الخطأ في قراءة أجهزة القياس الكهربائية نتيجة تعرضها للاهتزازات الميكانيكية

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يتناول هذا البحث أسلوب دراسة أثر الاهتزازات الميكانيكية على قراءات أجهزة القياس الكهربائية وذلك بتعريض السطح المثبت عليه الجهاز لاهتزازات ميكانيكية واعتبار أن الجزء المتحرك من جهاز القياس ذو ستة حريات للحركة وقد استعملت معادلات أويلر ولاقرانج لوضع معادلات وصف الحركة ثم استخدمت تحويلات (لابلاس) لحل مجموعة المعادلات الناتجة مع التجاوز في بعض التقريبات .

وقد أمكن التوصل الى نتيجة هامة وهي أن القراءة الثابتة للجهاز تزداد نتيجة لوقوع الجهاز تحت تأثير قوة الاهتزازات كما أمكن ايجاد معامل حسابي لتقدير هذه الزيادة في القراءة .

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ERROR IN READING OF ELECTRICAL MEASURING INSTRUMENTS DUE TO MECHANICAL VIBRATIONS

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The present work gives a proposed method for studying the effect of mechanical vibration on the reading of electrical measuring instruments. In this method the surface of fixation of the measuring instrument is exposed to mechanical vibrations and thus the moving part (or the measuring part) of the instrument is considered free to move with six degrees of freedom. The Euler-Lagrange formulation is used to describe the motion of the moving part. The Laplace Transformation is then used to solve the resulting set of equations with some reasonable approximations. It is found that the steady-state d-c reading of the instrument is increased due to the applied vibrating force. A factor evaluating this increase has been calculated.

1. Introduction

In recent years considerable attention has been given to the principles involved in the design and construction of electrical measuring instruments. A tremendous progress has been achieved in such a way that all the basic quantities, namely, voltage, current, impedance, frequency, and wavelength can now be measured all over the frequency spectrum via the use of suitably designed electrical instruments. Yet there remain many cases in which a special measuring equipment is needed. For example when it is required to measure an electrical quantity under the effect of a vibrating mechanical force, a specially designed instrument is needed in which the effect of this vibrating force is eliminated, or at least reduced.

2. Mathematical Analysis

In our analysis we will first derive the equations of motion of the moving part of the instrument. For simplicity, the moving part of the instrument will be represented by a rectangular box. The generalized system of coordinates will be used to describe the motion of this box. Each of the generalized coordinates will be independent of each other, that is the displacement in one direction does not produce any reaction in the other directions. As shown in Fig. 1. the faces of the box representing the moving part are taken parallel to the main planes of the cartesian system.

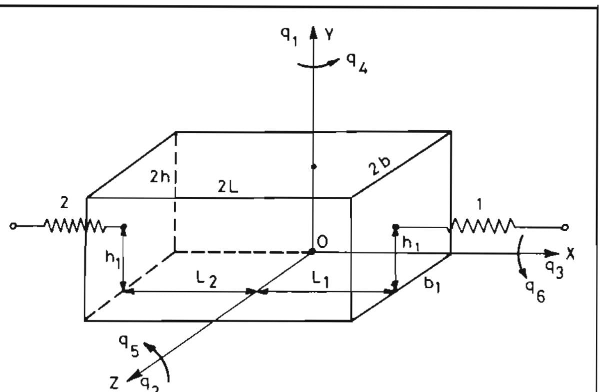

FIG. 1

The system with two springs in a general position can be replaced by three mutually perpendicular springs in the x, y, and z, directions. The springs used in the system are assumed to have constant stiffnesses in each of the above three directions. These components of the stiffnesses are denoted by

$$k_{1x} \quad , \quad k_{1y} \quad , \quad k_{1z}$$
$$k_{2x} \quad , \quad k_{2y} \quad , \quad k_{2z}$$

where the subscipts 1 and 2 denote the first and second springs respectively.

