J. King Saud Univ., Vol. **3**, *Agric. Sci.* (1), pp. 29-42 (1411/1991)

Factor and Image Analysis of Type Traits of Dairy Cows

Ahmed Kamal Ahmed Ali
Department of Animal Production, College of Agriculture,
King Saud University, Riyadh, Saudi Arabia

Abstract. Data on sixteen linear type traits and six udder measurements were simulated for an arbitrary lactation of 10.000 unrelated cows. Factor analysis and image analysis were carried to investigate the relationship between these traits. The first four principal components accounted for 59% of the total variance in type traits. However, first and second principal components accounted for 64% of the total variability of udder conformation traits. A principal factor analysis followed by a factor rotation was used to determine new factors. Image and anti-image covariance matrices were derived for body measurements and udder measurements. Image coefficients were computed to estimate a predicted value for any trait from n-1 other traits.

Introduction

The relationships among body measurements of the dairy cow represent a special interest to the dairy industry for several reasons; 1) Dairy farmers usually judge the merit of dairy cow, to a certain extent, on the basis of body conformation. Brum and Ludwick [1] and Wilk *et al.* [2] found that measurements of body capacity such as body length, heart girth and withers height are related to milk production. Lin *et al.* [3] found that: 1) rump length is the most important trait among all body measurements studied for prediction of first lactation performances. 2) Body measurements are interrelated because of the physiology and genetics of the cow. Some traits refer to the same part of the body, like basic form and strength of the body, which have a high genetic correlation of .91, [4]. 3) The udder of the cow is one of the most important criteria that can be used to predict production performance, Lin *et al.* [3] found that high producing heifers have lower udder than low producing heifers. Udder height was more closely related genetically and phenotypically to first lactation yield. 4) The udder confirmation and teat attachments are related to udder health and the efficiency of machine milking. 5) Cow with high score of body measurements live linger in the herd and not being culled for a health problem, Rogers *et al.* [5] found that udder depth and teats rear view are the traits most related to survival. 6) Selection based on body measurements and milk production could result in a greater genetic gain in milk yield than single selection for milk yield.

Factor model

The factor model can be written as $\underline{X} = \Lambda\ \underline{Y} + \underline{E}$, where $\underline{X}$ is a vector of P, responses observed random variables have a non-singular multinormal distribution. $\Lambda = ((\lambda_{i}))$ is a matrix of leadings of the i^{th} response on the j^{th} common factor Y and E are normally distributed with mean zero and variances, $var(E_{i}) = \Psi_{i}$, where Ψ is the specific variance of the j^{th} response, and $\sum = \Lambda\Lambda' + \Psi$ where $\sum$ is the variance-covariance matrix of the responses. The diagonal elements of $\Lambda\Lambda'$ are called the commuonalties of the responses. The primary defect of the model is that it fails to provide explicit definitions for the common and unique parts of variables.

Image model

Guttman [6] and Kaiser [7] developed the image analysis theory and explicitly defined: 1) the components of observed value P, which are the image, G, (predictable) and the anti-image, V, (unpredictable) part P = G + V; 2) the matrix $W = (I - R^{-1} D^{2})$ where R is the correlation matrix among the observed values and $D^{2} = (diag\ R^{-1})^{-1}$. The matrix W could be used to predict the image part, G, from the observed value, P. In other words, G = W'P where W is $p \times p$ multiple correlation weight matrix for predicting each trait in p from p - 1 random traits (See Appendix 1 for the basic definitions, restrictions and consequences of both image and factor analysis). The purposes of this study is twofold; 1) use factor analysis to explain several observed derived variables factors; 2) in the absence of pedigree information one can use image analysis to investigate the relationship, or covariances among the components of the observed traits.

