الأساسات المستمرة

فاروق الزين*

ان فرض ضغط تربة منتظم تحت الأساسات المستمرة يعطي نتائج اما خطرة أو مكلفة ، فعندما لا تسمح حدود الأرض بزيادة القواعد أبعد من نهاية الأعمدة الخارجية يجب اغفال فرض ضغط تربة منتظم كما يجب عمل تحليل انشائي مرن ، وعندما يختلف امتداد تلك القواعد فان التحليل بطريقة توزيع العزوم يعطي فقط شكل منحنى العزم العام .

وفي حالة الفتحات الداخلية بين الأعمدة فان نتائج التحليل الانشائي المرن يطابق بشكل تقريبي نتائج التحليل بطريقة توزيع العزوم .

ويعتبر مقدار التوافق بين الأحمال الحقيقية للأعمدة وتلك الناتجة عن التحليل بطريقة توزيع العزوم قياساً لدقة التحليل الانشائي المرن ، أما العزوم الناتجة بتحليل القوى فانها تعطي نتائج عالية وغير حقيقية ، ويقترح عمل تحليل كامل مبني على نظرية الأساسات المرنة في حالة المشاريع الهامة .

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THE PROBLEM OF CONTINUOUS FOUNDATIONS

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The assumption of a uniform soil pressure under continuous foundations is either dangerous or too expensive. When the boundary restrictions do not permit the extension of a foundation beyond the edge of the exterior columns, a uniform soil pressure distribution assumption should be discarded and an elastic analysis could be performed. When overhangs are possible, moment distribution will only give the general trend of the moment curve. Rough agreement of moment distribution results with elastic analysis in interior spans. The agreement between column reactions obtained by moment distribution and the actual column loads is a measure of the elastic analysis. Moments obtained by statics analysis are unrealistically high. It is suggested that a rigorous analysis based on the elastic foundation theory may be used in important projects.

Nomenclature

$A_{\lambda x} = e^{-\lambda x} (\cos \lambda x + \sin \lambda x)$

$B_{\lambda x} = e^{-\lambda x} (\sin \lambda x)$

$C_{\lambda x} = e^{-\lambda x} (\cos \lambda x - \sin \lambda x)$

$D_{\lambda x} = e^{-\lambda x} (\cos \lambda x)$

B = Footing Width

EI = Flexural Rigidity of Footing Section

[E] = 4 x 4 matrix of coefficients of the 4 end conditioning equations.

{F} = 4 element vector consisting of computed shears and moments, M_a, V_a, M_b, V_b, at both ends of the footing when infinite length is assumed.

$\{F_E\}$ = 4 element vector consisting of the end conditioning forces P_{oa}, P_{ob}, M_{ob}, M_{ob}, required to correct for the infinite length assumption.

K_s = Modulus of Subgrade Reaction

l = Total Length of Footing

M_o = Externally Applied Moment

M = Moment at any section along footing length.

P = Concentrated Applied Column Load

p = Soil pressure ordinate at any point under the footing.

q = Uniformly distributed applied load.

V = Shear at any section along footing length.

x = Distance along the footing as measured to the right of the point of application of a concentrated load.

y = Vertical deflection of the footing at any point.

λ = Characteristic Length = $\sqrt[4]{\frac{BK_s}{4EI}}$

θ = Slope of the elastic line of the footing at any point.

1. Introduction

In any continuous foundations problem, the major unknown is the soil pressure distribution pattern. Once this distribution is determined, the problem reduces to simple statics calculations. Some engineers tend to overlook this fact by accepting a uniform soil pressure distribution as a reasonable assumption. It turns out, however, that this assumption can be either dangerous or too expensive (in terms of reinforcement required) even in the case of the fairly rigid foundations. It is clear that the problem becomes statically determinate once the soil pressure distribution is obtained. It would, therefore, seem contradictory to proceed with an indeterminate analysis after having assumed a soil pressure diagram. Dunham (1) has suggested an approach similar to that of twoway or flat slabs in designing mat foundations. The mat is assumed to be loaded by a uniform soil pressure with moments and shears computed as in a floor slab. In many cases results obtained by this approach can be completely in error and computed column reactions would be incompatible with actual applied

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column loads. The analogy does not hold, and a mat foundation should be treated as a slab supporting concentrated column loads where the reactions to these loads are provided by soil pressure and not as an inverted beam or slab supporting an assumed uniform soil pressure.