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For small displacement about the equilibrium position, the potential energy of the moving part can be written in a quadratic form [1,2]

$$P = \frac{1}{2} \sum_{i,j=1}^{6} b_{ij}\, q_i \, q_j \qquad (1)$$

where $q_i, i = 1, 2, 3, \ldots, 6$ are the generalized coordinates,

and $b_{ij}, i,j = 1, 2, 3, \ldots, 6$, are constant coefficients.

A similar expression can be obtained for the kinetic energy of the system

$$E = \frac{1}{2} \sum_{i,j=1}^{6} a_{ij}\, \dot{q}_i \, \dot{q}_j \qquad (2)$$

where $\dot{q}_i$, $i = 1, 2, 3, \ldots, 6$, are the generalized velocities, and a_{ij}, $i,j = 1, 2, 3, \ldots 6$ are constant coefficients containing the inertia properties of the system.

Since the potential energy of a spring under an extension x is $\int \frac{1}{2} kx^2$ the expression for the potential energy P of the system under consideration (see Fig.2) is

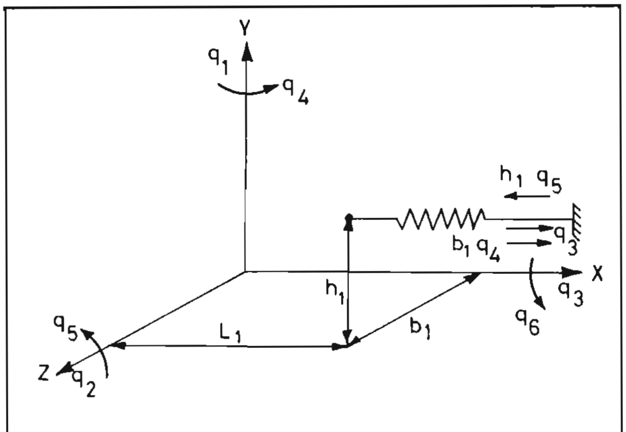

FIG. 2

$$P = \tfrac{1}{2}[k_{1x}(q_3 + b_1q_4 - h_1q_5)^2 + k_{2x}(q_3 + b_1q_4 + h_1q_5)^2 + k_{1y}(q_1 + b_1q_6 - L_1q_5)^2 + k_{2y}(q_1 - b_1q_6 - L_2q_5)^2 + k_{1z}(q_2 - h_1q_6 + L_1q_4)^2 + k_{2z}(q_2 - h_1q_6 - L_2q_4)^2\,] \quad (3)$$

This expression can be explained as follows: Consider spring 1 in Fig.2. It undergoes three extensions, the extension q_3, the extension b_1q_4 in the x-direction and the extension h_1q_5 in the negative x-direction. The other bracketed terms in Eq. (3) can be explained in similar manners. Note that the quantity in each bracket represents the extension in the corresponding direction with the second order terms neglected.

The kinetic energy E of the system consists of two parts. The first part is due to translation along the main axes, and the second part is due to rotation about these axes.

$$2\in = M(\dot{q}_1^2 + \dot{q}_2^2 + \dot{q}_3^2) + I_{xx}\,\dot{q}_6^2 + I_{yy}\,\dot{q}_4^2 + I_{zz}\,\dot{q}_5^2 - I_{xy}\,\dot{q}_6\,\dot{q}_4 - I_{xz}\,\dot{q}_5\,\dot{q}_6 - I_{yz}\,\dot{q}_4\,\dot{q}_5 \qquad (4)$$

Where M is the mass of the moving part and I_{xx}, I_{yy}, I_{zz}, I_{xy}, I_{xz} and I_{yz} are the principal moments of inertia about the main axes. The equation Euler-Lagrange of motion of the moving parts is given by,

$$\frac{\partial P}{\partial q_i} + \frac{d}{dt}\left(\frac{\partial E}{\partial \dot{q}_i}\right) = 0, i = 1,2,3,\ldots,6 \quad (5)$$

the equation can be written in the form

$$\frac{d}{dt}\left(\frac{\partial E}{\partial \dot{q}_i}\right) = -\frac{\partial P}{\partial q_i}, \; i = 1,2,3,\ldots,6 \qquad (6)$$