Materials and Methods

The data consist of sixteen type traits and six udder measurements of 10.000 cows. Animals were assumed to be judged at the same stage of arbitrary lactation. Animals are assumed to be unrelated. The phenotypic value of the first trait, p_{1}, was generated with mean μ_{1} and standard deviation σ_{1}. This was done by repeated calculation of:

$$P_{1j} = \mu_{1} + a_{1j}\,\sigma_{1}\ (j = 1,\ldots 10000) \qquad 1)$$

where a_{1j}'s were "normal deviates", independent drawings from a set of normal distributed random digits with mean zero and standard deviation one [8]. So, $E_{p1} = \mu_{1}$ and var $p_{1} = \sigma_{1}^{2}$. Trait p_{2} was simulated with mean μ_{2} and standard deviation σ_{2}, which has a correlation r_{12} with trait p_{1}. So,

$$p_{2j} = \mu_2 + a_{1j} r_{12} \sigma_2 + a_{2j} b_{21} \sigma_2 \qquad 2)$$

where a_{2j} were normal deviates, uncorrelated with a_1 and var p_{2j} has to be equal σ^2_2. $E_{p2} = \mu_2$, var $p_2 = r_{12} \sigma^2_2 + b^2_{12} \sigma^2_2$, cov $p_1 p_2 = r_{12} \sigma_1, \sigma_2$, cov $a_1 a_1 = 1$ and cov $a_1 a_2 = 0$, so $b^2_{21} = (1 - r^2_{12})$.

This principle was extended to n cows and m traits which obtained recursively such that

where
$$p_{ij} = \mu_i + a_{ij} r_{i1} \sigma_i + \sum_{k=2}^{i} a_{kj} b_{ik} \sigma_i,$$
$i = 2,\ldots, m$ ($m = 16$ or 6)
$j = 1,\ldots, n = 10{,}000$

where b_{ik}'s were obtained by transforming the correlation matrix R into a triangle matrix B with b_{ik}'s, $k \leq i$, using Cholesky decomposition, [9].

Tables 1, 2, 3 and 4 shows the mean, standard and correlation coefficients of body measurements and udder traits, used as parameters in simulating the data.

Table 1. Means and standard deviations of type traits

Trait	Mean	Standard deviation
1. Final score (FS)	78.2	4.6
2. Stature (ST)	24.6	7.3
3. Strength (SR)	20.7	6.5
4. Body depth (BD)	22.5	6.1
5. Angularity (AN)	27.6	7.3
6. Rump angle (RA)	26.4	5.3
7. Rump length (RL)	24.3	5.3
8. Rump width (RW)	20.4	6.0
9. Rear legs side view (RV)	26.3	6.3
10. Foot anlge (FA)	24.2	6.6
11. Fore udder attachment (FU)	24.6	7.8
12. Rear udder height (RH)	24.1	7.8
13. Rear udder width (RU)	22.5	7.8
14. Udder support (US)	28.0	6.9
15. Udder depth (UD)	29.8	6.1
16. Teat placement (TP)	25.0	6.5

Adapted from Thompson *et al.* [11] and Lawstuen *et al.* [12].

Table 2. Means and standard deviations of udder measurements

Trait	Mean	Standard deviation
1. Front teat length (FTL)	4.6	.8
2. Front teat diameter (FTD)	2.0	.3
3. Rear teat length (RTL)	4.0	.7
4. Rear teat diameter (RTD)	2.0	.3
5. Teat distance (TD)	9.7	1.8
6. Udder height (UH)	58.0	4.3

Adapted from Lin *et al.* [3]

