سلوك الخيوط على البكرات المدارة

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يشتمل هذا البحث على دراسة الخيوط التي تلامس البكرات بزاوية ميل عند دخولها وخروجها الى ومن البكرات ، وقد تم التوصل الى المعادلات التفاضلية اللاخطية التي تمثل الحركة ، واستخدمت الطرق العددية لحل هذه المعادلات.

وحصلت النتائج في هذا البحث باستخدام الحاسب الألكتروني – وهي تمثل طريقة التغير من الشد وشكل تواجد الخيط على البكرات .

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THE BEHAVIOUR OF THREADS OVER ROTATING ROLLS

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A study for threads entering or leaving rotating rolls with some inclination is presented. Ordinary non-linear equations representing the behaviour of threads were deduced. Numerical method was used for solving the equations. The results presented in this work were obtained by using digital computer. The thread distribution and the tension variation is presented.

Nomenclature

$\vec{A}$	=	acceleration
V	=	speed of thread
v	=	peripheral speed of the roll
r, θ ,z	=	cylindrical coordinates
$\vec{u}_r$, $\vec{u}_\theta$, $\vec{k}$	=	unit vectors in cylindrical coordinates
R	=	radius of drum
T	=	tension
ρ	=	mass per unit length
μ	=	coefficient of friction

1. Introduction

In textile industries, thread lines move at very high speed (over 3000 meter/min) from spinning to wind up. During the process, the thread passes over several rolls that makes it change direction. The thread enters and leaves the roll with some inclination. Obviously, the location of the thread on the roll is not in one plane, and Euler equation cannot be applied. For this reason a different approach should be evaluated.

In spinning processes the speed of the thread line is constant. However, when the thread is in contact with a roll there are constraint forces which induce acceleration. At the same time the forces affect the physical properties of the thread. The purpose of this work is to derive equations representing the behaviour of the thread at the entrance and exit region of a rotating roll, then to find their proper solution; finally, to show how the thread is laid on the roll, and how different factors affect the tension distribution along the thread.

2. Equations of Motion:

2.1 Acceleration of a thread particle:

The acceleration of a thread particle moving with a constant speed, in cylindrical coordinates, is given by (See Appendix)

$$\vec{A} = \frac{V^2}{(r^2+\dot{r}^2+\dot{z}^2)}\Big[\,[\,r^2(\ddot{r}-r) - r(2\dot{r}^2+\dot{z}^2) - \dot{r}\,\dot{z}\,\ddot{z}$$

$$+\ \ddot{r}\dot{z}^2]\,\vec{u}_r + [r\,(r\dot{r} - \dot{r}\ddot{r} - \dot{z}\ddot{z})$$

$$+\ 2\dot{r}\,(\dot{r}^2+\dot{z}^2)\,]\,\vec{u}_\theta + [\,r\,(r\ddot{z} - \dot{r}\dot{z})$$

$$+\ \dot{r}\,(\dot{r}\ddot{z} - \ddot{r}\dot{z})\,]\,\vec{k}\,\Big] \qquad (1)$$

where the dot means differentiation with respect to θ. The acceleration of a particle moving on the circumference of a roll is obtained by substituting into Eq.

(1) the following conditions:

$$r = R = \text{constant}$$

$$\dot{r} = \ddot{r} = 0$$

$$x = R\theta$$

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Thus

$$\vec{A} = \frac{V^2}{(1+z'^2)^2}\left[-\frac{1+z'^2}{R}\vec{u}_r\right.$$

$$\left. - z'z''\vec{u}_\theta + z''\vec{k}\right] \quad (2)$$

where the dash means differentiation with respect to x.