It might, therefore, seem reasonable on this basis to solve the continuous foundations problem by statics after having assumed some soil pressure diagram. The fact remains, however, that the results can be expected to be only as good as the assumed pressure diagram. If a uniform soil pressure is assumed, an uneconomical design will result and the moment diagram will mostly maintain one sign throughout the foundation causing tension on the upper fibers only. This situation can be aggravated when the exterior column load exceeds half that of the average interior column load, or when overhangs beyond the limits of exterior columns are either small or absent. In general, it would be difficult to guess beforehand at a realistic soil pressure distribution pattern.

The interaction between the supporting subgrade and the foundation should not be overlooked. The analysis should also reflect the effect of the foundation size and its flexural stiffness. Such approach would result in a unique solution that will satisfy equilibrium and both the properties of the supporting subgrade and the foundation. The elastic subgrade reaction theory is an attempt in this direction [2]. The subgrade is assumed to be homogeneous and elastic. It is simulated to a group of closely spaced independent identical elastic springs. Any horizontal continuity in the subgrade is often ignored. The equivalent spring stiffness provides the constant of proportionality between pressures and deformations in the soil, namely the coefficient or modulus of subgrade reaction. Clearly, this theory oversimplifies the subgrade properties but also simplifies the elastic foundation solution. Some elasticity of the soil is, of course, undeniable and Kramrisch-Rogers [5] formulated an empirical simplified version of the elastic foundations theory in an effort to bypass the tedious calculations involved in the exact solution. A.C.I. Committee 436 [6] proposed the use of this simplified empirical approach in designing continuous foundations. However, one simplification made in developing this empirical procedure restricts it from general application to most practical situations. It states that moments caused by settlement deformations (dishing) of the continuous footing as a whole are neglected. Thus only localized effects of the applied loads are computed and no attempt is made to determine the overall behavior of the footing under the effect of the loads as a group. The procedure assumes that the resultant of subgrade reaction pressures under a column is equal to and concentric with the applied column load. It also assumes that these pressures have maximum ordinates under the column and minimum ordinates at the center of the bay or at the tip of the overhang in case of an exterior column. That these assumptions can be totally or partially in error is illustrated by the solved examples at the end of this presentation. In many cases, the engineer is faced with designing a mat foundation where property lines do not permit any overhangs beyond exterior columns. Maximum soil pressure ordinates do not necessarily have to occur right under the column nor does the minimum pressure ordinate have to occur at the middle of the bay. In any event, no real advantage is gained if the designer would still be faced with the problem of solving for the effects of the overall foundation dishing and superposing it on the results obtained by the empirical method. It should be noted here that in many practical cases the "dishing" moments can be of such a magnitude as to offset completely and reverse the sign of the localized moments. Negative moments may no longer occur under the column load nor will positive moments occur necessarily at midspan.

With the current trend of using electronic computers in many engineering offices where, there remains little reason for not applying the more exact solution of the elastic foundation theory as provided by Hetenyi [7]. In the remainder of this paper, the elastic foundation solution is briefly described and presented in the form of a computer program. Three solved examples at the end provide a comparison of results from different methods.

2. Elastic Foundation Formulation

Sign convention:

The following forces are considered positive when acting as described:

1. Applied loads acting downwards.

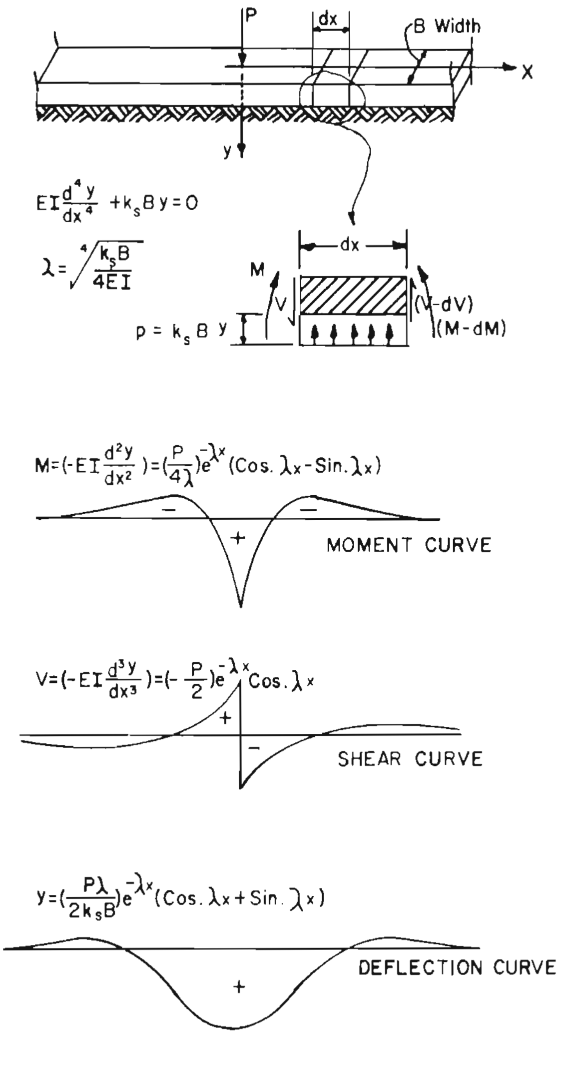


FIGURE (1)
AN INFINITE LENGTH BEAM ON ELASTIC FOUNDATIONS ACTED UPON BY ONE CONCENTRATED LOAD

2. Externally applied moments acting clockwise.
3. Internal moments when causing tension on the bottom fibres of a section.
4. Shears when acting downwards on the right end of an elemental section.

Deflections and slopes are referred to the coordinate system shown on Fig. 1.

Equilibrium considerations of an element of a beam on elastic foundations results in the well known differential equation of the elastic line of that beam:

$$EI\,\frac{d^4y}{dx^4}+K_s\,By = qB \qquad (1)$$

(assuming no horizontal continuity of the subgrade).

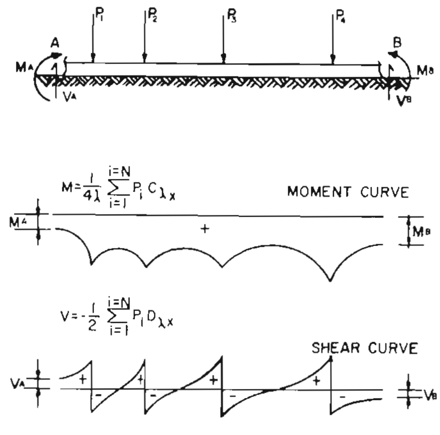


FIGURE (2)
BEAM ON ELASTIC FOUNDATIONS (INFINITE LENGTH ASSUMED) ACTED UPON BY A SERIES OF CONCENTRATED LOADS

In the absence of a distributed load "q" and assuming a beam that has an infinite length acted upon by one concentrated load "P", the solution to Eq. (1) becomes (with origin of coordinates considered at the point of load application);

$$y = \frac{P\lambda}{2K_sB}\,e^{-\lambda x}\,(\cos\lambda x+\sin\lambda x) \qquad (2)$$

$$\lambda = \sqrt[4]{\frac{K_s\,B}{4EI}}$$

The constant λ is termed the characteristic length of the beam and has the units of (1/unit length). By successively differentiating "y" with respect to "x" three times one obtains expressions for the slope, moment and shear at any point along the beam as follows:

$$\frac{dy}{dx} = \theta = -\frac{P\lambda}{BK_s}\,e^{-\lambda x}\,(\sin\lambda x) \qquad (3)$$

$$-EI\,\frac{d^2y}{dx^2} = M = \frac{P}{4\lambda}\,e^{-\lambda x}\,(\cos\lambda x-\sin\lambda x) \qquad (4)$$

$$-EI\,\frac{d^3y}{dx^3} = V = -\frac{P}{2}\,e^{-\lambda x}\,(\cos\lambda x) \qquad (5)$$

Eqs. (2), (3), (4) and (5) involve the first power of the column load "P" and it is clear therefore, that the effect of several concentrated loads applied at different points on the beam, can be obtained by direct superposition of the separate effect of individual concentrated loads as obtained from Eqs. (2), (3), (4) and (5) See Fig. 2.

3. Correction for Finite Length Effect

Let a continuous footing having a finite length extending between two points "A" and "B" be analyzed according to the procedure discussed above (i.e. assuming it has infinite length). This procedure will result in moments and shears (M_a, V_a) and (M_b, V_b) at both ends "A" and "B" respectively (Fig. 2).