Table 3. Correlation coefficients between body measurement

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	1.00	0.44	0.39	0.43	0.31	-0.15	0.33	0.34	-0.04	0.29	0.49	0.49	0.52	0.42	0.28	0.41
2	0.44	1.00	0.49	0.62	0.22	0.06	0.54	0.37	-0.03	0.11	0.13	0.12	0.16	0.07	0.16	0.08
3	0.39	0.49	1.00	0.72	-0.07	0.01	0.41	0.42	-0.08	0.18	0.15	0.15	0.22	0.04	-0.01	0.06
4	0.43	0.62	0.72	1.00	0.16	0.02	0.48	0.45	-0.04	0.16	0.12	0.15	0.23	0.06	-0.03	0.07
5	0.31	0.22	-0.07	0.16	1.00	-0.02	0.14	0.07	0.10	0.01	0.04	0.13	0.14	0.15	-0.03	0.10
6	-0.15	0.06	0.01	0.02	-0.02	1.00	0.04	-0.06	-0.01	-0.07	-0.01	-0.13	-0.11	-0.07	-0.02	0.05
7	0.33	0.54	0.41	0.48	0.14	0.04	1.00	0.44	0.00	0.01	0.01	0.11	0.16	0.05	0.05	0.07
8	0.34	0.37	0.42	0.45	0.07	-0.06	0.44	1.00	-0.03	0.15	0.11	0.15	0.24	0.06	0.00	0.06
9	-0.04	-0.03	-0.08	-0.04	0.10	-0.01	0.00	-0.03	1.00	-0.14	-0.03	-0.06	-0.05	0.02	0.00	0.02
10	0.29	0.11	0.18	0.16	0.01	-0.07	0.01	0.15	-0.14	1.00	0.14	0.14	0.16	0.09	0.07	0.08
11	0.49	0.13	0.15	0.12	0.04	-0.01	0.01	0.11	-0.03	0.14	1.00	0.42	0.40	0.31	0.41	0.40
12	0.49	0.12	0.15	0.15	0.13	-0.13	0.11	0.15	-0.06	0.14	0.42	1.00	0.71	0.32	0.20	0.26
13	0.52	0.16	0.22	0.23	0.14	-0.11	0.16	0.24	-0.05	0.16	0.40	0.71	1.00	0.32	0.11	0.29
14	0.42	0.07	0.04	0.06	0.15	-0.07	0.05	0.06	0.02	0.09	0.31	0.32	0.32	1.00	0.29	0.44
15	0.28	0.16	-0.01	-0.03	-0.03	-0.02	0.05	0.00	0.00	0.07	0.41	0.20	0.11	0.29	1.00	0.33
16	0.41	0.08	0.06	0.09	0.10	-0.05	0.07	0.06	0.02	0.08	0.40	0.26	0.29	0.44	0.33	1.00

Table 4. Correlation coefficients between udder measurements

(1)	(2)	(3)	(4)	(5)	(6)
1.00	.24	.71	.18	.04	.01
.24	1.00	.16	.62	.14	- .07
.71	.16	1.00	.24	.03	- .01
.18	.62	.24	1.00	.16	- .09
.04	.14	.03	.16	1.00	- .25
.01	- .07	- .01	- .09	- .25	1.00

Results and Discussion

Principle components (PC) were computed from both phenotypic and image correlation matrices (Table 5). Use of the correlation matrix ensure that body measurements have equal weights in principle components analysis. The first PC of phenotypic correlation components analysis. The first PC of phenotypic correlation matrix represents the highest proportion of total variability, (accounts 31% of total variability). The first four PC accounted for 59% of the total variability and any additional PC will not account more than any standardized type trait (eigenvalue < 1.00). The first principle components of image correlation matrix account for 64% of the total variability. The first six principle components account for 95% of the total variability, so the dispersion of image matrix is more than that of the phenotypic correlation matrix. Sieber *et al.* [10] found that a principle component analysis showed that the first eight components accounted for more than 69% of the total variance in type score.

Table 5. Eigenvalues and proportion of total variance explained by principle components from a phenotypic and image matrices.

Principle component	Proportion of total phenotypic variance		Phenotypic eigenvalue	Proportion of total image variance		Image eigenvalue
	Proportion	Cumulative		Proportion	Cumulative	
F_1	31.1	31.1	4.977	63.630	63.63	4.500
F_2	13.3	44.4	2.131	18.420	82.05	1.300
F_3	7.8	52.2	1.239	5.6000	87.64	.395
F_4	7.2	59.3	1.476	3.3800	91.02	.239
F_5	6.2	65.5	.992	2.4000	93.44	.170
F_6	5.9	71.5	.948	1.9500	95.39	.138
F_7	5.1	76.5	.812	1.7000	97.08	.119
F_8	4.6	81.1	.731	.0097	98.06	.069
F_9	4.3	85.4	.692	.0062	98.70	.044
F_{10}	3.5	88.9	.554	.0040	98.11	.031
F_{11}	3.4	92.9	.535	.0038	99.49	.027
F_{12}	3.1	95.3	.495	.0021	99.70	.015
F_{13}	1.8	97.1	.282	.0019	99.80	.013
F_{14}	1.6	98.7	.262	.0010	99.98	.007
F_{15}	1.3	99.9	.200	.0001	1.000	.001
F_{16}	0.01	01.0	.002	.00001	1.000	.0001