2.1 Forces acting on an element

Consider a thread element as shown in Fig. 1. It is subjected to tensions at both ends, the normal reaction dN, and the friction force μdN. The direction of the friction force depends upon the velocity of the thread relative to the roll, (see Fig. 2). The unit vector $\vec{u}$ parallel to the friction force is

$$\vec{u} = \frac{1}{\sqrt{(v\sqrt{1+z'^2} - V^2) + V^2z'^2}} \times$$

$$[(v\sqrt{1+z'^2} - V)\vec{u}_x - Vz'\vec{k}] \quad (3)$$

$\vec{u}_x$ is a unit vector tangent to the roll and is normal to its axis. The component of the tension along $\vec{u}_x$, and $\vec{k}$ are $-\frac{T}{\sqrt{1+z'^2}}$ and $-\frac{Tz'}{\sqrt{1+z'^2}}$ respectively. The resultant of the tension components are denoted by dT_r, dT_x, and dT_z and are given by:

$$dT_r = -\frac{T}{\sqrt{1+z'^2}}\,dx$$

$$dT_x = \frac{\partial}{\partial x}\left(\frac{T}{\sqrt{1+x'^2}}\right)dx = \left(\frac{T'}{\sqrt{1+z'^2}} - \frac{T\,z'\,z''}{(1+z'^2)^{3/2}}\right)dx$$

$$dT_z = \frac{\partial}{\partial x}\left(\frac{Tz'}{\sqrt{1+z'^2}}\right)dx = \left(\frac{T'\,z'}{\sqrt{1+z'^2}} + \frac{Tz''}{(1+z'^2)^{3/2}}\right)dx$$

$$(4)$$

2.3 Equations of motion

Applying Newton's second law and making use of Eqs. (2), (3) and (4), then

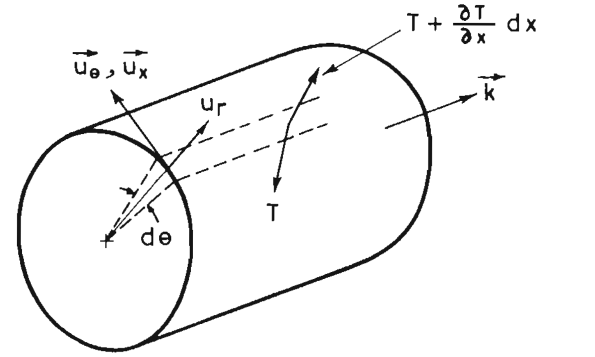

FIG. 1 POSITION OF AN ELEMENT ON THE ROLL

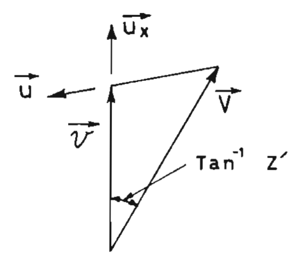

FIG. 2 DIRECTION OF THE FRICTION FORCE

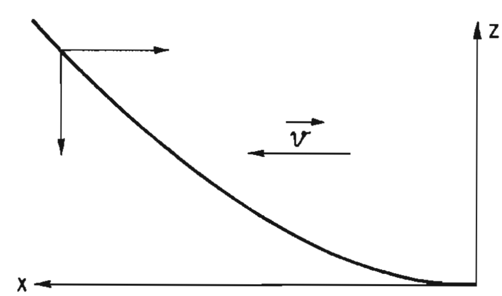

FIG. 3 CHANGE OF COORDINATES

$$dN - \frac{T}{\sqrt{1+z'^2}}\frac{dx}{R} = - \frac{\rho \quad V2}{R\sqrt{1 + z'^2}}dx \quad (5)$$

$$\left(\frac{T'}{\sqrt{1+z'^2}} - \frac{T\, z'\, z''}{(1+z'^2)^{3/2}}\right)dx$$

$$+ \mu dN \frac{v\sqrt{1+z'^2} - V}{\sqrt{(v\sqrt{1+z'^2} - V)^2 + V^2 z'^2}}$$

$$= - \frac{\rho V^2\, z'^2\, z''}{(1 + z'^2)^{3/2}}dx \quad (6)$$

$$\left(\frac{T'\, z'}{\sqrt{1 + z^2}} + \frac{T\, z''}{(1+z'^2)^{3/2}}\right)dx$$

$$- \mu dN \frac{V\, z'}{\sqrt{(v\sqrt{1+z'^2} - V)^2 + V^2 z'^2}} = \frac{\rho V^2\, z''}{(1+z'^2)^{3/2}}dx \quad (7)$$

