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Calculating *F* Factor for Center-pivots Using Simplified Formula and Modified Christiansen Equation

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Abstract. A single-accurate exponential equation is presented to estimate the friction correction factor for center-pivot systems. The proposed equation is simpler than and well agreed with the previously developed equations of the correction friction factors. For a large number of outlets, two simplified forms have also been presented. The presented equation and its subsequently derived forms are uniquely functions of the number of outlets N and the flow exponent m of the used friction formula. Also, the Christiansen friction correction factor, originally devised for systems where discharge decreases linearly with distance, was adapted to simply and accurately estimate the friction correction factor and, consequently, to facilitate the head loss calculation in the center-pivot laterals. The developed formulas of the friction correction factors were compared to the previously proposed equations. The results showed well agreement between the present and the previous equations for determining the friction correction factors for center-pivots laterals. Numerically documented and field examples have been used to test the presented equations. The examples outputs revealed that the equations can be used to determine the friction correction factors for a pipe of non-uniformly discharging outlets with insignificant errors. For practical number of outlets for center-pivot laterals, the errors were mostly in the vicinity of ± 2 %.

Introduction

The use of center-pivot systems has spread worldwide. The wide use of center-pivot systems might be attributed to the desire of water and energy conservation. Although different center-pivot systems have been invented, the hydraulics attains similarity and subjects to the mechanism of water flow in pipes. In other words, when water flows into a pipeline, an energy head loss occurs due to friction. The friction head loss is essential to the analysis (design, evaluation, and management) of pressurized irrigation systems. It is essential due to the fact that it has strong impact on the irrigation efficiency and, consequently, on the water conservation. It is known that the proper estimate of the friction head loss will help in determining the pressure distribution along the irrigation system laterals. The flow distribution will also be known and the system can accordingly be judged.

Basically, the energy loss in a pipe is directly related to the flow rate. Thus, the energy head loss for pipes with no outlets is higher than for pipes with outlets, as the case with laterals in the pressurized irrigation systems. In pipes with outlets, the friction head loss is usually determined via a stepwise technique or by computing the head loss with no outlets, and then a multiplier called friction correction factor (*F*) is used to account for the decrease in the discharge along the lateral pipe. Since the discharge along the lateral pipes may change linearly (periodic-move laterals) or non-linearly (center-pivot laterals) with the distance, the friction correction factor will differ subsequently. The general mathematical formulation of computing friction head loss in a multiple-outlet pipe with flow decreasing linearly is as follows:

$$\hat{h}_f = F \cdot h_f \tag{1}$$

And the formulation for non-linearly decreased flow is written as:

$$\hat{h}_{fcp} = F_{cp} \cdot h_f \tag{2}$$

In (1), $\hat{h}_f$ is the energy loss due to friction with multiple outlets of linear decline in discharge, F is the corresponding correction friction factor, and h_f is the total friction head loss where there is only one outlet. In (2), $\hat{h}_{fcp}$ is the energy loss due to friction with multiple outlets of varied discharge and F_{cp} is the corresponding correction friction factor.

The friction head loss h_f can be computed from an appropriate friction loss equation. Several equations for the estimate of the friction head loss have been proposed. While some equations used to compute h_f were developed based on physical considerations, others were empirically based. The Darcy-Weisbach is the most popular friction head loss that was physically based and may be of the following form:

$$h_f = \frac{K \cdot f \cdot L}{D^5} \cdot Q^2 \tag{3}$$

where K is the units conversion factor equal to 0.0826, f is the Darcy-Weisbach friction factor, L is the pipe length (m), Q is the flow rate (m^3/s), D is the inside pipe diameter (m). Due to tedious computation of f, particularly for routinely quick tasks, alternative equations for h_f estimate have been developed. Williams and Hazen [1] empirically derived an alternative equation called Hazen-Williams equation, which is easier than Darcy-Weisbach and commonly used in irrigation fields. The Hazen-Williams equation for friction head loss estimate has the following from:

$$h_f = \frac{K \cdot L}{C_{HW}^{1.852} \cdot D^{4.871}} \cdot Q^{1.852} \tag{4}$$

