Some Studies in The Spin Orbit Splitting in The P–Shell

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The spin orbit splitting in P-shell nuclei is investigated using sussex matrix elements and different oscillator constants for core and valence orbits where harmonic oscillator wave functions are used as basis.

Section 1

Introduction

Several calculations have been carried out for the P-shell nuclei to study their level scheme as other physical properties *e.g.* Cohn and Kurath (1965) and others. Both phenomenological and realistic interactions were used. In general the fit to the data is good when considering about one hundred level or more. However, the spectrum of certain nuclei in this region displayed some anamolies which can be seen in the above references. Most of the calculations carried out so far took the interaction only up to second order. Also the input for the spin orbit splitting is either varied in an ad-hoc manner through the shell or fitted to the experimental values. As for the choice of the oscillator parameter, only one constant was taken for each nucleus or in some cases an optimal oscillator length for the whole region.

Although it was shown (Elliott and Lane, 1954), that the splitting may increase systematically through the shell, it is hard to justify the large values taken for the splitting in these calculations as for example in reference 2. Even with reasonable choice

for such quantity and an effective realistic interaction (Paul and Maripuu, 1973, Elliott and Lane 1954), we still can see discrepancies in the spectrum.

Arima and Terasawa (1966) in an early paper studied the effect of the second order tensor force on the splitting. By using mesonic potential they found out that half of the experimental value was accounted for if such effect was included. In a recent paper (Sanderson *et al*. 1974), however, by using Sussex interaction showed that the role of the tensor force in second order to spin-orbit splitting is negligible. To improve the spectrum one perhaps need to include three-body effective forces. This has been done (Evans *et al*. 1976), however, the authors still needed to increase the spin-orbit splitting by certain amount to get better results in their calculations.

In this paper, we propose to investigate the spin-orbit splitting and its dependence on the size parameter of the nucleus. It is well known that the actual size parameter is rather ill-determined. This fact in itself makes it difficult to determine the splitting since the latter is a sensitive quantity to changes in the oscillator constant in first order (Elliott *et al*. 1967). Even in second order there is still an appreciable dependence on such parameter (Sanderson *et al*. 1974). In section 2 we give some general remarks while in section 3 some outline of the method of calculation is given. In section 4, results and discussions are presented.

Section 2

Most previous calculations have been performed using one size parameter for all orbitals. A more realistic approach would be to choose different size parameters for the different orbitals and minimise the energy with respect to these parameters. In this calculation we adopt such model to investigate and calculate the spin-orbit splitting in the P-shell using Sussex interation. We denote by ΔP the splitting and the input which is being used is

$$\Delta P = <S^4P_{1/2}|H|S^4P_{1/2}> - <S^4P_{3/2}|H|S^4P_{3/2}>$$

Where H is the hamiltonian. This is exactly the Spin-orbit Splitting for He^5.

For $A > 5$, $\Delta P \rightarrow \Delta P +$ effective 2-body interaction. Instead of including a 2-body effective interaction we use a simple model, namely (S^4P^m). $m = 1,2....,12$. This is explained in section 3. In this first order calculation ΔP is varied as a function of the

three size parameters b_o, b_s and b_p where they denote the size of the nucleus as a whole, the core size and the valence size respectively. Another quantity of interest and which will be considered is

$$X = \frac{b_p^2 - b_s^2}{b_o^2}$$

One should point out that no attempt is being made here to determine optimal oscillator constant. However, to have a feeling of what sort of splitting one might have in the P-shell nuclei, we take for b_0 the values reported by Wilkinson and Mafethe, (1966), which are deduced from analysing experimental data. Values for b_s and b_p are taken from Volkov's paper (1965), where a different interaction from ours was used.

Section 3

Method of calculation

To study the splitting through the P-shell, we make use of the formula for $A = 5$

$$\Delta P = \frac{3}{8}\{2e(^3P_o) + 3e(^3P_1) - 5e(^3P_2)\}\frac{2b_p^2}{b_s^2 + b_p^2}$$

Where $e(^{2s+1}l_j)$ are the relative sussex matrix elements interpolated at the b-values given by $b^2 = \frac{b_s^2 + b_p^2}{2}$ Since these matrix elements are only given for a certain set of b-values.

