انتقال الحرارة في مجرى كوييت في حالة وجود فرق ضغط اضافي وتشتت لزج

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تقدم هذه الدراسة تحليلا لانتقال الحرارة في مجرى كوييت في حالة وجود فرق ضغط اضافي وتشتت لزج .
أمكن بالتحويل البسيط تطبيق أثر التشتت اللزج على الحل المعطى من قبل (بروين) (٤) من أجل تشتت لزج قدره صفر ، لقد قدمت النتائج العددية لمجموعة أساسية من الشروط المحددة لاظهار تطور هياكل (مرتسمات) درجة الحرارة وعدد " نوسلت " .
لقد وجد أن التشتت اللزج يمكن أن يقود لاقلال بين في الانتقال الحراري من اللوحة (القطعة) الحارة المتحركة، خاصة عند ميل ضغط منخفض .

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HEAT TRANSFER IN PLANE COUETTE FLOW WITH PRESSURE GRADIENT AND VISCOUS DISSIPATION

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The paper presents an analysis for heat transfer in plane couette flow in the presence of additional pressure gradient and viscous dissipation. By a simple transformation, the effect of viscous dissipation can be superimposed on the solution given by Bruin [4] for zero viscous dissipation. Numerical results are presented for a typical set of boundary conditions showing the development of temperature profiles and Nusselt number. It is found that viscous dissipation can lead to significant reduction in heat transfer from the hot moving plate particularly at low pressure gradient.

Nomenclature

a	= distance between the plates
c_p	= specific heat
h	= heat transfer coefficient
k	= thermal conductivity
p	= pressure
P	= dimensionless pressure gradient
T	= temperature
u	= fluid velocity in x-direction
U	= velocity of moving plate
x	= streamwise coordinate
X	= dimensionless x coordinate = $x/a\,P_e$
y	= transverse coordinate
Y	= dimensionless y coordinate = y/a
Ec	= Eckert number = $U^2/c_p(T_2 - T_o)$
Pe	= Peclet number = $U\,a/\alpha$
Pr	= Prandtl number = $\mu c_p/k$
Nu	= Nusselt number = h a/k
α	= thermal diffusivity = $k/\rho c_p$
θ,φ	= dimensionless temperature
μ	= dynamic viscosity
ρ	= density

Superscripts

*	= quantity with viscous dissipation

Subscripts

0	= at entrance, x=0
1	= at bottom plate y=0
2	= at top plate, y=a
m	= cup-mixing
x	local
∞	at very large x

1. INTRODUCTION

Heat transfer in Couette liquid flow has been investigated extensively for a number of flow situations and boundary conditions. For example, Sestak and Rieger [1] considered plane Couette flow in the absence of pressure gradient and viscous dissipation and optained temperature distributions for four combinations of uniform temperature and zero heat flux boundary conditions. Vogelpohl [2], on the other hand, studied plane flow with zero pressure gradient and reported the temperature profiles resulting from the viscous dissipation of energy. The effect of additional pressure gradient was analysed by Hudson and Bankoff [3] and more recently by Bruin [4]. In these analysis the effect of viscous dissipation was neglected. In certain applications, the knowledge of combined effect of pressure gradient and viscous dissipation on heat transfer is more useful. The purpose of this paper is to present an analysis for such applications and to provide representative results showing the development of temperature profiles and Nusselt number.

2. ANALYSIS

The physical situation analysed is sketched as inset in Fig. 1. The bottom plate is stationary while the top plate moves with a velocity U. The plates are maintained at uniform temperatures T_1 and T_2. The fluid enters the channel with uniform temperature T_0 and flows under the influence of negative pressure gradient. The velocity distribution is given by [5]

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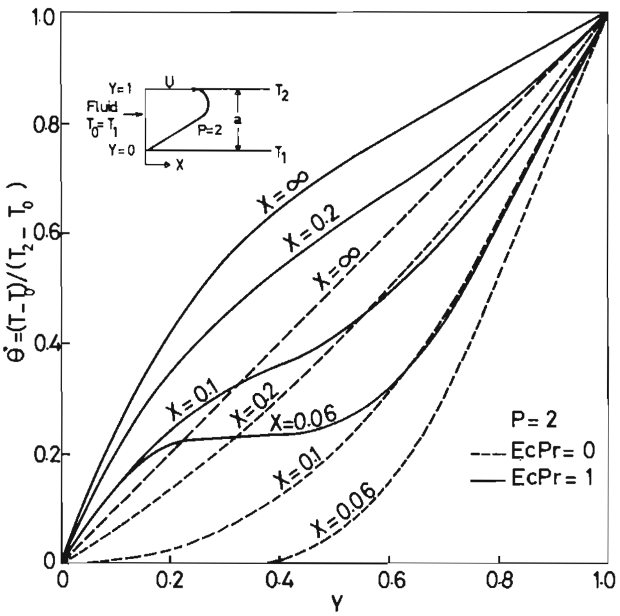

Fig.1 Temperature distribution in Couette flow with pressure gradient and viscous dissipation.