where K is equal to 1.2×10^{10} for L in m, D in mm, and Q in L/s; and C_{HW} is the Hazen-Williams friction factor that depends on the flow media. The other common methods for f estimates are the equations of Scobey [2] and Watters and Keller [3]. The Scobey equation has the following form:

$$h_f = \frac{K \cdot C_S \cdot L}{D^{4.9}} \cdot Q^{1.9} \tag{5}$$

where K equals 2.05×10^{12} for L in m, D in mm, and Q in m^3/s; and C_S is the Scobey coefficient of retardation equal to 0.40 for portable aluminum and 0.42 for steel. Watters and Keller [3] combined the Darcy-Weisbach equation with Blasius [4] equation of f factor and obtained the following:

$$h_f = \frac{K \cdot L}{D^{4.75}} \cdot Q^{1.75} \tag{6}$$

in which K is equal to 1.387×10^{11} for L in m, D in mm, and Q in m^3/s.

The friction correction factors in Eqs. 1 and 2 have been estimated using different techniques. Those techniques take into consideration that the friction head loss in a pipe with water discharging out from the pipe lateral is less when the pipe has no side outlets, but one at the end.

For equally spaced and uniformly discharging outlets, the most widely and commonly used formula is that proposed by Christiansen [5]. When the distance from the lateral inlet to the first outlet is equal to the distance between the two successive sprinklers, F formula takes the following form:

$$F = \frac{1}{m+1} + \frac{1}{2N} + \frac{\sqrt{m-1}}{6N^2} \tag{7}$$

If the distance from the lateral inlet to the first outlet is equal to the half distance between the two successive sprinklers, F is expressed as follows:

$$F_{0.5} = \frac{2NF-1}{2N-1} \tag{8}$$

where m is the velocity exponent in the used friction equation and N is the number of outlets. The magnitude of the exponent *m* depends on the equation used to compute the total head loss for pipes.

Neither equation 7 nor 8 is suitable for estimate of the friction correction factor in center-pivot systems, as previously mentioned. Therefore, the friction correction factor for a center-pivot system is differently computed by several proposed methods. Reddy and Apolayo [6] developed a friction correction factor for center-pivots, which is of the following form:

$$F_{cp} = \frac{1}{N}\left[1 + \sum_{i=2}^{N}\left(1 - \frac{2}{N^2}\sum_{j=1}^{i-1} j\right)^m\right] \tag{9}$$

The above equation is obviously valid for number of outlets greater than or equal to 2. Anwar [7] proposed two equations that are valid for any number of outlets, $N \geq 1$. For constant spacing (increasing varied outflow), the equation takes the form:

$$F_{cp} = \frac{1}{N^{2m+1}}\sum_{i=1}^{N}\left(2Ni - i^2\right)^m \tag{10}$$

And for constant outlets discharge (decreasing varied space), the equation has the following form:

$$F_{cp} = \frac{1}{N^{m+0.5}}\sum_{i=1}^{N} i^m\left(\sqrt{N - i + 1} - \sqrt{N - i}\right) \tag{11}$$

As seen from Eqs. 10 and 11, the calculated friction factor for a given number of outlets depends on the previous computations. In other words, the F_{cp} for a given N requires the computation of N–1, which, in turns, requires the computation of N–2 and so on. Compared to F obtained from Eq. 7, the computation of F_{cp} using either Eq. 10 or Eq. 11 is quite tedious and somewhat cumbersome. Therefore, the objectives of this research were:

1. To develop a simple formula to calculate the friction correction factor for center-pivots, and
2. To modify the Christiansen equation to be suitable for center-pivots.