If one assumes that the properties of such muclei are given by the lowest configuration (OS^4OP^m), then one has using oscillator basis

$$<r^2> = \frac{1}{A}\sum_{i=1}^{A}(2n_i + l_i + +\tfrac{3}{2})b_i^2 = \frac{1}{4+m}(6b_s^2 + \tfrac{5}{2}mb_p^2) \qquad (2)$$

We can also write

$$<r^2> = \frac{b_0^2}{2(4+m)}(12 + 5m) \qquad (3)$$

by equating (2) & (3) and writing

$$\delta^2 = b_p^2 - b_s^2$$

$$X = \frac{\delta^2}{b_0^2}$$

$$b_0^2 = \frac{12b_s^2 + 5mb_p^2}{12 + 5m}$$

One can express b_s, b_p and b as a function of X, b_0 and m where m is the number of practicles in the P-shell $i.e.$

$$b_s^2 = b_0^2 \{ 1 - \frac{5m}{12 + 5m} X \}$$

$$b_p^2 = b_0^2 \{ 1 + \frac{12}{12 + 5m} X \}$$

$$b^2 = b_0^2 \{ 1 + \frac{12\text{-}5m}{24 + 10m} X \}$$

$$X = (1 + \frac{12}{5m})(1 - \frac{b_s^2}{b_o^2})$$

Put

$$F_m(X) = \frac{1 + \dfrac{12X}{12 + 5m}}{2 + \dfrac{12 - 5m}{12 + 5m} X} = \frac{b_p^2}{b_s^2 + b_p^2}$$

Hence equation (1) can be rewritten as

$$\Delta P = \frac{3}{4} \left\{ 2e(^3P_0) + 3e(^3P_1) - 5e(^3P_2) \right\}_b F_m(x) \ (4)$$

Now let us assume for simplicity that the expression inside the bracket of equation (4) is of the order Vob^{-n} where $n \simeq 4$ or 6. This assumption is based on the behaviour of the spin-orbit splitting as a function of b. Hence one can write ΔP as

$$\Delta P = \tfrac{3}{8} Vob^{-n} \quad Fm(x) \tag{5}$$

where one defines

$$\Delta P(o) = \tfrac{3}{8} \quad Vob^{-n}$$

and

$$Fm(X) = \frac{2b_p^2}{b_p^2 + b_s^2}$$

We shall call $\Delta P(o)$ the H_e^5 input splitting and $Fm(x)$ is a factor which depends on the particle number m in the P-shell, it depends also on b_s and b_p. The behaviour of $fm(x)$ as a function of x and m is shown in Fig.1 where we take $n = 4$. We also give the values of $fm(x)$ in Table 1 for each m as a function of X where we include $+$ Ve and $-$ Ve values of X. The latter gives values for b_s greater than b_p.

Section 4

Results and Discussion

By looking at Table 1 and Fig. 1, we notice that the splitting increases as X increases for $m > 1$, *i.e.* for $b_p > b_s$ but for $b_s > b_p$ it drops again. In Fig.1 we also include a plot of X against m (solid line) where X is calculated from Volkov's Curves. The dips in this curve mark the minima and the alpha structure of the corresponding nuclei. Also from Fig.1 we notice that X is large at the beginning of the shell and small and negative at the end of the shell.

In Table 2 we give results for the spin-orbit-splitting for $m = 2,......,12$ where we use Volkov's values for b_s and b_p. As for b_o we use the values deduced by Wilkinson and Mafethe (1966), We shall call those $\Delta P (W + V)$. We also compute ΔP as a function of $b_o(W)$ where W refers to Wilkinson results for b_o, call those $\Delta P(W)$. The two calculations are compared in Table 2. The formula used here is equation (4) where the interpolated Sussex matrix elements also used as input for the interaction. Finally in Table 3 we list the values of b_s, b_p and b_o as well as X. For the sake of completeness we plot in Fig. 2, $\Delta P(V + W)$ and $\Delta P(W)$ as a function of m.