Since the actual beam should have zero moments and shears at both free ends we now require that M_a, V_a, M_b, V_b, be reduced to zero. If this is achieved the net effect between points "A" and "B" on the infinitely long beam will be that of a beam of finite length extending between "A" and "B". To achieve this purpose additional end conditioning forces (M_{oa} P_{oa}) and (M_{ob}, P_{ob}) should be applied at points "A" and "B" respectively on the infinitely long beam.

The magnitudes of these end conditioning forces are computed as follows: At each end "A" and "B" the sum of all moments and shears due to the applied concentrated column loads (Fig. 2) and due to the added and conditioning forces (Fig. 3) should add up to zero. Thus, four conditions are available to compute the four unknown end conditioning forces:

$$M_a + \frac{P_{oa}}{4\lambda} + \frac{P_{ob}}{4\lambda} C_{\lambda l} + \frac{M_{oa}}{2} - \frac{M_{ob}}{2} D_{\lambda l} = 0 \quad (6)$$

$$V_a - \frac{P_{oa}}{2} + \frac{P_{ob}}{2} D_{\lambda l} - \frac{\lambda M_{oa}}{2} - \frac{\lambda M_{ob}}{2} A_{\lambda l} = 0 \quad (7)$$

$$M_b + \frac{P_{oa}}{4\lambda} C_{\lambda l} + \frac{P_{ob}}{4\lambda} + \frac{M_{oa}}{2} D_{\lambda l} - \frac{M_{ob}}{2} = 0 \quad (8)$$

$$V_b - \frac{P_{oa}}{2} D_{\lambda l} + \frac{P_{ob}}{2} - \frac{\lambda M_{oa}}{2} A_{\lambda l} - \frac{\lambda M_{ob}}{2} = 0 \quad (9)$$

The four equations can be solved simultaneously for P_{oa}, P_{ob}, M_{oa}, M_{ob}. The separate effects of these new forces are computed (Fig.3) and superimposed on the results of (Fig. 2) The results will be that of a finite beam with two free ends.

4. COMPUTER PROGRAM

The program performs six different interdependent tasks. These are indicated on the General Flow Chart (Fig. 4) and will be described below.