Development of Equations

Exponential equation

Different analysis tools can be used to trace the trend of a dependent variable affected by one or more independent variables. The non-linear regression analysis is a valuably useful mathematical tool that can be used for that purpose. Direct use of the regression analysis may however lead to undesired formulations. In the present research, the concept of segregate analysis was considered and used to obtain a more suitable formulation of the friction correction factor F_{cp}. As can be seen from Eqs. 9-11, the F_{cp} is a function of the number of outlets N and the velocity or flow exponent m in the equation used for friction loss calculation. Mathematically:

$$F_{cp} = f\,(N\,, m) \tag{12}$$

For a certain value of m and a changing N, an equation for F_{cp} was obtained using non-linear regression and found to be of the following type:

$$F_{cp} = \frac{\alpha_1 + N\,\alpha_2}{1 + N\,\alpha_3} \tag{13}$$

where α_1, α_2 and α_3 are coefficients that are functions of the flow exponent m. To quantify the coefficients in Eq. 13, data were generated using the average values obtained by Eqs. 10 and 11. The average values were used since Eqs. 10 and 11 gave almost identical results [7]. The ranges of m and N used to generate data via Eqs. 10 and 11 were 1.5 to 2.5 with increment of 0.25 and 1 to 150 with increment of 1 for m and N, respectively. From non-linear regression and with algebraic manipulation, the resulted equations for the coefficients in Eq. 13 were as follows:

$$\left.\begin{aligned} \alpha_1 = \ \alpha_2 &= \frac{e^{\frac{5m}{\pi}}}{\pi} \\ \alpha_3 &= \frac{e^{\frac{6m}{\pi}}}{\pi} \end{aligned}\right\} \tag{14}$$

Replacing the coefficients of Eq. 14 into Eq. 13, the F_{cp} is to be computed by the following equation:

$$F_{cp} = \frac{\dfrac{e^{\frac{5m}{\pi}}}{\pi} + \dfrac{Ne^{\frac{5m}{\pi}}}{\pi}}{1 + \dfrac{Ne^{\frac{6m}{\pi}}}{\pi}} \tag{15}$$

With rearrangement, Eq. 15 might be written as:

$$F_{cp} = \frac{\left(1+N\right)e^{\frac{5m}{\pi}}}{\pi + Ne^{\frac{6m}{\pi}}} \tag{16}$$

Equation 16 indicates that when the number of outlets increases, the term π in the dominator becomes negligible and can be deleted. Thus, Eq. 16 can now be written as:

$$F_{cp} = \frac{\left(1+N\right)e^{\frac{m5}{\pi}}}{Ne^{\frac{m6}{\pi}}} \tag{17}$$

which can be reduced to a simpler form as follows:

$$F_{cp} = \frac{\left(1+N\right)}{Ne^{\frac{m}{\pi}}} \tag{18}$$

Eq. 18 can further be rearranged and written as:

$$F_{cp} = \frac{1+\frac{1}{N}}{e^{\frac{m}{\pi}}} \tag{19}$$

By eliminating the term $1/N$ in (19) as the case for a large N, the F_{cp} can ultimately be computed through the following simplest form of Eq. 16:

$$F_{cp} = \frac{1}{e^{\frac{m}{\pi}}} \tag{20}$$

Modified Christiansen equation

It is probably alike that the Christiansen equation, Eq. 7, is more convenient to use than other equations. This is because it has been used for decades and the users are familiar with. In its present form, the Christiansen F factor, however, is not applicably valid for center-pivot systems. This is because the F is considerably smaller than F_{cp}. The latter is larger because the flow reduction in the laterals of center-pivots is much less near the pivot-end. Therefore, an effort has been made to modify Eq. 7 such that it can suitably be used with center-pivot systems. When plotting F_{cp} values computed from Eq. 16 versus F values obtained by using Eq. 7, not shown here, a relationship has been realized and was of the following form:

$$F_{cp} = \beta_1 F^{\beta_2} \tag{21}$$

From the regression analysis, the coefficients in Eq. 21 were found to be generally independent of the flow exponent parameters *m*. Thus, the magnitudes of the coefficients β_1 and β_2 were rationally considered constant and equal to 1.0 and 0.567, respectively. Thus, the modified Christiansen friction correction factor for center-pivots has the following formulation:

$$F_{cp} = \left[\frac{1}{m+1} + \frac{1}{2N} + \frac{\sqrt{m-1}}{6N^2}\right]^{0.567} \tag{22}$$

In which all terms have previously been identified.