The right hand side represents the force acting on the system represented as the potential gradient of the potential energy function P. It is clear that Eq. (6) describes the motion of the moving part of the instrument under the forces of inertia and torsion. The effect of an external force could be taken into account by adding a force term to the right-hand side of Eq. (6). Taking this external force as

$$\vec{f} = f_1 \, \vec{i} + f_2 \, \vec{j} + f_3 \, \vec{k} \qquad (7)$$

acting at the point

$$\vec{r} = a \, \vec{i} + b \, \vec{j} + c \, \vec{k}$$

with the origin at the centre of gravity of the moving system.

This force produces a moment

$$\vec{T} = \vec{r} \times \vec{f}.$$

$$= (bf_3 - cf_2)\, \vec{i} + (cf_1 - af_3)\, \vec{j} + (af_2 - bf_1)\, \vec{k} \quad (8)$$

So that the force term to be added to the right hand side of Eq. (6) is

$$\bar{F} = (f_1, f_2, f_3, bf_3 - cf_2, cf_1 - af_3, af_2 - bf_1) \qquad (9)$$

Substituting from Eqs. (3) and (4) into Eq(5) and adding the force term one obtains:

$$M\ddot{q}_1 + (k_{1y} + k_{2y})\,q_1 + (L_2k_{2y} - L_1k_{1y})q_5 - b_1(k_{1y} + k_{2y})\,q_6 = f_1 \tag{10}$$

$$M\ddot{q}_2 + (k_{1z} + k_{2z})\,q_2 + (L_1k_{1z} - L_2k_{2z})\,q_4 - h_1(k_{1z} + k_{2z})\,q_6 = f_2 \tag{11}$$

$$M\ddot{q}_3 + (k_{1x} + k_{2x})\,q_3 + b_1(k_{1x} + k_{2x})\,q_4 + (k_{1x} + k_{2x})\,q_5 = f_3 \tag{12}$$

$$I_{yy}\ddot{q}_4 - I_{xy}\ddot{q}_6 - I_{yz}\ddot{q}_5 + h_1b_1\,(k_{1x} + k_{2x})\,q_5 + h_1\,(L_2k_{2z} - L_1k_{1z})\,q_6 + (L_1k_{1z} - L_2k_{2z})\,q_2 + b_1^2\,(k_{1x}+k_{2x})q_4 + b_1\,(k_{1x}+k_{2x})q_3 = bf_3 - cf_2 \tag{13}$$

$$I_{zz}\ddot{q}_5 - I_{xz}\ddot{q}_6 - I_{yz}\ddot{q}_4 + h_1^2(k_{1x} + k_{2x}) + L_1^2 k_{1y} + L_2^2 k_{2y}\,q_5 + h_1 b_1 (k_{1x} - k_{2x})\,q_4 + (L_2 k_{2y} - L_1 K_{1y})\,q_1 + b_1 (L_1K_{1y} - L_2 k_{2y})\,q_6 + h_1 (h_{1x} + k_{2x})\,q_3 = cf_1 - af_3 \tag{14}$$

$$I_{xx}\ddot{q}_6 - I_{xy}\ddot{q}_4 - I_{xz}\ddot{q}_5 - (b_1k_{1y} + b_2k_{2y})\,q_1 + L_1(b_1k_{1y} - b_2k_{2y})\,q_5 + [\,b_1^2(k_{1y} + k_{2y}) + h_1^2(k_{1z} + k_{2z})\,]\,q_6 - h_1(L_1k_{1z} - L_2k_{2z})\,q_4 - h_1(k_{1z} + k_{2z})q_2 = af_2 - bf_1 \tag{15}$$