From Eqs. (5), (6), and (7) it can be shown that

$$z'' = \frac{\mu z' v}{R}\sqrt{\frac{1 + z'^2}{(v\sqrt{1+z'^2} - V)^2 + V^2 z'^2}} \quad (8)$$

$$T' = \frac{\mu(T - \rho V^2)\left(V - \frac{v}{\sqrt{1 + z'^2}}\right)}{R\sqrt{(v\sqrt{1+z'^2} - V)^2 + V^2 z'^2}} \quad (9)$$

Let $1 + z'^2 = y^2$ then

$$y' = \frac{\mu v\,(y^2 - 1)}{R\sqrt{(V^2 + v^2)\, y\text{-}2V\, v\, y}} \quad (10)$$

$$T' = \frac{\mu\,(T - \rho V^2)\,(Vy - v)}{Ry\sqrt{(V^2 + v^2)\, y^2 - 2V\, v\, y}} \quad (11)$$

3. Algorithm for the numerical solution

The inlet tension and the angle of inclination are specified. The circumference of the roll is divided to small intervals each of length h. An interative scheme [1] is used. The starter values are obtained by using the formulas

$$Y'_{-1} = Y'_{\circ} - h\, Y''_{\circ}$$
$$Y'_{1} = Y'_{\circ} + h\, Y''_{\circ}$$
$$Y_{-1} = Y_{\circ} - \frac{h}{24}(Y'_{1} + 16\, Y'_{\circ} + 7Y'_{-1}) + \frac{h\, Y''_{\circ}}{4}$$
$$Y_{1} = Y_{\circ} + \frac{h}{24}(7Y'_{1} + 16\, Y'_{\circ} + Y'_{-1}) + \frac{h^2\, Y''_{\circ}}{4}$$

The values of Y_{-1} and Y_1 can be iterated to any degree of accuracy. Convergence is assured when h is less than (3/F), F is the largest value of Y'' in the region.

The main calculations are carried out by using the formula

$$Y_{n+1} = Y_{n-1} + \frac{h}{3}(Y'_{n+1} + 4\, Y'_{n} + Y'_{n-1})$$

The truncation error in the algorithm is in the order of $\frac{h^5\, Y^{(5)}}{90}$.

4. Results

The mathematical model was obtained for positive roll and thread velocities as indicated in Fig. 2. This situation represents the behaviour of the thread during the exit region. The results were obtained for two cases. In the first, the roll is pulling the thread and its circumferencial speed is slightly larger than the speed of the thread. In the second case, the speed of the thread is slightly larger which represents the thread pulling the roll.

At the beginning of the exit region, the slope of the thread is zero which satisfies the differential equations. When the boundary conditions are taken at this position the numerical results show no change in the thread location which is trivial . Thus it is necessary to transfer the coordinates as shown in Fig. 3 first, to overcome this difficulty, and second, because the boundary conditions are always stated at the end of the region.

The tension distribution when the roll is pulling the thread is shown in Fig. 8. For the case when the thread is pulling the roll the tension distribution is