It is seen from Table 2, that ΔP can differ markedly when different oscillator constants are taken for the different orbitals from those when only one oscillator parameter is used for all in a particular   nucleus. This difference is pronounced in the first half of the shell, see third row of the table where we list the differences. We also quote for comparison the values of ΔP used by Paul and Maripuu (1973) where they

take on optimal oscillator constant of 1.7 fermi for all nuclei they studied where ΔP is fitted to the experimental data. It seems that if one takes second order correction to the above values, there will be a difference of about 3 MeV for the nuclei $A = 10$ to $A = 14$. One may conjecture that the inclusion of contribution resulting from three-body effective forces in a calculation which involves separate oscillator parameters for the various orbitals could result in better fit for the P – shell nuclei and consequently explain some of the difficulties arising in determining the energy levels of these nuclei.

Conclusion

The spin orbit splitting ΔP in the P-shell nuclei has been investigated using the harmonic oscillator model and Sussex interaction where the lowest configuration is assumed to be of the form $(S^4 P^m)$.

ΔP is studied as a function of three oscillator constants b_s, b_p and b_o where they refer to OS, OP orbitals and to both respectively. It is found that ΔP does not only depend on b_o as found in most previous calculations, but also depends on b_s and b_p when such model is adopted.

References

Arima, A. and T. Terasawa (1966) *Prog. Theor. Phys., Osaka*, **23**, 87.

Cohen and Kurath (1965) *Nuclear Physics*. **73**, 1.

Dirim, H. (1972) *Ph.D. Thesis* Univ. of Sussex England.

Elliott, J.P. and Lane (1954) *Phys. Rev.*, **96**, 1160.

Elliott, J.P., E. Sanderson and H.A. Mavromatis (1967) *Phys. Letters*, **24B**, 358.

Evans, T. and J.P. Elliott, *Nuclear Physics*, (Under publication).

Halbert, E.C., Y.E. Kim and T.T.S. Kuo (1966) *Physics Letters*, **20**, 657.

Paul Goldhammer *et. al.* (1968) *Nuclear Physics* **A106, 62.**

Paul S. Hauge and S. Maripuu (1973) *Phys. Rev.* **C5**, 1609.

Sanderson, E., J.P. Elliott, H.A. Mavromatis and B. Singh (1974) *Nuclear Physics* **A219**, 190.

Terasawa, T. (1966) *Prog. Theor. Phys.*, **23**, 87.

Volkov, A.B. (1965) *Nuclear Physics* **74**, 33.

Wilkinson, D.H. and M.E. Mafethe (1966) *Nuclear Physics*, **85**, 97.

Table 1. Values of fm(x) for each m as a function of X.

X m	.2	.4	.6	.8	1.0	−.1	−.2
1	1.01	1.01	1.01	0.99	0.975	0.99	0.974
2	1.08	1.16	1.23	1.30	1.36	0.96	0.915
4	1.16	1.34	1.55	1.79	2.06	0.93	0.86
8	1.24	1.54	1.94	2.46	3.15	0.905	0.82
12	1.27	1.64	2.15	2.88	3.92	0.89	0.80

Table 2. Spin-orbit values Δ P(V + W) and Δ P(W) as well as the difference between them. Also included the Spin-orbit values used in reference q, these are indicated by a stan.

m	2	3	4	5	6	7	8	9	10	11	12
P(V − W)	3.14	3.70	5.30	4.04	5.12	5.11	3.80	4.30	3.96	3.76	3.38
P(W)	2.43	2.94	4.50	3.47	4.25	4.64	4.00	4.37	4.09	4.17	3.14
Ref. 5	2.60	3.40	4.80	2.80	7.50	6.70	6.80	7.20	6.70[*]		
P(V − W) − P(W)	.71	.76	.80	.57	.87	.47	−.20	−.07	−.13	−.41	.24

Table 3. Input values for b_o, b_s, b_p and X which are used to calculate Δ P(V + W) and Δ P(W).