The diagonal elements of image variance-covariance matrix is the square multiple correlation coefficients (R^2) of each trait with a linear function of other traits. Final score (FS) and Fore udder attachments (FU) have almost a perfect square multiple correlation (Table 6). In other words FS and FU have the highest image variance ($R^2 = .996$), consequently, the lowest anti-image variance (Image Var + Anti-image Var = 1). Rump angle and rear leg view have the lowest image variances ($R^2 = .05$ and .03).

Table 6. Image and anti-image variances of body measurements

Trait	Image variance	Anti-image variance
Final score (FS)	.995700	.004287
Stature (ST)	.706840	.293165
Strength (SR)	.588270	.412726
Body depth (BD)	.650290	.349708
Angularity (AN)	.251550	.748445
Rump angle (RA)	.054674	.945326
Rump length (RL)	.378000	.622113
Rump width (RW)	.311146	.688854
Rear legs side view (RV)	.036573	.963427
Foot angle (FA)	.101536	.898464
Fore udder attachment (FU)	.996000	.004208
Rear udder height (RH)	.535457	.464543
Rear udder width (RU)	.557719	.442281
Udder support (US)	.305490	.694510
Udder depth (UD)	.286412	.713588
Teat placement (TP)	.317800	.682200

After the decision was made on how many PC to extract and retain from the original set of variables, a principle factor analysis was followed by a varimax rotation to redistribute the variance of the retained factors so the rotation make the factors as intuitively meaningful as possible [13,14]. Table 7 shows that traits having the largest coefficients contributed the most to the value of a factor. For factor 1, these traits are final score, stature, strength, body depth, rump length, rump width, fore udder attachment, rear udder height and udder support. Animal with highest score for this factor would be a big strong cow with wide rump and tall stature and good udder conformation. F_2 included udder support, udder depth and teat placement. Angularity, rear legs view and foot angle loaded heavily on F_3. Finally F_4, concern mainly about balance of the cow, rump angle and udder depth was explained mostly by factor 7.

Communality estimate (Table 7) is the proportion of the response variance of type traits which is shared with the other traits via the common-factor variates. communality of Final Score (FS), stature, Rear Udder Height (RH) and Strength were the highest. In other words, the specific variance (Ψ_i) of these traits are the lowest. However, Rear Legs View (RV), Foot Angle (FA), and Rump Length (RL) have the lowest communalities and consequently the highest specific variance. Two common estimates of communalities are: 1) the square of the multiple correlation coefficient of the ith trait with all other traits; 2) the largest correlation coefficient between the ith trait and one of the other traits, that is, that $\max_{j \neq i} = | r_{ij} |$.

Table 7. Factor pattern (*100) and communalities of body measurements

Trait	F_1	F_2	F_3	F_4	Communality
Final score (FS)	89.6	17.5	5.1	-3.5	.836
Stature (ST)	81.2	-26.5	18.3	20.9	.775
Strength (SR)	56.7	-54.6	-17.4	12.5	.677
Body depth (BD)	64.7	-56.1	4.5	3.6	.737
Angularity (AN)	30.9	9.0	58.8	-40.0	.670
Rump angle (RA)	11.8	-20.4	20.0	45.9	.306
Rump length (RL)	52.9	-46.7	17.8	9.6	.539
Rump width (RW)	53.0	-40.6	-6.8	-7.2	.546
Rear legs side view (RV)	-3.2	10.0	62.9	-18.9	.443
Foot angle (FA)	29.8	-4.8	-43.9	0.2	.284
Fore udder attachment (FU)	89.7	19.5	4.2	-2.3	.845
Rear udder height (RH)	57.3	39.3	-26.6	-30.0	.644
Rear udder width (RU)	62.3	31.0	-25.4	-33.1	.664
Udder support (US)	45.4	51.8	7.1	19.2	.516
Udder depth (UD)	29.2	42.2	5.1	59.0	.613
Teat placement (TP)	43.5	48.9	10.1	33.6	.531