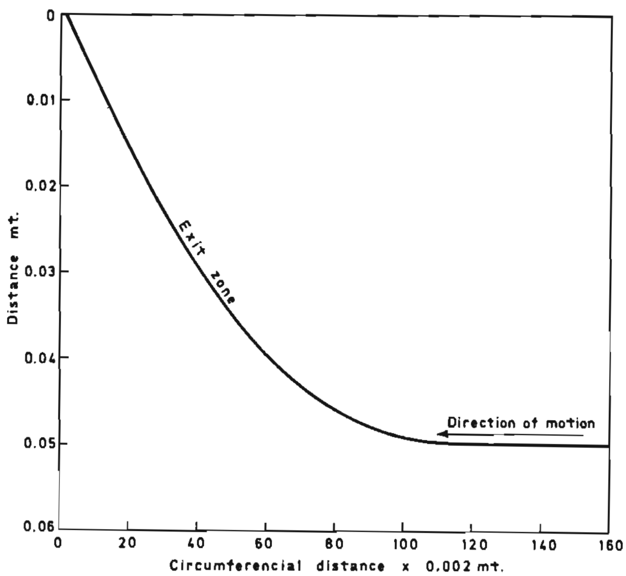

FIG. 4 LOCATION OF THREAD IN EXIT ZONE

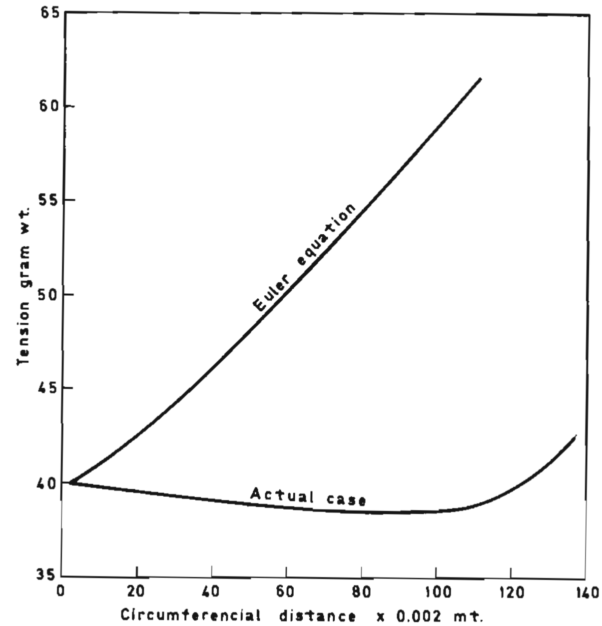

FIG. 8 TENSION DISTRIBUTION – ROLL IS PULLING THE THREAD

shown in Fig. 9. For both cases the tension distribution for a thread normal to the roll axis (represented by Euler equation [2] is indicated for comparison. The effect of the speed on the tension distribution is shown in Figs. 10 and 11. Fig. 12 shows the effect of the coefficient of friction upon the tension distribution. The effect of mass per unit length on the value of tension at the beginning of the exist zone is shown in Fig. 13.

5. Discussion

The location of the thread on the roll is shown in Fig. 4. This location is represented by Eq. (10), and is affected by the thread and the roll circumferencial speeds, the coefficient of friction, and the radius of the roll. Since the speed of the thread and the circumferencial speed of the roll are nearly equal, the term in Eq. (10) containing these speeds is approximately equal to one. This explains why the thread distribution is not affected by the magnitude of the velocities.

As the coefficient of friction increases, the exit zone is reduced, and the location of the thread varies as shown in Fig.5. The relation between the coefficient of friction and the arc angle of the exit zone is illustrated in Fig . 6. The curve representing this relation becomes less steep with the coefficient of friction.

According to the results obtained the arc length of the exit zone is directly proportional with the radius of the roll, and the arc angle is constant. This can be easily explained from Eq. (10). If the derivative of the left hand side of Eq. (10) is made with respect to θ rather than with respect to x, the location of the thread is not affected by the radius of the roll. This means that arc angle of the zone is constant. At the same time the arc length is proportional to the radius of the roll.

As was mentioned in the previous section, positive values of thread and roll circumferential velocities represent the behavior of the thread in the exit zone. For the inlet region, the velocities can be assumed negative. In this situation, the behavior of the thread is rather peculiar. Mathematically speaking, the slope of the thread decreases with x as can be concluded from Eq. (10). If the thread approaches the roll with some inclination, its location will be similar to curve a b c, Fig. 7. However, after some point b, a thread element cannot be in equilibrium under the effect of tensions at both ends, and the friction force. The element is accelerated in the direction of the resultant until the thread finally becomes normal to the axis of the roll at point d. The region b d cannot be represented by Eqs. (10), and (11) since the thread does not have a uniform velocity.