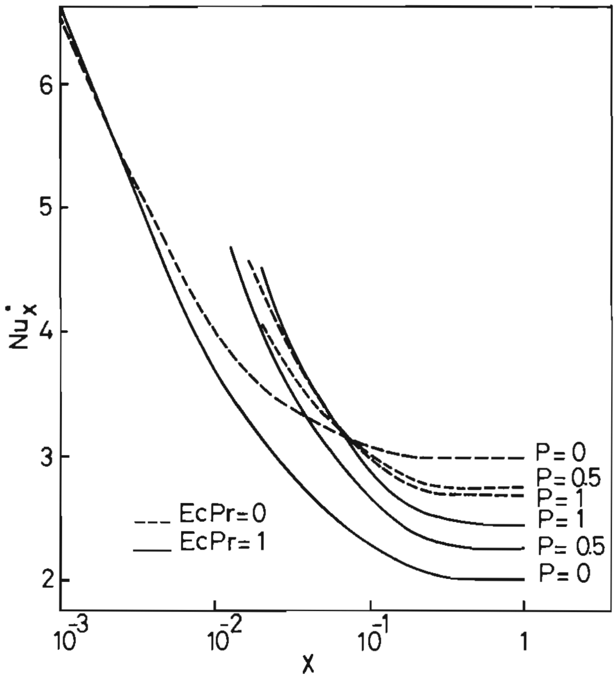

Fig.2 Variation of local Nusselt number with longitudinal distance at different pressure gradients.

$$\frac{u}{U} = \frac{y}{a} - \frac{a^2}{2\mu U}\,\frac{dp}{dx}\,\frac{y}{a}\,(1 - \frac{y}{a}) \quad (1)$$

Neglecting streamwise conduction compared to streamwise convective heat transport but retaining the viscous dissipation term, the simplified energy equation for steady flow of constant-property fluid becomes

$$u\,\frac{\partial T}{\partial x} = \alpha\,\frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho c_p}\,(\frac{du}{dy})^2 \quad (2)$$

To obtain the solution for temperature distribution with pressure gradient and viscous dissipation, the dependent variable T is transformed to θ according to

$$\theta = \frac{T - T_0}{T_2 - T_0} - \varphi\,(Y) \quad (3)$$

Introducing Eq. (1) into Eq. (2) and using the above transformation the energy equation in dimensionless form becomes

$$Y\,[1 + P\,(1 - Y)\,]\,\frac{\partial \theta}{\partial X} = \frac{\partial^2 \theta}{\partial Y^2} +$$

$$\varphi'' + Ec\,Pr\,[1 + P\,(1 - 2Y)\,]^2 \quad (4)$$

where

$$X = \frac{x}{a\,P_e}\,,\quad Y = \frac{y}{a}\,,\quad P = -\,\frac{a^2}{2\mu U}\,\frac{dp}{dx}$$

$$Pr = \frac{\mu c_p}{k},\quad Pe = \frac{Ua}{\alpha},\quad Ec = \frac{U^2}{c_p(T_2 - T_0)} \quad (5)$$

If φ (Y) is chosen such that

$$\varphi'' + EcPr\;[\,1 + P(1 - 2Y]^2 = 0 \quad (6)$$

Eq. (4) reduces to

$$Y\;[\,1 + P(1 - Y)\,]\,\frac{\partial \theta}{\partial X} = \frac{\partial^2 \theta}{\partial Y^2} \quad (7)$$

which is analogous to energy equation without viscous dissipation and has been solved by Bruin [4] in terms of hypergeometric functions. The solution for φ can readily be obtained from Eq. (6) and superimposed on Bruin's solution according to Eq. (3) to give the temperature distribution.

To display representative numerical results, solutions for $P = 2$ were obtained for the following boundary conditions:

$$\begin{aligned} &Y = 0\,,\ X > 0\,,\ \theta = 0 \\ &Y = 1\,,\ X > 0\,,\ \theta = 1 \\ &X = 0\,,\ 0 \leq Y \leq 1,\ \theta = 0 \end{aligned} \quad (8)$$

These boundary conditions are the same as chosen by Bruin and for which numerical results are available in his paper. The corresponding boundary conditions on φ become

$$Y = 0 , \varphi = 0 , Y = 1 , \varphi = 0 \quad (9)$$

and the solution for φ is

$$\varphi = \frac{1}{6} \text{Ec Pr} [(P^2 + 2 P + 3) Y - 3 (1 + P)^2 Y^2 + 4 P(1 + P) Y^3 - 2 P^2 Y^4] \quad (10)$$

Denoting the dimensionless temperature $(T - T_0) / (T_2 - T_0)$ by θ^*, the cup mixing temperature based on the usual definition can be obtained from Eqs. (1), (3) and (10) as

$$\theta^*_m = \theta_m + \frac{1}{420} \text{Ec Pr} \times \left[\frac{8 P^3 + 21 P^2 + 28 P + 105}{3 + P}\right] \quad (11)$$

where θ_m is the dimensionless cup-mixing temperature calculated by Bruin. As usual, the local Nusselt number Nu^*_x at the moving plate is defined as

$$Nu^*_x = \frac{h_x a}{k} = - \frac{1}{1 - \theta^*_m} \left(\frac{\partial \theta^*}{\partial Y}\right)\Big|_{Y = 1} \quad (12)$$

It can be shown that for the fully developed temperature profile the Nusselt number approaches the limiting value

$$Nu^*_\infty = \frac{3 + P}{1 + \frac{1}{2}P} \left[\frac{1 - \frac{1}{6} \text{Ec Pr} (P^2 - 2 P + 3)}{1 - \frac{1}{210} \text{Ec Pr} \left[\frac{8P^3 + 21 P^2 + 28 P + 105}{2 + P}\right]}\right] \quad (13)$$

3. RESULTS

Fig. 1 shows a typical set of temperature profiles for P = 2 and Ec Pr = 1 together with the corresponding profiles for zero viscous dissipation, EcPr = 0 based on [4]. As expected, the addition of viscous dissipation term enchances the temperature level throughout and the effect is quite significant for Ec Pr of the order of unity. The variation of Nusselt number with longitudinal distance is shown in Fig. 2 for parametric values of P in the range 0 - 1. It can be seen that viscous dissipation can lead to significant reduction in heat transfer from the hot moving plate particularly at low values of P.

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