Discussion of Developed Equations

The errors in F_{cp} computed from Eqs. 16, 19, 20 and 22 relative to the average values obtained from Eqs. 10 and 11 versus the number of outlets *N* are depicted in Figures 1-4, respectively. In each figure, the parameter distinguishing one curve from another is the flow exponent *m*. Three values of *m*, 1.852, 1.9 and 2 associated with the most common h_f methods, were considered. As depicted from Figure 1, the errors in computing F_{cp} using Eq. 16 relative to the averaged values of Eqs. 10 and 11 are apparently insignificant for all *m* values. The low relative errors support the use of Eq. 16 over either Eq. 10 or Eq. 11. The simplicity of Eq. 16 is not the only reason for its advantageous use, but the ease of remembrance compared to Eqs. 10 and 11 is also a merit. It can also be depicted from Fig. 1 that the relative errors for different *m* values, 1.852, 1.9 and 2, are not different and fall in the domain of + 2 to – 2%. It can, consequently, be stated that the relative error in the F_{cp} obtained by Eq. 16 compared to the averaged F_{cp} computed from Eqs. 10 and 11 is with in ±2 %.

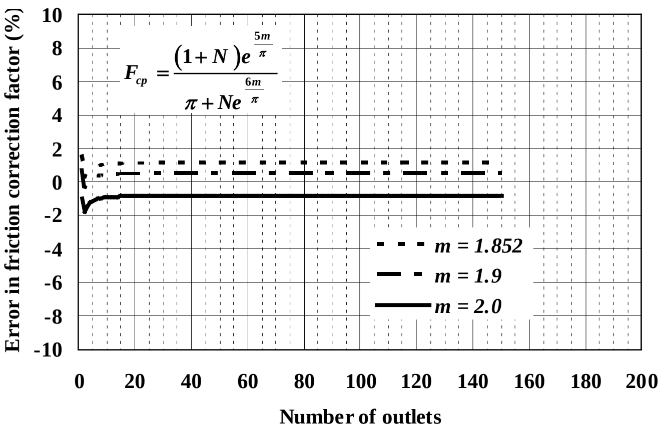

Fig. 1. Relative error in F_{cp} computed by using Eq. 16.

Figure 2 shows the relative error in F_{cp} versus the number of outlets N with m being the parameter distinguishing the curves. As depicted by the figure, the relative error in estimating F_{cp} using Eq. 19 compared to the averaged values of F_{cp} obtained from Eqs. 10 and 11 also is insignificant, particularly for N equal to or greater than 2. Nevertheless, the maximum relative error in F_{cp} obtained from Eq. 19 was found to be in

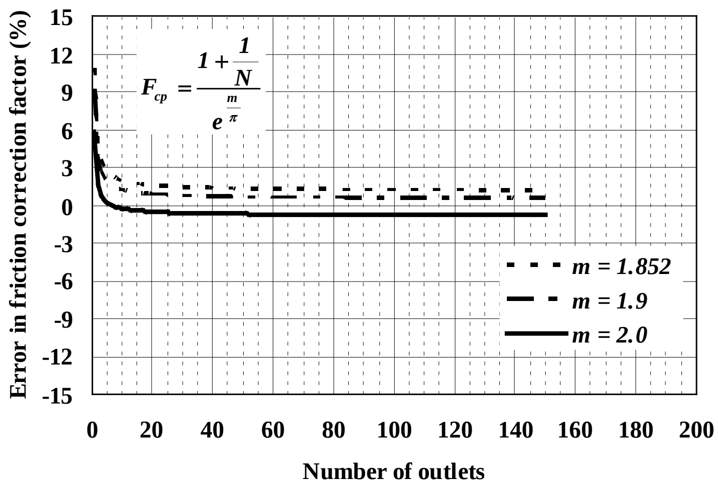

Fig. 2. Relative error in F_{cp} computed by using Eq. 19.

the vicinity of ± 3 % for N greater than unity. For N equal to one, the relative error was about 10.9 %.