The tension distribution in the thread in the exit zone is shown in Figs. 8, and 9. When the thread is normal to the axis of the roll the tension distribution follows Euler equation i.e. decreases when the roll is pulling the thread, Fig. 8, and increases when the thread is pulling the roll, Fig. 9. During the exit zone the tension increases in both cases but with a small rate as illustrated. However, there is a substantial difference between the tension distribution compared to that of a thread normal to the axis of the roll. When the roll is pulling the thread, the rate of change of tension becomes zero when $z' = \sqrt{\frac{v}{V} - 1}$ as given by Eq. (11). This is true because v is larger than V.

The magnitude of the velocities affect the tension distribution as shown in Figs. 10, and 11. When the thread is pulling the roll, Fig. 11, the curves become less steep as the speeds increase. This already can be concluded from Eq.(11). On the other hand, when the roll is pulling the thread, Fig. 10, the curves have minimum values at $z' = \sqrt{\frac{v}{V} - 1}$. The position of the minimum tension depends upon the difference between v, and V. In this work a difference of 0.1 m./sec was considered only for demonstration. Hence, there is a distinct difference in the position where the minimum tension occurs specially at small speeds. This difference decreases as the speeds are increased.

Fig. 12 shows the effect of the coefficient of friction on the tension distribution when the roll is pulling the thread. It is clear that as the coefficient of friction increases, a sharp drop in tension results. However, the minimum value of tension is the same for all values of coefficient of friction.

The value of tension at the beginning of the exit zone is proportional to the mass per unit length as shown in Fig. 13. The change is very small. In the analysis, the mass per unit length was considered constant and not affected by the tension variation. However, if an exact model is to be presented, the variation in the mass per unit length due to the change in tension will contribute a modification in the value of tension by less than 0.001% which is negligible.