m	2	3	4	5	6	7	8	9	10	11	12
b_o	1.76	1.69	1.54	1.63	1.56	1.53	1.58	1.55	1.573	1.566	1.666
b_s	1.38	1.49	1.45	1.52	1.45	1.48	1.60	1.56	1.59	1.64	1.78
b_p	2.13	1.83	1.59	1.68	1.60	1.55	1.57	1.55	1.57	1.55	1.64
X	0.84	0.39	0.19	0.19	0.19	0.08	−.04	−.02	−.03	−.12	−.17
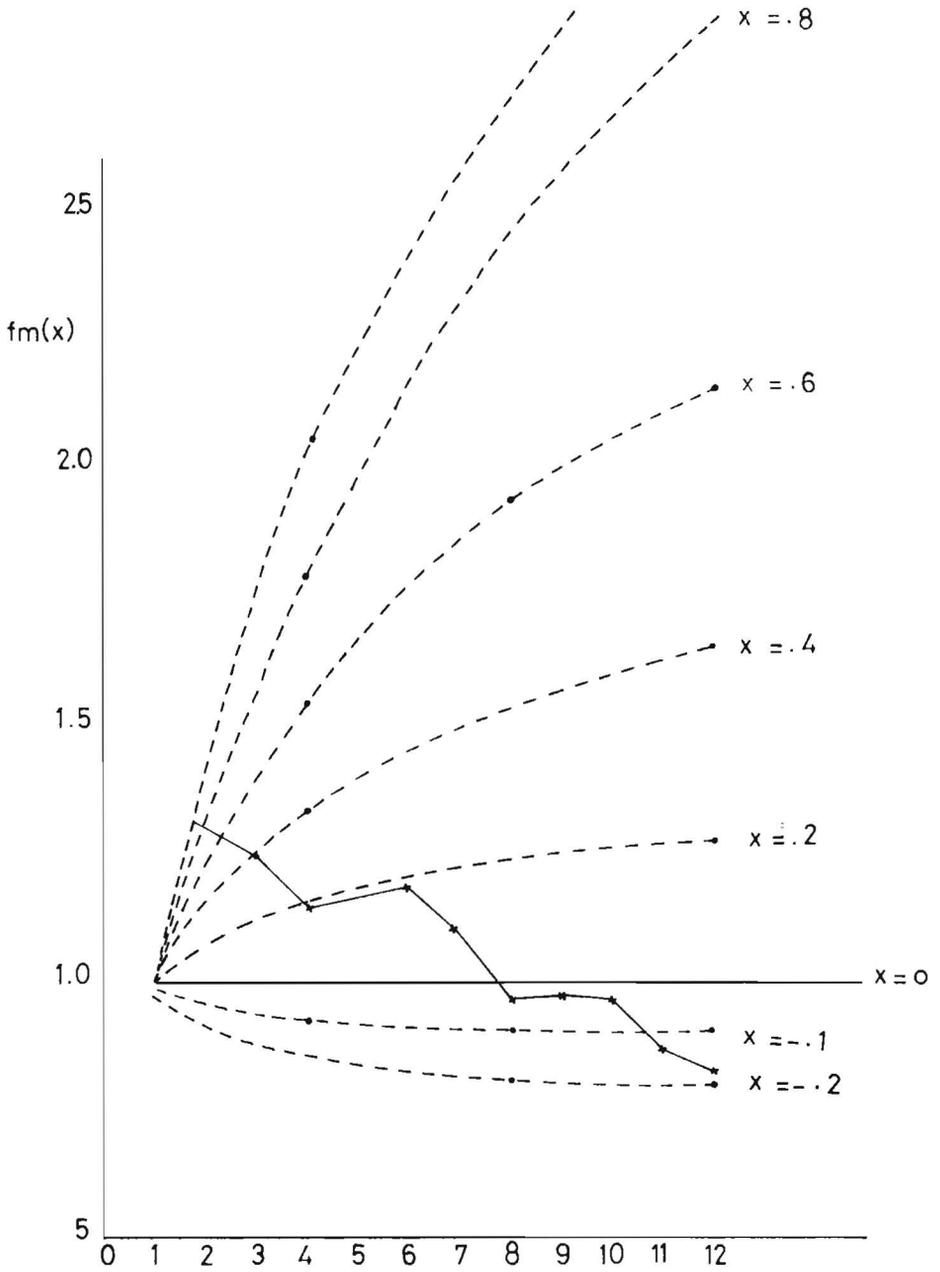
Fig. 1. This curve gives the variation of $\dfrac{\Delta\ P(x)}{\Delta\ P(o)}$ fm(x) as a function of x for each m. It also indicates the variation of x with m.