These represent a set of six equations governing the motion of the moving part of the measuring instrument under the effect of the mechanical force given by Eq. (7). This set of equations can be solved using the Laplace Transformation after assuming zero initial conditions namely,

$$\dot{q}_i(O) = O,\; q_i(O) = O,\; i = 1,2,3,\ldots,6 \tag{16}$$

Thus

$$\begin{bmatrix} S^2a_{11}+b_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} \\ a_{21} & S^2a_{22}+b_{22} & a_{23} & a_{24} & a_{25} & a_{26} \\ a_{31} & a_{32} & S^2a_{33}+b_{33} & a_{34} & a_{35} & a_{36} \\ a_{41} & a_{42} & a_{43} & S^2a_{44}+b_{44} & S^2a_{45}+b_{45} & S^2a_{46}+b_{46} \\ a_{51} & a_{52} & a_{53} & S^2a_{54}+b_{54} & S^2a_{55}+b_{55} & S^2a_{56}+b_{56} \\ a_{61} & a_{62} & a_{63} & S^2a_{64}+b_{64} & S^2a_{65}+b_{65} & S^2a_{66}+b_{66} \end{bmatrix} \begin{bmatrix} q_1(S) \\ q_2(S) \\ q_3(S) \\ q_4(S) \\ q_5(S) \\ q_6(S) \end{bmatrix} = \begin{bmatrix} f_1(S) \\ f_2(S) \\ f_3(S) \\ f_4(S) \\ f_5(S) \\ f_6(S) \end{bmatrix} \tag{17}$$

Or in abbreviated form

$$[\,A\,]\;[\,q\,] = [\,f\,] \tag{18}$$

The coefficients b_{ij}, a_{ij}, i, j = 1, 2, 3, ...,6 are constants involving the system inertia and spring stiffness. The matrix Eq. (18) may be solved to give

$$[\,q\,] = [\,a\,]^{-1}\,[\,f\,] \tag{19}$$

The time response of the system can be determined by taking the inverse Laplace Transformation of Eq. (19). This inverse Transformation is obtained either analytically or graphically using numerical methods [3,4].

3. Solution for practical problems

The solution of the set of Eqs. (10) - (15) may be simplified using the fact that the moving part of the instrument has symmetrical shape so that the products of inertia are all zeros, that is,

$$I_{xy} = I_{yx} = I_{zx} = \ldots = 0 \tag{20}$$

Also $L_1 = L_2 = L$, $b_1 = b_2 = b$, and $h_1 = h_2 = h$ (21)

Symmetric springs will also be considered, hence,

$$k_{1x} = k_{2x} = k_x,\; k_{1y} = k_{2y} = k_y,\; \text{and } k_{1z} = k_{2z} = k_z \tag{22}$$

The external vibrating mechanical force will be taken to lie in a plane normal to the axis of the main rotation of the moving part, i.e. the yz-plane. This force may be any periodic function of time

$$\vec{f} = f_1 \cos(ut)\,\vec{j} + f_2 \cos(ut)\,\vec{k} \tag{23}$$

Acting at the point (0, 1, 0) as shown in Fig. 3. This force causes a torque,

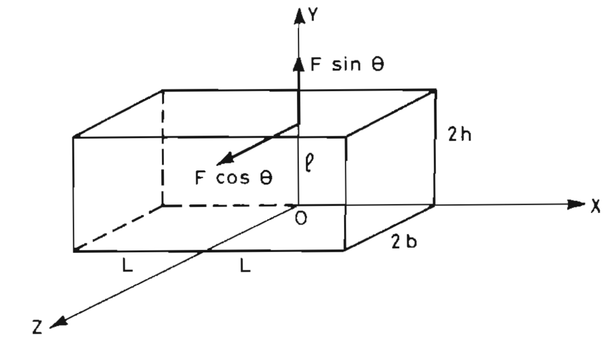

FIG. 3

$$\vec{r} \times \vec{f} = l\,\vec{j} \times \vec{f} = l\, f_2 \cos(ut)\, \vec{i} \quad (24)$$

Other torques acting on the moving part of the instrument are :

(i) The torque due to the d–c measuring current; it consists of two parts. The first T_1 is due to the d–c current through the instrument coil. The second part T_2 is a periodic one, it is due to the a–c ripple superposed on the d–c current.

$$T_2 = T_0 \cos(vt) \quad (25)$$

where $(v/2\pi)$ is the frequency of the ripple current.