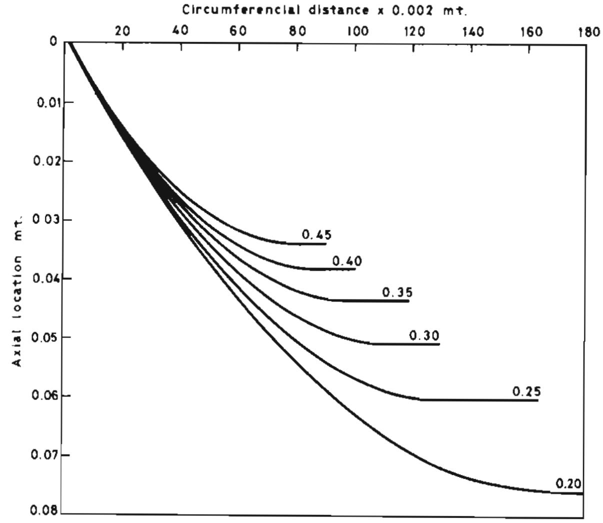

FIG. 5 EFFECT OF COEFFICIENT OF FRICTION ON THREAD LOCATION

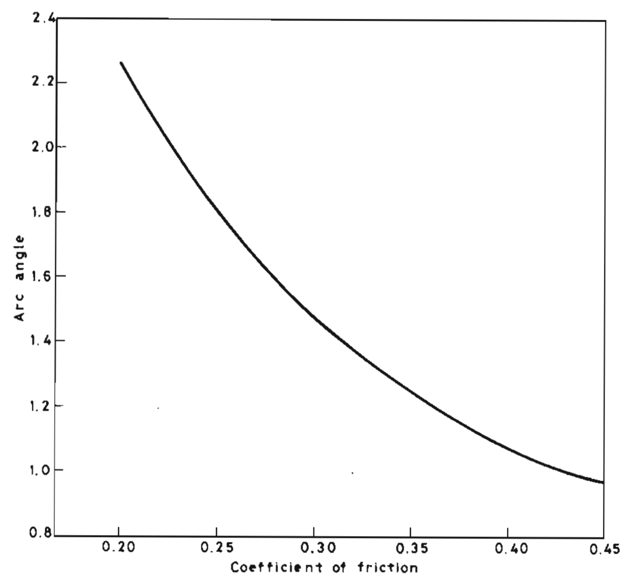

FIG. 6 EFFECT OF COEFFICIENT OF FRICTION ON ARC ANGLE OF EXIT ZONE

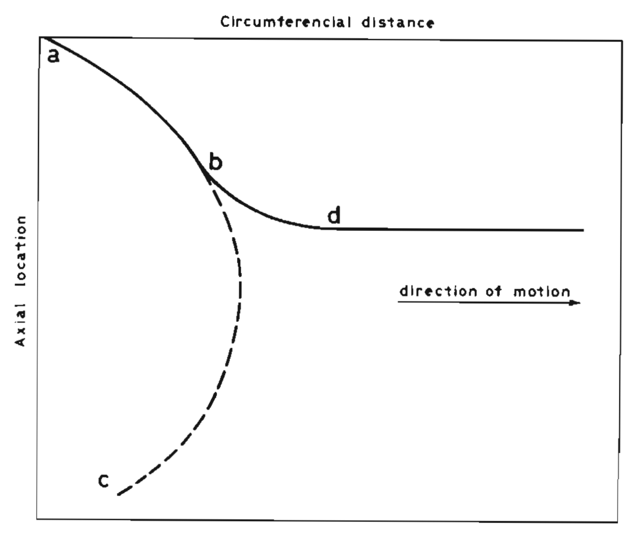

FIG. 7 LOCATION OF THREAD IN INLET ZONE

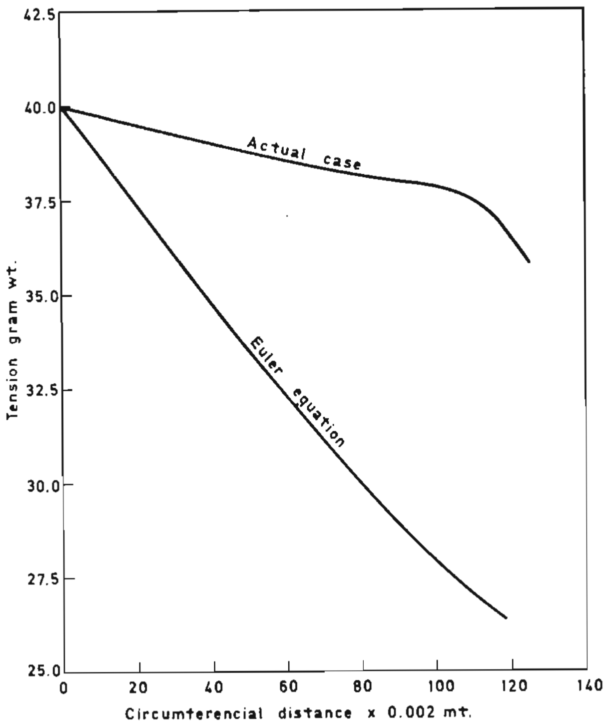

FIG. 9 TENSION DISTRIBUTION – THREAD IS PULLING THE ROLL

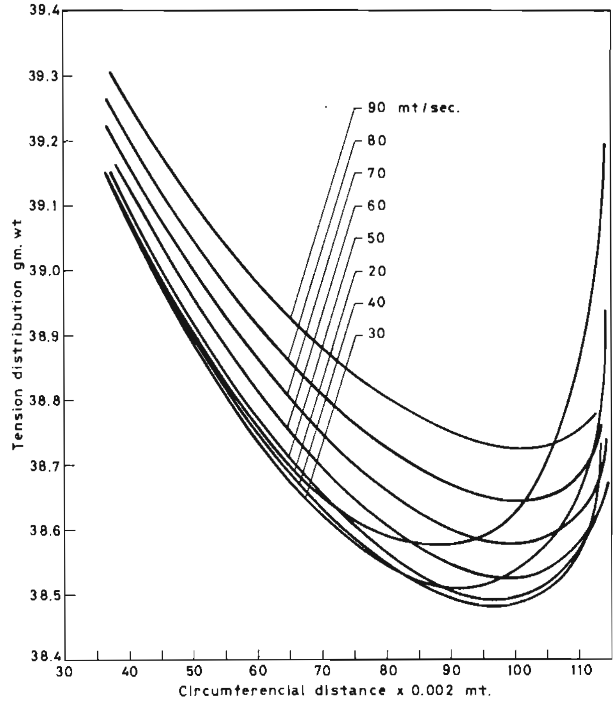

FIG. 10 EFFECT OF SPEED ON TENSION DISTRIBUTION ROLL IS PULLING THE THREAD

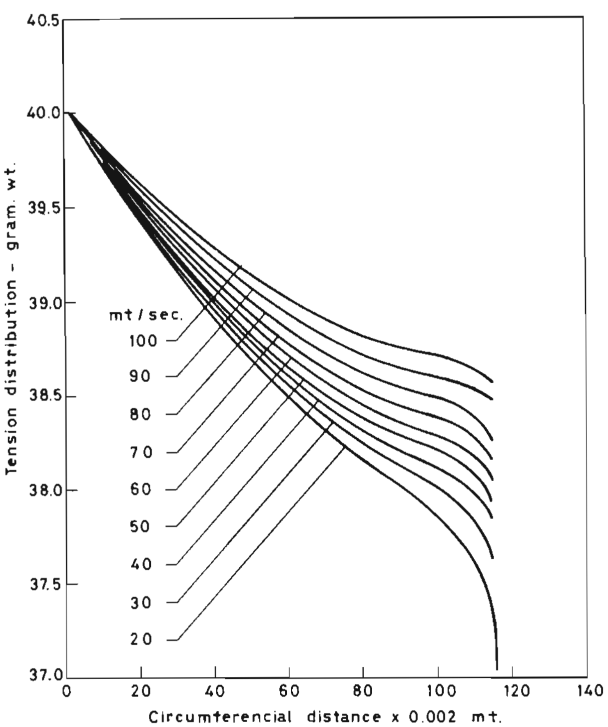

FIG. 11 EFFECT OF SPEED ON TENSION DISTRIBUTION–THREAD IS PULLING THE ROLL

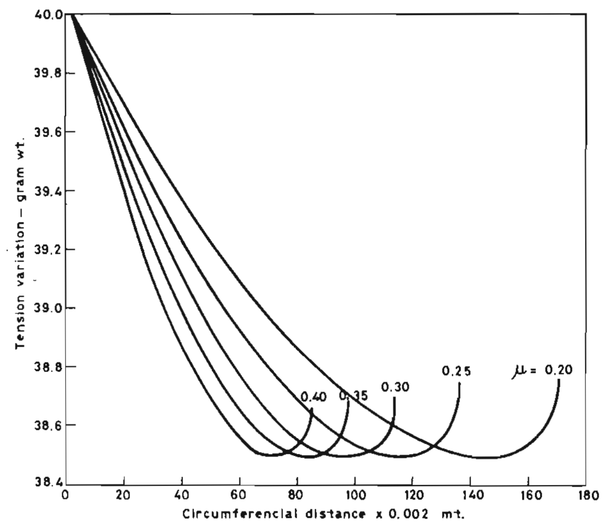

FIG. 12 EFFECT OF THE COEFFICIENT OF FRICTION ON TENSION DISTRIBUTION

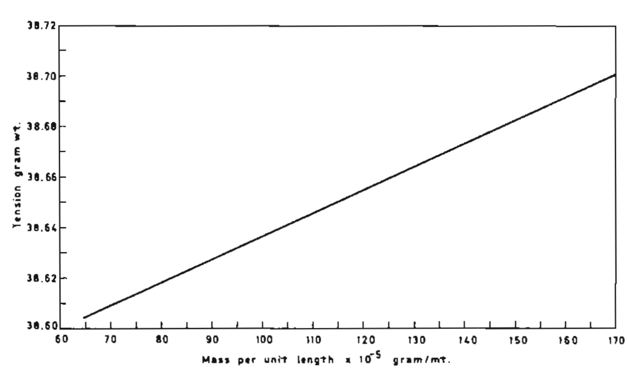

FIG. 13 EFFECT OF MASS PER UNIT LENGTH ON TENSION AT BEGINNING OF EXIT ZONE

APPENDIX

