, D. Z., Wieman, C. E. and Cornell, E. A. "Laser-guided Atoms in Hollow-core Optical Fibers." *Phys. Rev. Lett*., 75 (1995), 3253-3256.

[13] Ol'Shanii, M.A., Ovchinnkov, Y.B. and Letokhov, V.S. "Laser Guiding of Atoms in Hollow Optical

Fiber." *Opt. Commun*., 98 (1993), 77-79.
[14] Wright, E.M., Jessen, P.S. and Lapeyere, G.H. "Two-dimensional Motion of Cold Atom in a Near-resonant Annular Laser Beam: Artificial Two-dimensional Molecules." *Opt. Commu*., 129 (1996), 423-432.

مستويات الطاقة الاهتزازية لذرة مأسورة بين طبقين موصلين

سعود العوفي

أستاذ مساعد بقسم الفيزياء ،كلية العلوم،فرع جامعة الملك عبد العزيز

بالمدينة المنورة ، المملكة العربية السعودية

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ملخص البحث. لقد تم حساب مستويات الطاقة الاهتزازية لحركة مركز الكتلة لذّرة أسيرة بواسطة جهد ضوئي ثنائي القطب ناشىء عن نمط اهتزازي داخل تجويف. وعلى وجه الخصوص فقد تمت معالجة ذرة صوديوم مأسورة بين طبقين موصلين (صفيحتين) بحيث إن متجه عزم ثنائي القطب لها يكون موازيا لسطح الصفيحتين. و تم اختيار حالة توليف سالبة ومتغيرات أخرى مناسبة للحصول على عمق كاف للبئر الذي يمثل الجهد من أجل الحفاظ على أسر الذّرة. كما تمت دراسة التغيرات على مستويات الطاقة الاهتزازية للذّرة مع كل من شدة نمط الاهتزاز ، السرعة الطولية والمسافة بين الصفيحتين.