Figure 3 indicates that the use of Eq. 20, the simplest form of Eq. 16, may lead to high error in computing F_{cp}, specifically for N less than 15. However, the relative error considerably decreases for N equal to or larger than 20. Practically, the use of Eq. 20 would lead to insignificant error because the number of sprinklers of a center-pivot system will generally be larger than 20. For almost all standard pivots a value of F_{cp} = 0.555 (which occurs with 73 outlets) will give results that are accurate to within ± 1% [8]. The values of F_{cp} obtained from Eqs. 16, 19 and 20 for m equal to 1.852 and N equal to 73 are, respectively, 0.561, 0.562 and 0.555.

The error in computing F_{cp} using Eq. 22 relative to the averaged values of Eqs. 10 and 11 is shown in Fig 4. For the common m values and for N ranging from one to infinity, the figure indicates that the relative error is insignificant. Likely, the insignificant relative error supports the use of modified Christiansen, Eq. 22, over either Eq. 10 or Eq. 11. It is depicted from Fig. 4 that the relative errors for different m values, 1.852, 1.9 and 2, are identically small and fall in a domain approximately ranging from +

2 to – 2 %. In general, the relative error in F_{cp} obtained by Eq. 22 compared to the averaged F_{cp} computed from Eqs. 10 and 11 is with in ± 2 %.

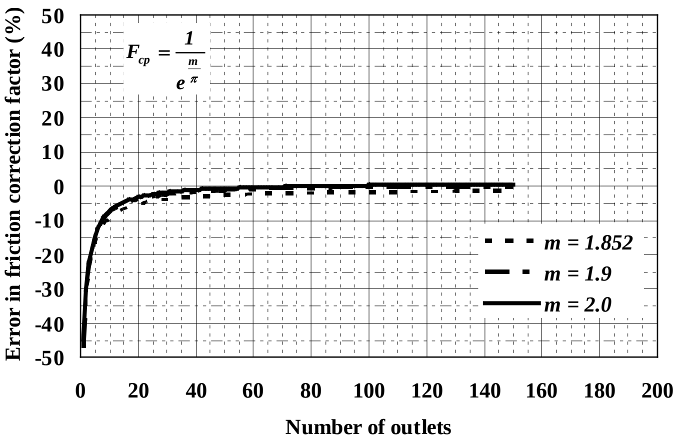

Fig. 3. Relative error in F_{cp} computed by using Eq. 20.

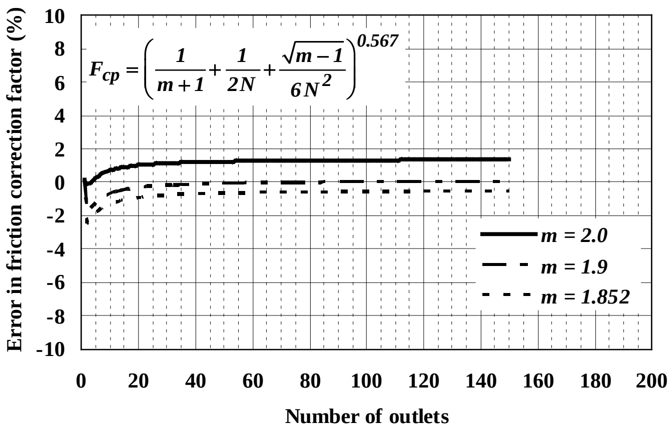

Fig. 4. Relative error in F_{cp} computed by modified Christiansen method.