The position vector, $\vec{S}$, of a point in cylindrical coordinates is

$$S = r\,\vec{u}_r + z\,\vec{k}$$

The velocity is given by

$$\vec{V} = \frac{dr}{dt}\,\vec{u}_r + r\frac{d\theta}{dt}\,\vec{u}_\theta + \frac{dz}{dt}\,\vec{k}$$

$$= (\dot{r}\,\vec{u}_r + r\,\vec{u}_\theta + \dot{z}\,\vec{k})\,\frac{d\theta}{dt}$$

Where the dot means differentiation with respect to θ

$$V = \sqrt{\dot{r}^2 + r^2 + \dot{z}^2}\,\frac{d\theta}{dt}$$

$$\frac{d\theta}{dt} = \frac{V}{\sqrt{\dot{r}^2 + r^2 + \dot{z}^2}}$$

Thus

$$\vec{V} = (\dot{r}\,\vec{u}_r + r\,\vec{u}_\theta + \dot{z}\,\vec{k})\,\frac{V}{\sqrt{\dot{r}^2 + r^2 + \dot{z}^2}}$$

The acceleration is

$$\vec{A} = \vec{V}\,\frac{d\theta}{dt}$$

$$= (\ddot{r}\,\vec{u}_r + \dot{r}\vec{u}_\theta + \dot{r}\vec{u}_\theta - r\vec{u}_r + \ddot{z}\vec{k})\,\frac{V}{\sqrt{\dot{r}^2 + r^2 + \dot{z}^2}}$$