Image matrix, was used to explain the relationship among the predicted parts of the phenotypic measurements in the absence of the pedigree information. Image correlation matrix is parallel to the genetic correlation matrix, in a sense that the magnitude of its value reflects the pairwise relationship among each pair of traits. A high pair wise image correlation was observed between final score and udder characteristics (r_g >.5). Final score has high image correlation with the stature of the cow (r_g = .83). Stature has a high image with rump length (r_g = .82) and rump width (r_g = .86). Udder conformation traits have a mutual close relationship. For example, udder attachment and rear udder had image correlation of .70, image correlation of rear udder width with udder support was .80. Image correlation was also substantial for udder height and udder width r_g > .72. Foster *et al.* [15] found a high genetic correlation between udder traits ranged from .60 to .86. High positive image correlation between two traits indicate the same behaviour of the two traits. However, negative image correlation indicate different directions of the behaviour of two traits. Anti-image correlation coefficients were low (< .1) among all body measurements.

Anti-image correlation coefficients were lower than environmental correlation coefficients reported by Lawstuen *et al.* [12].

The image coefficients of body measurements, the matrix W, can be used to predict the image value from the observed measurements, G = WP. The image matrix W has some characteristics, first, it is a p*p multiple correlation weight matrix for predicting eact trait (measurement) in P from p-1 traits, where P is an P*1 random vector whose coordinates are p random traits. 2) W is asymmetric with diagonal elements zero, so a trait being predicted from the p-1 other traits will receive a weight zero in the prediction equation involving the full set of n traits. For example, to predict the final score, one multiply the first row (.000 .1283 -.1679 -.0060 .1069 .2527 .0985 -.1322 -1.3657 -.30523 1.0206 -.0998 .0372 -.0151 -.1998 .3010) by the observed measurements of the sixteen traits and add these products together. 3) If R can be partitioned, such

$$\text{that } R = \begin{bmatrix} 1 & R_{12} \\ R_{21} & R_{22} \end{bmatrix} \quad \text{then } W = (I\text{-}R_{22}\, D^2) = R_{22}\, (R_{22}\, \text{-}D^2)$$

so the matrix R_{22} -D^2, represents the covariance between the original measurements and their predictable parts. The matrix D^2 represents the errors of estimate of each measurement with respect to the p - 1 other measurements as predictors of it, and

D^2 = Diag (	.0048287	.293165	.411726	.349708	.748445
	.9453260	.622113	.688854	.963427	.898464
	.0042089	.464540	.442281	.694510	.713588
	.6822000).				

Covariance between components of phenotypic value could be obtained using R and D^2 (See Appendix II).

Udder measurements

The first PC of phenotypic correlation matrix accounted for 42%. However, the first PC of image matrix accounted for 73% (Table 8). Again the dispersion of the eigenvalue of the image matrix (G) is more than that of the eigenvalue of G the correlation matrix R. This of course, reflects the off-diagonals of G being substantially larger, in absolute value, than those of R. Table 9 shows that front and rear length, front and rear teat diameter, had high image variance (R^2). However, teat distance and udder height gave the lowest image variance and highest anti-image variances. These results, have been reflected in factor analysis (Table 10). Since front teat length, front teat diameter, rear teat length and rear teat diameter contributed the most to factor 1. However, teat distance and udder had the highest contribution in factor 2. Table 11 gives the correlation among images, front teat length had high positive phenotypic image correlation with front teat diameter, rear teat length and rear teat diameter. So cows with longer teats have larger teat diameter and cows with longer front teats have longer rear teats. Udder height had slight negative phenotypic and image correlation with teat lengths and teat diameters. These results are similar to those found by Lin *et al.* [3], Batra and McAllister [16], and Seykora and McDaniel [17]. Teat distanced showed small phenotypic and image correlations with all other udder traits. Image coefficients of udder measurements (Table 12) could be used to predict the observed value of any udder measurements from the other udder traits. The matrix of image coefficients has the same features of the image matrix of body measurements. D^2 = Diag(.272607 .305762 .469231 .574548 .912332 .925413).