To demonstrate and assess the applicability of the presented equations, well numerically documented examples and one field application are used. The numerical examples were taken from Anwar [7] and Schwab *et al.* [9]. The field example was provided by an agricultural company, which is Hail Agricultural Development Company (HADCO). The outputs of the three examples are summarized and shown in Table 1. In addition, the statistical parameters, average AVG, standard deviation STDEV and coefficient of variation CV, for F_{cp} are also provided in Table 1. The stepwise technique was the reference method used to compare and judge the F_{cp} methods. When the stepwise technique was not used, the method originally applied in the example was considered as the reference method for the comparison and judgment. The results of example 1 shown in Table 1 indicate that the highest error of F_{cp} estimate is expectantly associated with Eq. 20 and equal to -3.29 %. Surprisingly, the lowest error was associated with modified Christiansen method, Eq. 22 that is equal to -0.66 %. All F_{cp} methods apparently gave identical outputs as reflected by the low STDEV and CV values that are 0.006 and 0.012. This was anticipated because the number of outlets is very large, N = 67. The similar conclusion can be stated for outputs of example 2, except that Eq. 20 gave a high error in estimating F_{cp} that is attributed to small number of outlets, N =10. The modified Christiansen again has the lowest error in predicting F_{cp}. It is clear from Table 1 that the STDEV and CV values are higher for example 2 compared to those of example 1 due to the decrease in N from 67 to 10 outlets. The results of the field example don't differ much from the outputs of example 1 because of the large value of N, which is equal to 158 outlets. The considerably small values of STDEV and CV values reflect the close agreement of all used F_{cp} methods, as depicted in Table 1. In other words, the minor differences among the F_{cp} values obtained from the different equations (Eqs. 16, 19, 20 and 22) support the validity of them all.

Table 1. Comparison of F_{cp} methods using numerical and field examples

Method	F_{cp} (dimensionless)	$\hat{h}_f$ (m)	Error (%)
Numerical Example # 1 [7]: N = 67, m = 2, h_f = 11.1 m, F_{cp} = 0.547			
Stepwise	0.547	6.07	0.00
Eqs. 10 and 11 [7]	0.541	6.01	-0.99
Eq. 16	0.536	5.95	-1.98
Eq. 19	0.537	5.96	-1.81
Eq. 20	0.529	5.87	-3.29
Modified Christiansen, Eq. 22	0.543	6.03	-0.66
AVG = 0.539, STDEV = 0.006, CV = 0.012			
Numerical Example # 2 [9]: N = 10, m = 1.9, h_f = 9.81 m, F_{cp} = 0.579 (average of all F_{cp} values).			
Schwab et al. [9]	0.540*	5.30	-6.69
Eqs. 10 and 11[7]	0.593	5.82	2.46
Eq. 16	0.596	5.85	2.99
Eq. 19	0.604	5.93	4.40
Eq. 20	0.546	5.36	-5.63
Modified Christiansen, Eq. 22	0.592	5.81	2.29
AVG = 0.579, STDEV = 0.028, CV = 0.048			
Field Example # 3 (HADCO): N = 158, m = 1.852, h_f = 14.86 m, F_{cp} = 0.555			
Stepwise	0.555	8.25	0.00
Keller & Bliesner [8]	0.553	8.22	-0.32
Eqs. 10 and 11[7]	0.551	8.19	-0.68
Eq. 16	0.558	8.29	0.53
Eq. 19	0.558	8.29	0.53
Eq. 20	0.555	8.25	0.00
Modified Christiansen, Eq. 22	0.554	8.23	-0.19
AVG = 0.555, STDEV = 0.003, CV = 0.015			

* It is felt that 0.54 is too low and it may was intended to use 0.59 instead.
AVG = Average of F_{cp}
STDEV = Standard deviation of F_{cp}

CV = Coefficient of variation of F_{cp}

It should be emphasized that the presented equations can accurately be used to compute F_{cp} for flow cases of equally and variably spaced outlets. This is because the friction correction factor for center-pivots with constant outlets spacing was found to be similar to the friction correction factor for center-pivots with constant outlets discharge [7]. In fact, the close agreement between Eqs. 10 and 11 has also been studied.