$$+ (\dot{r}\,\vec{u}_r + r\,\vec{u}_\theta + \dot{z}\,\vec{k})\left[-\frac{V(\dot{r}\ddot{r} + r\dot{r} + \dot{z}\ddot{z})}{(\dot{r}^2 + r^2 + \dot{z}^2)^{3/2}}\right] \times \frac{V}{\sqrt{\dot{r}^2 + r^2 + \dot{z}^2}}$$

After rearranging the terms, therefore

$$A = \frac{V^2}{(r^2 + \dot{r}^2 + \dot{z}^2)^2} \times$$

$$\Big[\left[(r^2(\ddot{r} - r) - r(2\dot{r}^2 + \dot{z}^2) - \dot{r}\dot{z}\ddot{z} + \ddot{r}\dot{z}^2)\right]\vec{u}_r$$

$$+ \left[r(r\dot{r} - \dot{r}\ddot{r} - \dot{z}\ddot{z}) + 2\dot{r}(\dot{r}^2 + \dot{z}^2)\right]\vec{u}_\theta$$

$$+ \left[r(r\ddot{z} - \dot{r}\dot{z}) + \dot{r}(\dot{r}\ddot{z} - \ddot{r}\dot{z})\right]\vec{k}\Big]$$

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(1) *Milne, W.E., Numerical Calculus, Princeton University Press, Princeton, New Jersey,* 1949.

(2) *Phelan, R.M., Fundamentals of Mechanical Design, McGraw-Hill,* 1962, *p.* 272.