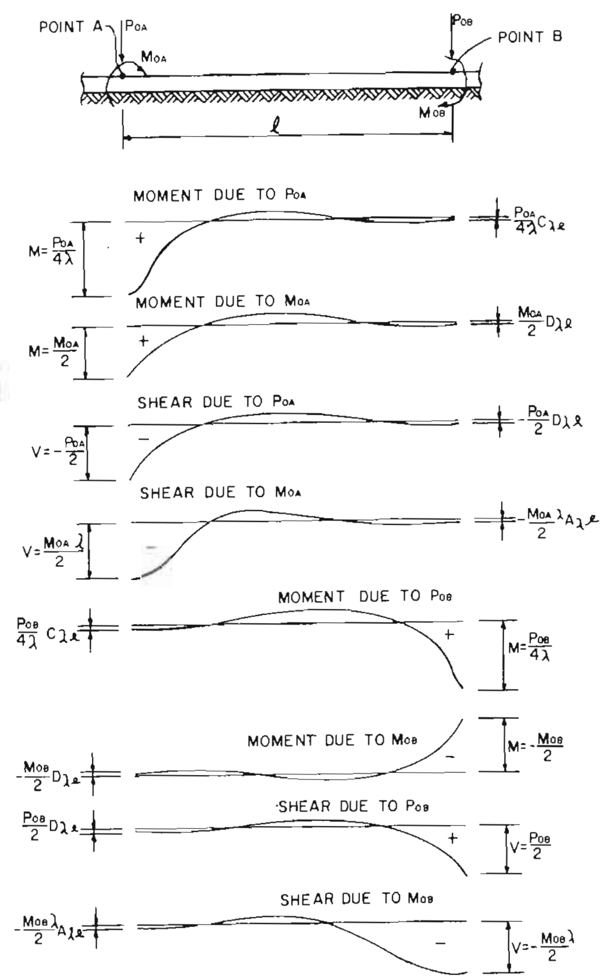


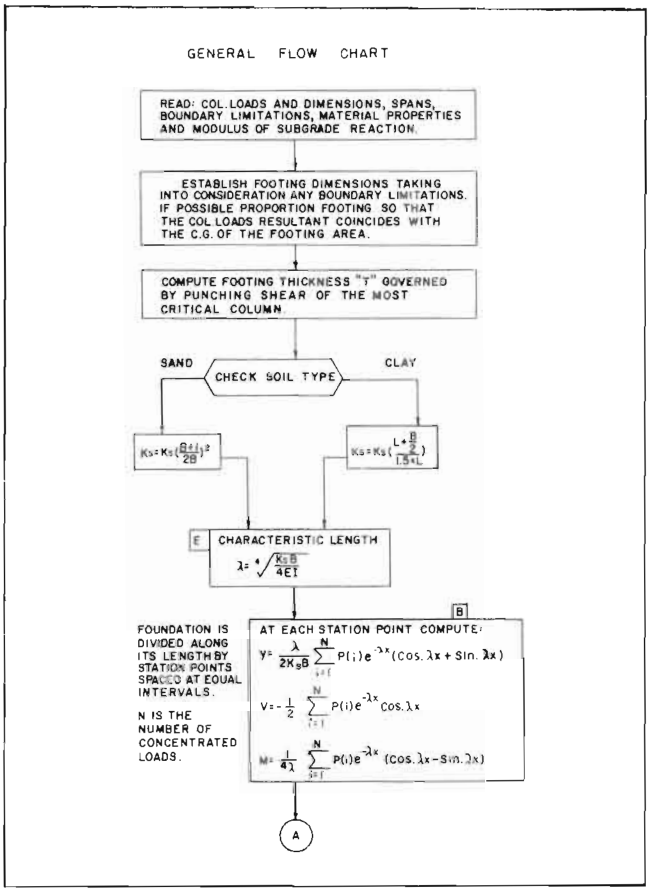
FIGURE (3)

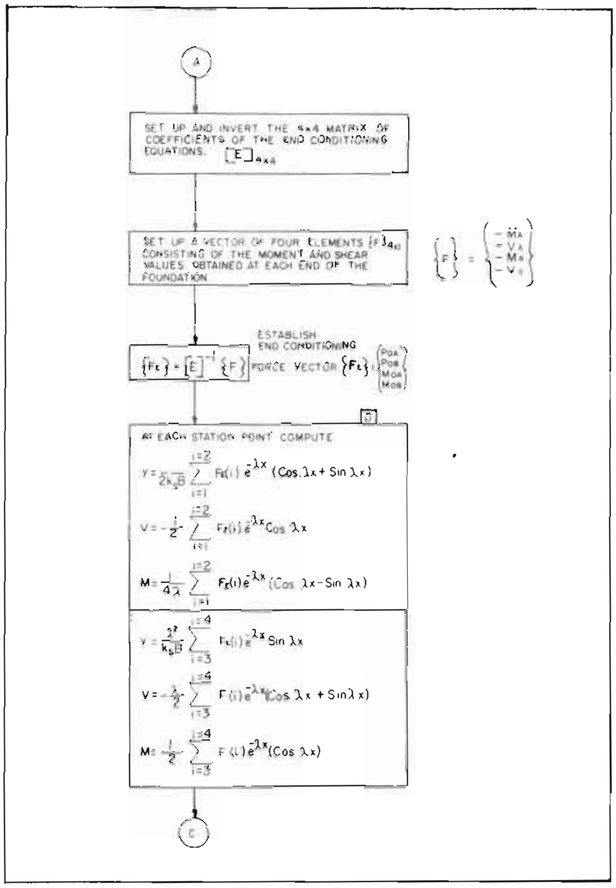
SEPARATE EFFECTS OF END CONDITIONING FORCES