Estimating *F* from F_{CP}

It is worth mentioning that Eq. 16 can easily be adjusted and accurately used to determine the friction correction factor *F* for linear-moved sprinkle systems. This can be achieved by equating Eqs. 16 and 22, that is:

$$F_{cp} = \frac{(1+N)e^{\frac{5m}{\pi}}}{\pi + Ne^{\frac{6m}{\pi}}} = \left[\frac{1}{m+1} + \frac{1}{2N} + \frac{\sqrt{m-1}}{6N^2}\right]^{0.567} \qquad (23)$$

The term between brackets is merely the *F* factor and Eq. 23 can thus be written as:

$$F_{cp} = \frac{(1+N)e^{\frac{5m}{\pi}}}{\pi + Ne^{\frac{6m}{\pi}}} = [F]^{0.567} \qquad (24)$$

Accordingly, the exponential function for *F* factor is obtained and takes the following form:

$$F = \left[\frac{(1+N)e^{\frac{5m}{\pi}}}{\pi + Ne^{\frac{6m}{\pi}}}\right]^{\frac{1}{0.567}} \qquad (25)$$

The same concept can also be applied for Eqs. 19 and 20, which can, respectively, be written as follows:

$$F = \left[\frac{1+\frac{1}{N}}{e^{\frac{m}{\pi}}}\right]^{\frac{1}{0.567}} \qquad (26)$$

and

$$F = \left[\frac{1}{e^{\frac{m}{\pi}}} \right]^{\frac{1}{0.567}} \tag{27}$$

In addition, the F can be estimated by adjusting Eqs. 10 and 11 that can, respectively, be written as follows:

$$F = \frac{1}{N^{\frac{2m+1}{0.567}}} \left[\sum_{i=1}^{N} i^m (2N - i)^m \right]^{\frac{1}{0.567}} \tag{28}$$

and

$$F = \frac{1}{N^{\frac{m+0.5}{0.567}}} \left[\sum_{i=1}^{N} i^m \left(\sqrt{N-i+1} - \sqrt{N-i} \right) \right]^{\frac{1}{0.567}} \tag{29}$$

Figure 5 illustrates the relationship between F computed by the different methods for m equal to 1.852 versus the number of outlets N. It is obviously depicted from the figure that the F can be determined by Eqs. 7, 25 and 28 or 29 with insignificant errors. The close agreement of the curves in Fig. 5 indirectly supports the accuracy of the developed equations for F_{cp} determination.

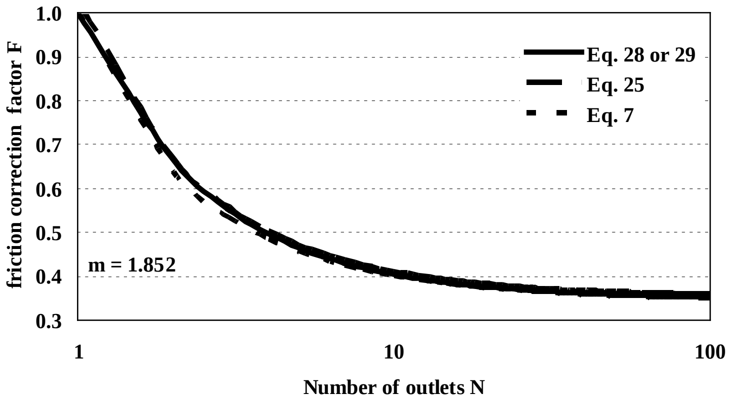

Fig. 5. Comparison of *F* methods for flow exponent *m* = 0.1852.

Three numerical examples have been used to demonstrate the applicability of the equations used to estimate *F*. While one example was taken from Schwab *et al.* [9], the other two examples were taken from [8]. The summarized results of the three examples, in addition to AVG, STDEV and CV of *F* are presented in Table 2. The results of example 1 depicted in Table 2 illustrate that the highest errors of *F* estimate are associated with Eqs. 25 and 26 that equal 3.05 and 3.61 %, respectively. The lowest error was associated with Eqs. 28 and 29. The Christiansen method, Eq. 7, has an error of 1.66 that is equal to the absolute error associated with Eq. 26. It can generally be stated that all presented *F* methods gave almost identical outputs as reflected by the low STDEV and CV values that are 0.007 and 0.019, respectively. This was anticipated because the number of outlets is large, $N = 33$.