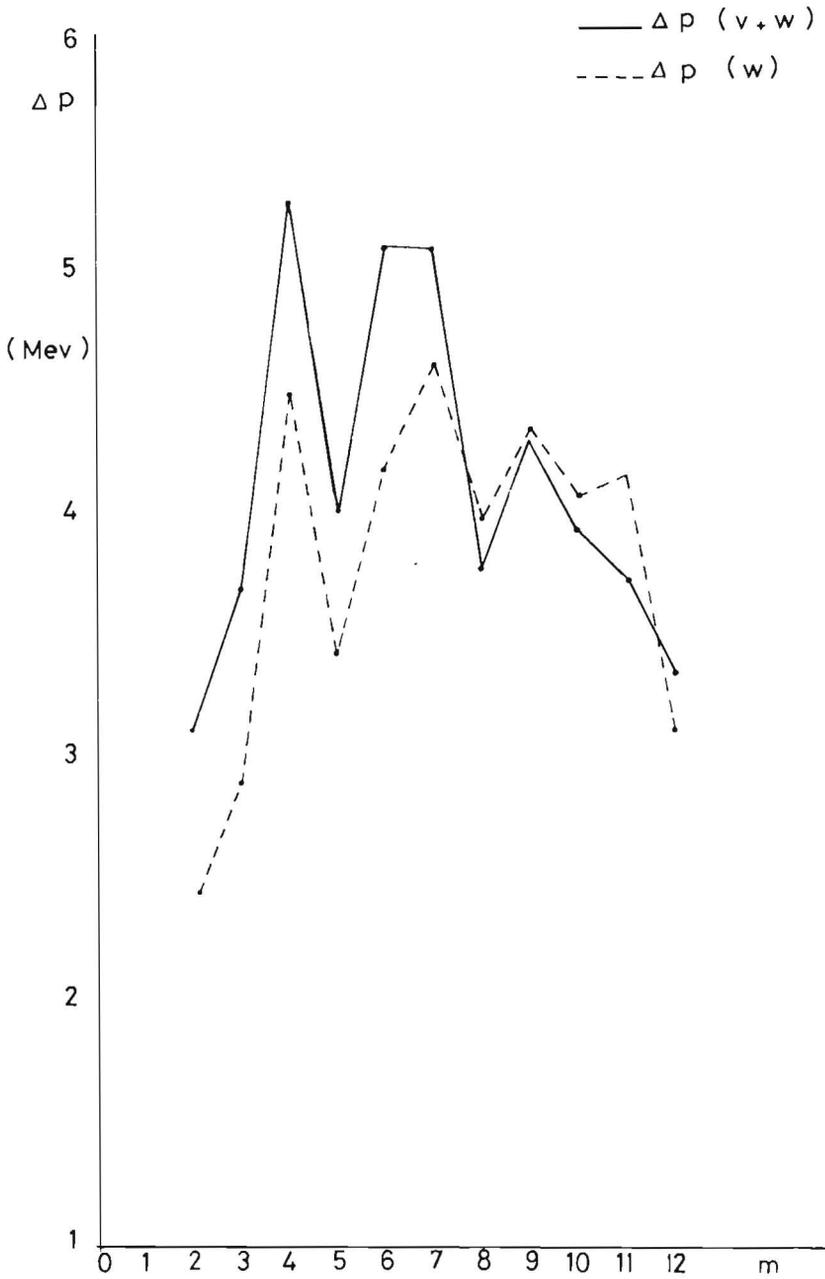
Fig. 2. The spin-orbit values as deduced from eq. (4) using the values given by Wilkinson and Volkov for bo, bs and bp.

بعض الـدراسات على الانشـطار المداري المغـزلي في الطبقة P

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درس الانشطار المداري المغزلي للنويات في المدار P باستخدام مصـفوفات Sussex للقوى النووية والموجات الهرمونية بثوابت هرمونية مختلفة لكل من المدارات OS و OP للنويات في هذه الطبقة .

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