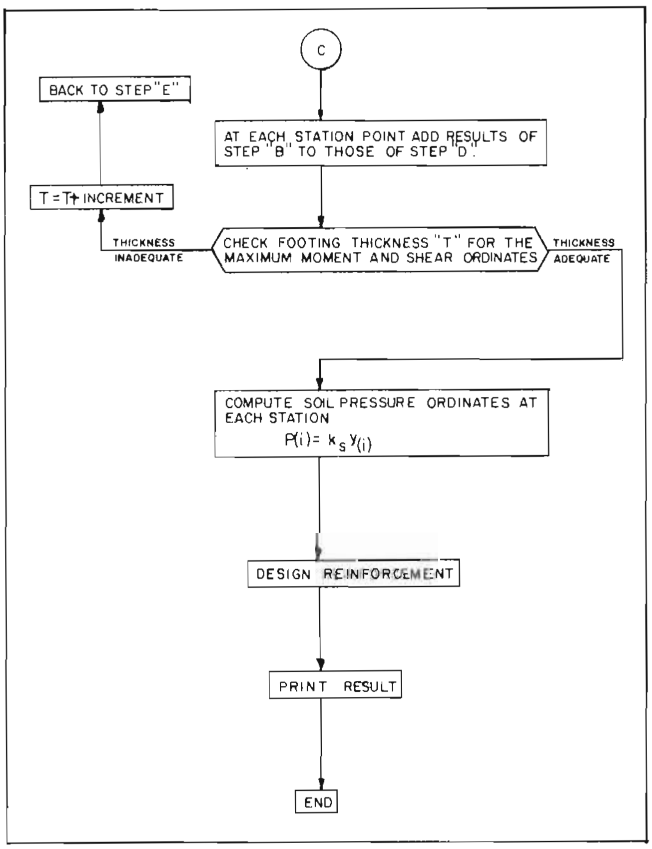
1. Read all input data and establish the dimensions of the footing considering any boundary limitations. If possible the footing is proportioned such that the resultant of column loads coincides with the center of gravity of the footing area.

2. Compute footing thickness as governed by the allowable punching shear stress at the most critical column.

3. With a footing thickness established from step 2, the characteristic length λ is computed. For the purpose of analysis the foundation length is divided by (m + 1) stations into "m" number of equal segments. Deflections, moments and shears, due to each one of the applied column loads are computed from Eqs. (2), (4) and (5) at each one of these stations







and at the end of the footing in case it does not coincide with a station point. Each time the point of application of the column load in question is considered the origin of coordinates. The effects of all column loads at each station are added, the assumption being now is that of an infinite length beam.

4. Eqs. (6), (7), (8) and (9) can be written in Matrix Form as:

$$\begin{Bmatrix} \dfrac{1}{4\lambda} & \dfrac{C_{\lambda l}}{4\lambda} & \dfrac{1}{2} & -\dfrac{D_{\lambda l}}{2} \\ -\dfrac{1}{2} & \dfrac{D_{\lambda l}}{2} & -\dfrac{\lambda}{2} & -\dfrac{\lambda A_{\lambda l}}{2} \\ \dfrac{C_{\lambda}}{4\lambda} & \dfrac{1}{4\lambda} & \dfrac{D_{\lambda l}}{2} & -\dfrac{1}{2} \\ -\dfrac{D_{\lambda l}}{2} & \dfrac{1}{2} & -\dfrac{\lambda A_{\lambda l}}{2} & \dfrac{\lambda}{2} \end{Bmatrix} \begin{Bmatrix} P_{oa} \\ P_{ob} \\ M_{oa} \\ M_{ob} \end{Bmatrix} = \begin{Bmatrix} -M_a \\ -V_a \\ -M_b \\ -V_b \end{Bmatrix}$$

or concisely:

$$[E] \quad \{F_E\} \quad = \quad \{F\}$$

inverting (E) one can write:

$$\{F_E\} = [E]^{-1} \quad \{F\}$$

with the elements of the $\{F_E\}$ vector known, (P_{oa}, M_{oa}) and (P_{ob}, M_{ob}) are now applied at points "A" and "B" of the infinite beam respectively. Their separate effects are computed as in step 3, at each station and their total effect at each station is added to the results of step 3. The results now reflect the behavior of the actual foundation.

5. The biggest moment ordinate is now picked and the footing thickness is checked for it. If the thickness is inadequate it is incremented and control returns to step 3.

6. As a final step, soil pressure ordinates at all station points are computed from the relationship:

$$p = K_s y$$

and printed out along with moment, shear and deflection ordinates at each station point. Steel reinforcement is computed under each column and at the point of maximum moment within each span. When a column load does not coincide with a station point, interpolation is required between the two adjacent stations to establish ordinates at the column point.