(ii) The damping torque, proportional to the angular velocity $\dot{q}_6$ of the moving system. Its magnitude is $\alpha\dot{q}_6$ where α is the damping coefficient the value of which should be chosen in such a way as to allow a stable state of the moving instrument. The damping torque is usually obtained via a damping arrangement which in practice can be achieved in many ways [5] .

Under the conditions described above the set of equations given by Eqs. (10) to (15) becomes

$$M\ddot{q}_1 + 2k_y q_1 - 2b\, k_y q_6 = f_1 \cos(ut) \quad (26)$$

$$M\ddot{q}^2 + 2k_z q_2 - 2h\, k_z q_6 = f_2 \cos(ut) \quad (27)$$

$$M\ddot{q}_3 + 2k_x q_3 - 2b\, k_x q_4 + 2k_x q_5 = 0 \quad (28)$$

$$I_{yy}\ddot{q}_4 + 2hbk_x q_5 + 2b^2k_x q_4 + 2bk_x q_3 = 0 \quad (29)$$

$$I_{zz}\ddot{q}_5 + 2(h^2k_x + L^2k_y) q_5 + 2hq_3 = 0 \quad (30)$$

$$I_{xx}\ddot{q}_6 - 2bk_y q_1 + 2(b^2k_y + h^2k_z) q_6 - 2hk_z q_2$$

$$= l f_2 \cos(ut) + T_1 + T_0 \cos(vt) - \alpha\dot{q}_6 \quad (31)$$

These equations can be divided into two sets. The first set contains Eqs. (28), (29), and (30) and relates q_3, q_4, and q_5 to each other. These can be written in the form

$$\ddot{q}_3 + \delta q_3 + \lambda q_4 + \eta q_5 = 0$$

$$\ddot{q}_4 + \rho q_5 + \zeta q_4 + \xi q_3 = 0$$

$$\ddot{q}_5 + \mu q_5 + \sigma q_3 = 0 \quad (32)$$

From this set q_3, q_4, and q_5 can be shown to be identically equal to zero. Taking the Laplace Transformation of Equation (32) (with zero initial conditions), one obtains

$$(s^2 + \delta)\, q_3 + \lambda q_4 + \eta q_5 = 0$$

$$\xi\, q_3 + (S^2 + \zeta)\, q_4 + \rho q_5 = 0$$

$$\sigma\, q_3 + 0 + (s^2 + \mu)\, q_5 = 0$$

According to Cramer's Rule this set of equations has a non-trivial solution only when the matrix of the system is singular, i.e. when

$$\begin{vmatrix} s^2 + \delta & \lambda & \eta \\ \xi & s^2 + \zeta & \rho \\ \sigma & 0 & s^2 + \mu \end{vmatrix} = 0$$

But this occurs only for certain eigenvalues in the s-plane, which is not a practical situation, since for a practical problem s varies all over the complex plane as the time varies. Thus the possibility of a singular matrix cannot be considered and the solutions for q_3, q_4, and q_5 will be identically zero (which are the same as the initial values).