Table 8. Eigenvalues and proportion of total variance explained by principle components from a phenotypic and image matrices.

Principle component	Proportion of total phenotypic variance		Phenotypic eigenvalue	Proportion of total image variance		Image eigenvalue
	Proportion	Cumulative		Proportion	Cumulative	
F_1	4.21	4.21	2.523	.731	.731	1.857
F_2	.219	.639	1.314	.145	.876	.367
F_3	.145	.785	.870	.087	.963	.222
F_4	.122	.907	.734	.019	.982	.049
F_5	.069	.976	.414	.018	1.000	.046
F_6	.024	1.000	.145	.000	1.000	.0009

Table 9. Image and anti-image variances of udder measurements

Trait	Image variance	Anti-image variance
Front teat length	.727	.273
Front teat diameter	.694	.306
Rear teat length	.531	.469
Rear teat diameter	.425	.575
Teat distance	.088	.912
Udder height	.075	.925

Table 10. Factor pattern of udder measurements

Trait	Factor 1	Factor 2
Front teat length	85.99	24.89
Front teat diameter	88.69	-1.20
Rear teat length	78.75	23.99
Rear teat diameter	57.88	-32.83
Teat distance	18.17	-73.46
Udder height	09.30	73.95

Table 11. Image (below diagonal) and Anti-image (above diagonal) correlations for udder measurements

1.0000000	-.6642980	-.6103370	.3882160	.0041380	-.0094351
.7298670	1.0000000	.1359270	-.6095200	-.3075449	-.0091788
.7993040	.9069220	1.0000000	-.1994890	.0161093	-.0067316
.6175760	.5515100	.2796730	1.0000000	-.0819080	.0679565
.1657430	.3915790	.1801210	-.0819080	1.0000000	.2486290
.0037223	.0422828	-.0020481	.0679565	.2486290	1.0000000

Table 12. Image coefficients of udder measurements

.00000	.70354	.80075	-.56360	-.00757	.01738
.62725	.00000	-.16839	.83553	.06485	.01597
.46521	-.10972	.00000	.22074	-.02246	.00945
-.26741	.44465	.18028	.00000	.10321	-.08625
-.00226	.02174	-.01155	.06500	00000	-.25040
.00512	.00528	.00479	-.05355	-.24687	.00000

The relation between factor analysis and image analysis exists, since Mulaik [18] showed that in a universe of responses of traits one can assume that $\lim_{n \to \infty} d_j^2 = u_j^2$ i.e. in a matrix form $\lim_{n \to \infty} D^2 = u^2$ i.e. as $n \to \infty$ as the number of traits increases without bound, the error of estimate for predicting a trait from n - 1 other traits, (image analysis) approaches as a limit to the unique variance of that trait (factor analysis). If the previous limits holds then $\lim_{n \to \infty} R_j^2 = h_j^2$ i.e. the square of the multiple correlation coefficients for predicting a trait from the other n - 1 other triats (image analysis), approaches as a limit to the communality of the trait (factor analysis). The immediate consequence of these two limits is that in the universe of the traits, image analysis and factor analysis are the same. One can also conclude that $\lim_{n \to \infty} G = R - u^2$ in other words, as the number of traits increases without bound, the image covariance matrix approaches as a limit to the reduced correlation matrix ($R\text{-}u^2$) with communality coefficients in the principle diagonal. And finally $\lim_{n\to\infty} Q = D^2 = u^2$ or as the number of traits increases ($n\to\infty$) without bound the anti-image covariance matrix approaches as a limit to the diagonal matrix of errors of estimate and in turn the diagonal matrix of unique variances.

References

[1] Brum, E.W., and Ludwick T.M. "Heritabities of Certain Immature and Mature Body Measusements and Their Correlations with First-Lactation Production of Holstein Cows." *J. Dairy Sci.*, **52** (1969), 352.

[2] Wilk, J.C., Young, C.W. and Cole, G.L.. "Genetic and Phenotypic Relationships between Certain Body Measurements and First-Lactation Milk Production in Dairy Cattle." *J. Dairy Sci.*, **46** (1963), 1273.