The computer program prepared by the writer consists of one mainline program that calls in six different subroutines to perform the six different steps outlined above. The two subroutines that perform steps (3) and (4) will require auxiliary storage in case of a limited core capacity. Auxiliary storage enables the program to handle footings of almost any length with a small chosen segment length. Accuracy of the results are of course independent of the length of the segment chosen, but it may be desirable sometimes to obtain results at closely spaced station points.

While the program is mainly designed for foundations continuous in one direction only (i.e. supporting a group of columns along one line) it can be applied with reasonable accuracy for the analysis and design

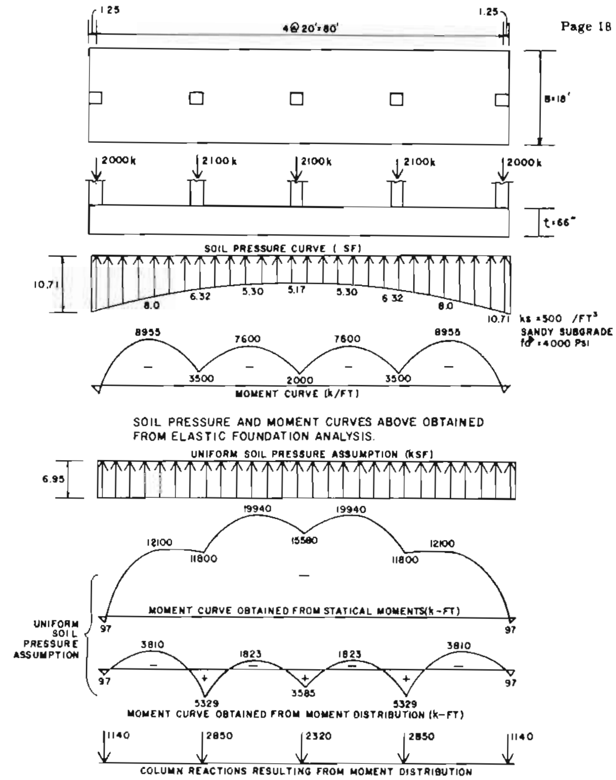


FIGURE (5)
EXAMPLE SHOWING COMPARISON OF THREE METHODS OF ANALYSIS OF A RELATIVELY RIGID CONTINUOUS FOOTING WITH NO OVERHANGS BEYOND EXTERIOR COLUMNS

of mat foundations. Each bay of the mat in each direction supporting any one column line can be treated independently as before.

5. SOLVED EXAMPLES

On Fig. 5 and 6 are shown two examples of two combined footings supporting identical column loads. The footing on Fig.5 does not extend beyond the edge of the exterior columns (as may be required by some boundary restriction). In spite of the relative rigidity of the footing (t=5.5ft) the soil pressure distribution pattern obtained from the elastic foundation analysis is not a uniform one. The deflected shape of the footing is shown to be concave down with no inflection points. Flexural tension is caused on top fibres only with increasing magnitudes in-between columns. Maximum soil pressure ordinates are mobilized at both ends where maximum deflection occurs and then decrease towards the middle. This result is characteristic of continuous footings which are not allowed to extend beyond the edge of exterior columns and when the exterior column load exceeds in magnitude about 40% of the typical interior column load (all spans being approximately equal).

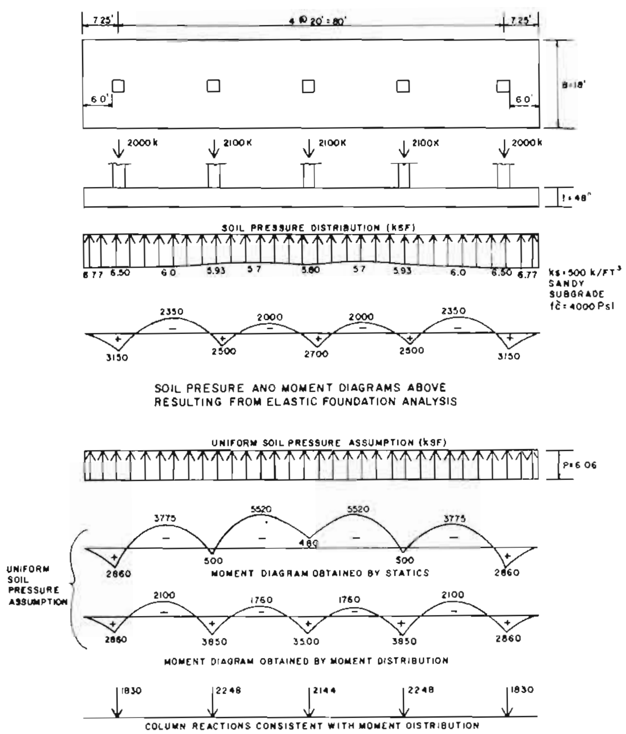


FIGURE (6)
EXAMPLE SHOWING COMPARISON OF THREE METHODS OF ANALYSIS OF A RELATIVELY RIGID CONTIUOUS FOOTING WITH OVERHANGS BEYOND EXTERIOR COLUMNS