Table 2. Comparison of *F* methods using numerical examples

Method	*F*(dimensionless)	$\hat{h}_f$ (m)	Error (%)
Numerical Example # 1 [8]: $N = 33$, $m = 1.852$, $h_f = 20.04$ m, $F = 0.360$			
Keller & Bliesner [8]	0.360	7.21	0.00
Eqs. 28 and 29	0.363	7.27	0.83
Eq. 25	0.371	7.43	3.05
Eq. 26	0.373	7.47	3.61
Eq. 27	0.354	7.09	-1.66
Christiansen [5], Eq. 7	0.366	7.33	1.66
AVG = 0.365, STDEV = 0.007, CV = 0.019			
Numerical Example # 2 [8]: $N = 10$, $m = 1.852$, $h_f = 86.43$ m, $F = 0.4023$			
Stepwise	0.4023	34.77	0.00
Keller & Bliesner [8]	0.4000	34.57	-0.57
Eqs. 28 and 29	0.4040	34.92	0.42
Eq. 25	0.4120	35.61	2.41
Eq. 26	0.4180	36.13	3.90
Eq. 27	0.3540	30.60	-12.00
Christiansen [5], Eq. 7	0.4020	34.75	-0.07
AVG = 0.399, STDEV = 0.023, CV = 0.057			
Numerical Example # 3 [9]: $N = 72$, $m = 1.75$, $h_f = 0.79$ m, $F = 0.38$			
Schwab et al. (9)	0.380	0.300	0.00
Eqs. 28 and 29	0.367	0.290	-3.33
Eq. 25	0.383	0.303	1.00
Eq. 26	0.384	0.303	1.00
Eq. 27	0.374	0.295	-1.67
Christiansen [5], Eq. 7	0.371	0.293	-2.33
AVG = 0.377, STDEV = 0.007, CV = 0.018			

AVG = Average of *F*
STDEV = Standard deviation of *F*
CV = Coefficient of variation of *F*

The outputs of example 2 shown in Table 2 show the all *F* methods, except Eq. 27, have lead to low errors. The large relative error, -12.00 %, associated with Eq, 26 is because of the small *N*, 10 outlets. Example 3 shows that the highest relative errors are related to Eq. 7, Eq. 28 and 29. These high errors may be attributed to the fact that these methods give high errors for low *m* values.

Conclusion

The current work showed that a simple equation written in different forms can accurately be used to determine the friction correction factor for center-pivot systems. The equation and its derived forms are of the exponential type and functions of the velocity exponent *m* and the number of outlets *N*. With slight adjustment, it has also been shown that the friction correction factor can accurately be computed from the Christiansen *F* factor originally developed for equally spaced and uniformly discharging outlets.

Numerical and field examples were used to demonstrate the applicability of the presented formulas. The examples outputs encourage the use of the developed equations to compute the friction correction factors for center-pivots and for similar type flow circumstances. It is expected that the proposed equations will facilitate the hydraulic analysis of the laterals of center-pivots and systems similarly characterized.

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حساب معامل التصحيح F لنظم الري المحوري باستخدام صيغ مبسطة ومعادلة كرستيانسن المعدلة

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ملخص البحث. تم في هذا البحث تطوير معادلة أسية-لوغارتمية لتقدير معامل تصحيح فاقد الاحتكاك F في نظم الري المحوري، كدالة في عدد المخارج N و أس التصرف m، وتم أيضا استنباط صيغتين أخريين من تلك المعادلة أكثر بساطة وملائمتان للحالات التي يكون فيها عدد المخارج N كبير. كما تم في هذا البحث تعديل معادلة كرستيانسن المستخدمة في حساب معامل التصحيح F، والتي تم استنباطها أساسا للحالات التي يتناقص التصرف خطيا مع المسافة. تم عمل مقارنة بين الصيغ المطورة في هذا البحث والمعادلات التي طورت من قبل باستخدام أمثلة نظرية وأخرى حقلية، وتبين وجود تطابق كبير في النتائج، وكان مقدار الخطأ في التقدير في حدود ± ٢ %.