[3] Lin, C.Y., Lee, A.J., McAllister, A.J., Batra, T.R. Roy, G.L., Veseley, J.A., Wauthy, J.M. and Winter, K.A. "Intercorrelations among Milk Production Traits and Body and Udder Measurements in Holstein Heifers." *J. Dairy Sci.*, **70** (1987), 2385.

[4] Foster, W.W. "Effects of First Lactation Linear Type Scores on First Lactation Production and Herd Life of Holstein Dairy Cattle. *M.S. Thesis*, Parks Lib., Iowa State Univ., Ames (1985).

[5] Rogers G.W., B.T. McDaniel, Dentine, M.R. and L.P. Johnson, "Relationships among Survival Rates, Predicted Differences for Yield and Linear Type Traits." *J. Dairy Sci.*," (Suppl. 1) (1981), 908 (Abstr).

[6] Guttman, L. "Image Theory for the Structure of Quantitative Variates." *Psychometrika*, **18** (1953), 277.

[7] Kaiser, H.F. "Image and Anti-image Covariance Matrices from a Correlation Matrix that May be Singular." *Psychometrika*, **41** (1976), 295.

[8] Rubinstein, R. Y. *Simulation and the Monte Carlo Method.* London: John wiley and Sons, 1981.

[9] Rao, C.R. *Linear Statistical Inference and its Applications.* 2nd ed. New York: John wiley & Sons, Inc. 1973.

[10] Seiber, M., Freeman A.E. and Hinz, P.N.. "Factor Analysis for Evaluating Relationships between First Lactation Type Traits Scores and Production Data of Holstein Dairy Cows." *J. Dairy Sci.*, **70** (1981), 1018.

[11] Thompson, J.R., Lee, K.L., Freeman, A.E. and Johnson, L.P. "Evaluation of a Linearized Type Appraisal System for Holstein Cattle." *Dairy Sci.*, **66** (1983), 325.

[12] Lawstuen, D.A., Hansen, L. B. and Johnson, L.P. "Inheritance and Relationships of Linear Type Traits for Age Groups of Holsteins." *J. Dairy Sci.*, **70** (1987), 1027.

[13] Anderson, T.W., *An Introduction to Multivariate Statistical Analysis*. New Work: John Wiley & Sons, Inc., 1985.

[14] Morrison, D.F., *Multivariate Statistical Analysis.* New York: McGraw Hill Book Company 1976.

[15] Foster, W.W., Freeman, A.E. and Berger, P.J.. "Linear Type Trait Analysis with Genetic Parameter Estimation." *J. Dairy Sci.*, **71** (1988), 223.

[16] Batra, T.R., and McAllister, A.J.. "Relationships Among Udder Measurements, Milking Speed, Milk Yield and CMT Scores in Young Dairy Cows." *Can. J. Anima. Sci.*, **64**(1984), 807.

[17] Seykora, A.J. and McDaniel, B.T., "Heritabilities and Correlations of Lactation Yields and Fertility for Holsteins." *J. Dairy Sci.*, **66** (1985), 1486.

[18] Mulaik, S.A.. *The Foundation of Factor Analysis.* New York: McGraw-Hill Book Company, 1972.

Appendix I: Comparison of characteristics of image theory and factor theory

Character	Image theory	Factor analysis
Basic partition	$P_{ji} = \overset{(n)}{G}_{ji} + \overset{(n)}{V}_{ji}$	$P_{ji} = C_{ji} + U_{ij}$
Basic definition	$\overset{(n)}{G}_{ji} = \sum_{k=1}^{n} \overset{(n)}{W}_{jk} \quad P_{ji}$	
Basic restrictions	a) $E\overset{(n)}{V}_{ji} \quad P_{ji} = 0\,(j \neq k)$	$EU_{ji}P_{ki} \quad = 0; (j \neq k)$
	b)	$E_i U_{ji} U_{ki} \quad = 0; (j \neq k)$
Consequences	a)	$E_i U_{ji} C_{ki} \quad = 0; (j \neq k)$
	b) $E\overset{(n)}{V}_{ji} \quad \overset{(n)}{P}_{ji} = 0$	$E_i U_{ji} C_{ji} \quad = 0\,(m<n)$
	c) $\sigma^2_{jn} + \sigma^2_{jn} = 1$	$\sigma^2_{cj} + \sigma^2_{uj} \quad = 1$
	d) $r_{jk} = g_{jk} - \overset{(n)}{\gamma}_{jk}\,(j/k)$	$r_{jk} = E_i C_{ij} C_{ik}$
	e) $r_{jk} = \overset{(n)}{g}_{jk} + \overset{(n)}{\Pi}_{jk}$	$\sigma_{jn}\,\sigma_{kn}\,(j \neq k)$