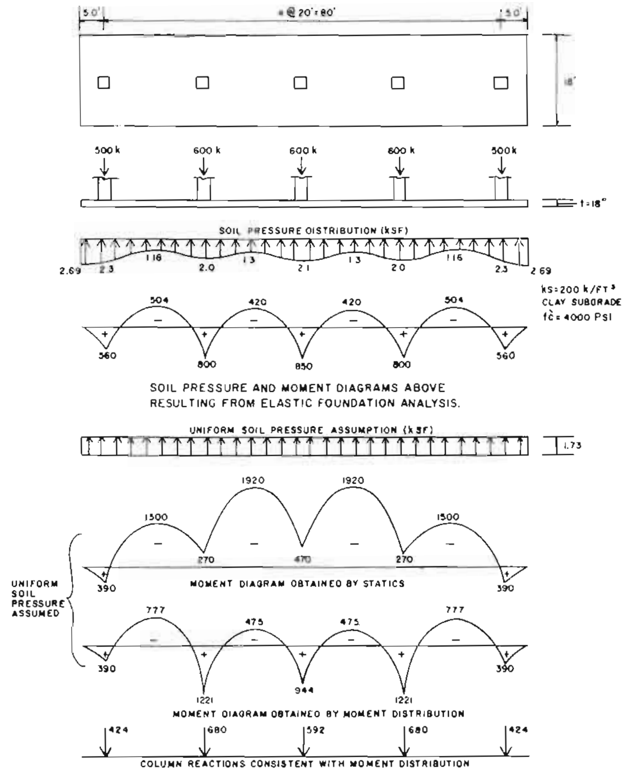


FIGURE (7)
EXAMPLE SHOWING COMPARISON OF THREE METHODS OF ANALYSIS OF A RELATIVELY NON-RIGID CONTINUOUS FOOTING WITH OVERHANGS BEYOND EXTERIOR COLUMNS.

The same footing when solved by statical moments assuming uniform soil pressure distribution yields again a concave downward deflected shape with no inflection points, but the moment ordinates become much higher than before. An inconsistency is observed here in that a uniform soil pressure distribution in reality is not compatible with a concave deflected shape. An indeterminate analysis by moment distribution with a uniform soil pressure diagram yields a completely erroneous result which is not even compatible with the actual column loads.

Fig. 6 shows the same footing extending 6 feet beyond both exterior columns. The general deflected shape is still concave down, but not as severely as before. Localized curvatures within each span are now more distinct because of the decreased footing rigidity (t = 4 feet) thus, higher soil pressure ordinates are now mobilized directly under the columns than within each span. This increased localized span curvature is also reflected on the moment diagram where the moment now reserves sign under each column.

The third example of Fig. 7 shows a footing 18 inches thick supporting lighter column loads. The elastic deformation of individual spans is now very clear. Most of the resisting soil pressure volume is concentrated under the columns directly.

6. Conclusions

In analyzing mat or continuous foundations it is mostly unjustified to assume a uniform soil pressure distribution pattern, even in the case of relatively rigid foundations. When boundary restrictions do not permit the extension of a foundation beyond the edge of exterior columns a uniform soil pressure distribution assumption should be discarded and the elastic analysis should be performed. Where overhangs are possible, moment distribution will only give the general trend of the moment curve. Rough agreement of moment distribution results with the elastic analysis is usually obtained in interior spans which are far enough from edge effects. The agreement between column reactions obtained from moment distribution method and the actual column loads can be considered as a measure of the accuracy of the indeterminate analysis procedure. On the other hand, the method of statical moments results in moments which are unrialistically high because of the unrealistic uniform soil pressure assumption. In any cause, a rigorous analysis based on the elastic foundation theory should be indispensible in important projects and irregular situations.

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