The second set, given by Eqs. (26), (27) and (31), can be written in the form.

$$\ddot{q}_1 + aq_1 - eq_6 = A \cos(ut)$$

$$\ddot{q}_2 + cq_2 - dq_6 = B \cos(ut)$$

$$\ddot{q}_6 + gp_6 + \theta q_6 - fq_1 - pq_2 = H + F \cos(ut) + G \cos(vt) \quad (33)$$

If the effect of q_6 on q_1 as indicated by Eq. (33) is neglected we put e and f equals zero. This may be considered a reasonable approximation since the main motion of the moving system is around the x-axis . Thus Eq. (33) gives

$$\ddot{q}_1 + aq_1 = A \cos(ut)$$

$$\ddot{q}_2 + cq_2 - dq_6 = B \cos(ut)$$

$$\ddot{q}_6 + gq_6 + \theta\dot{q}_6 - pq_2 = H + F \cos(ut) + G \cos(vt) \quad (34)$$

Taking the Laplace Transformation with zero initial conditions,

$$(s^2 + a)\, q_1 = \frac{AS}{s^2 + u^2} \quad (35)$$

$$(s^2 + c)\, q_2 - dq_6 = \frac{BS}{s^2 + u^2} \quad (36)$$

$$(s^2 + \theta S + g)\, q_6 - pq_2 = \frac{H}{S} + \frac{FS}{s^2 + u^2} + \frac{GS}{s^2 + v^2} \quad (37)$$

These are three linear equations in q_1, q_2, and q_6. Solving for q_6,

$$q_6(s) = \frac{Fs(s^2 + c + Bp/F)}{(s^2+u^2)[s^4 + \theta s^3 + (c+g)s^2 + c\theta s+(gc-dp)]}$$

$$+ \frac{H(s^2 + c)}{s[s^4 + \theta s^3 + (c+g)s^2 + c\theta s + (gc-dp)]}$$

$$+ \frac{Gs(s^2 + c)}{(s^2 + v^2)[s^4 + \theta s^3 + (c+g)s^2 + c\theta s + (gc-dq)]} \quad (38)$$

This equation can be factorized into simple fractions in the form

$$q_6(s) = \frac{q_0}{s} + \frac{\beta}{s-\beta'} + \frac{\gamma}{s-\gamma'} + \ldots\ldots \quad (39)$$

where q_0, β, β', γ, γ', ... are generally complex. By taking the inverse Laplace transform of Eq. (39) it can be concluded that the d-c steady state reading of the instrument is

$$q_6(t)_{d-c} = q_0$$

q_0 can be obtained by equating the right hand sides of Eqs. (38) and (39), multiplying by s and letting s tend to zero; thus

$$q_6(t)_{d-c} = q_0 = \frac{Hc}{gc - dp} \quad (40)$$

When no external mechanical force is present the moving part of the instrument is allowed to move around the axis only . Thus

$$q_6 + \theta q_6 + g q_6 = H + G\cos(vt)$$

where the constants θ, g, H, and G are same as before. The d−c steady state reading when no external force exists is

$$q_6(t)_{d-c} = H/g = q'_0 \quad (41)$$

From Eqs. (40) and (41) one sees that the d−c steady state reading of the instrument in the presence of an external vibrating mechanical force is greater than its reading in the force's absence. An error factor can be determined from Eqs. (40) and (41)

$$\text{Error factor} = \frac{q'_0}{q_0} = 1 - \frac{dp}{gc} \quad (42)$$

The value of (dp/gc) can be determined from Eqs. (33) and Eqs. (26) − (31).

$$\frac{dp}{gc} = \frac{1}{1 + \frac{b^2 k_y}{h^2 k_z}} \quad (43)$$

Another simplification can be considered by taking $k_y = k_z$, i.e. the stiffnesses of the springs in the z and y directions are the same. Note that this is a reasonable approximation since both directions are normal to the main axis of rotation. Thus,

$$\frac{dp}{gc} = \frac{1}{1 + \left(\frac{b}{h}\right)^2} \quad (44)$$

As a numerical example consider a practical coil of dimensions 2h = 0.6 cms, and 2b = 2.5 cms,

$$\frac{dp}{gc} = \frac{1}{1+\left(\frac{1.25}{0.3}\right)^2} = 0.0545 \text{ or } 5,4 \text{ percent}$$

The error given by Eq. (44) can be greatly reduced by selecting a coil such that b is much greater than h.

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