1) P^2_{jn} square remaining multiple correlation coefficient of P_j on the n-1 variables = variance of the image part.
σ^2_{jn} = variance of the errors (V_{ji}) image part.

2) $\overset{(n)}{g}_{jk}$ = Covariance between images.

3) $\overset{(n)}{\gamma}_{jk}$ = correlation between anti-images.

4) $\overset{(n)}{\pi}_{jk}$ = covariance between P_j and P_k.

5) $\overset{(n)}{r}_{jk}$ = the correlation among the common parts.

Appendix II: If the matrix $W=(I-R^{-1}D^2)$ where $R=E\,(PP')$ is pxp correlation matrix for the p traits and $D^2=[\text{diag}\,R^{-1}]^{-1}$. The matrix of covariances among p different images.

$$\begin{aligned} E(GG') &= E\,(W'PP'W) = W'RW \\ &= (I-D^2R^{-1})\,R\,(I-R^{-1}D^2) \\ &= R+D^2R^{-1}D^2-2D^2 \end{aligned}$$

The covariance matrix among n different anti-image.

$$\begin{aligned} E(VV) &= E(I-W')\,PP'\,(I-W) \\ &= (I-W')\,R(I-W) \\ &= [I-(I-D^2\,R^{-1})\,]\,R[I-(I-R^{-1}\,D^2)] \\ &= D^2\,R^{-1}\,RR^{-1}\,D^2 \\ &= D^2\,R^{-1}\,D^2 \end{aligned}$$

The covariance matrix between the image, and original linear score, phenotypic values.

$$\begin{aligned} E(GP') &= E\,(W'PP') \\ W'R &= (I-D^2\,R^{-1})\,R \\ &= R-D^2 \end{aligned}$$

The covariance matrix between the anti-image and phenotypic value.

$$\begin{aligned} E\,(VP') &= E[(I-W')\,PP'] = (I-W')\,R \\ &= R-W'R = R-(I-D^2\,R^{-1})\,R \\ &= R-(R-D^2) \\ &= D^2 \end{aligned}$$

The covariance matrix between images and anti-images.

$$\begin{aligned} E(VV') &= E[(I-W')\,PP'] = (I-W')\,RW \\ &= RW - W'RW = RW-G \\ &= R(I-R^{-1}\,D^2) - (R+Q-2D^2) \\ &= -Q + D^2 \text{ where } D^2 = \text{diag}\,[Q] \end{aligned}$$

التحليل العاملي والتخيلي للصفات الشكلية لأبقار اللبن

أحمد كمال علي

قسم الإنتاج الحيواني، كلية الزراعة، جامعة الملك سعود، الرياض،
المملكة العربية السعودية

ملخص البحث. استخدم التحليل العاملي والتخيلي في تحليل بيانات ستة عشر صفة شكلية لحيوانات اللبن وستة صفات للضرع لموسم حليب فرضي وذلك لعدد ١٠,٠٠٠ بقرة حليب ليس بينها أي صفة قرابة. وقد وجد أن ٥٩٪ من التباين الكلي في صفات الضرع ترجع إلى عاملين رئيسين. استخدم التحليل العاملي بعد إجراء التحويل المناسب للبيانات وذلك لتقدير عوامل جديدة تمثل الصفات الشكلية ولكنها أقل عدد. استخدم التحليل التخيلي لتقدير مصفوفات التباين المشترك بين مكونات القيم المظهرية للصفات الشكلية وكذلك صفات الضرع أمكن باستخدام التحليل التنبؤ بقيم صفة شكلية معينة أو صفة ضرع من باقي الصفات المرتبطة معها.