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Laminar Flow and Heat-Transfer on a Flat Plate with Approach-Flow of Parabolic Velocity and Temperature Distributions

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The laminar flow and heat - transfer on a constant temperature flat plate subjected to an approach - flow with parabolic velocity and temperature distributions are analysed using the kármán - Pohlhausen technique. It was found that positive approach - stream shear increases both the wall shear stress and the heat flux. Similarly, positive shear derivative of approach - flow increases wall shear stress and heat flux. Negative stream shear and negative shear derivative of the approach - flow decrease wall shear stress and heat flux. On the other hand, it was found that positive temperature gradient in the approach - flow increases the plate heat flux as compared to the case of approach - flow of uniform temperature. The opposite is true for approach - flow with negative temperature gradient. However, it was found that the second derivative of the approach - flow temperature distribution has a negligible effect on the plate heat flux.

Nomenclature

A approach - stream shear, $\frac{L}{U_o}\frac{dU_\infty}{dy}$

A' temperature gradient of approach - flow, $\frac{L}{t_o - t_w}\frac{dt_\infty}{dy}$

B approach - stream shear derivative, $\frac{L^2}{U_o}\frac{d^2U_\infty}{dy^2}$

B' second derivative of approach flow temperature, $\frac{L^2}{t_o - t_w}\frac{d^2t_\infty}{dy^2}$

C specific heat at constant temperature

D drag force

K thermal conductivity

L plate characteristic length

Pr prandtl number

Q overall heat - transfer rate

q local heat flux

Rx Reynolds number $= \rho U_o x/\mu$

r boundary - layer thickness ratio, $\delta t/\delta$

t temperature

t_w wall temperature

t_∞ approach - flow temperature

U_∞ approach - flow velocity

u,v velocity components

x,y coordinates

Greek Symbols

α see Eq. (23)

δ thickness of momentum boundary layer

δ_t thickness of thermal boundary layer

μ dynamic viscosity coefficient

ν kinematic viscosity

ρ fluid density

Subscripts

O approach - flow with uniform velocity and temperature

Introduction

The majority of boundary layer studies are concerned with bodies immersed in a free stream of uniform velocity and temperature. There are, however, situations in which the free stream flow cannot be considered uniform. For example the flow

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in the wakes of bodies, the flow in a propeller or jet slipstreams, and the flow in boundary – layers fall into the category of sheared flows. Each of these flows are characterized by a transverse velocity and temperature gradients which vary in magnitude depending on the particular configuration. Such transverse gradients affect the velocity and temperature boundary – layers development on the body, and consequently affect the body shear and heat transfer.

The effect of the approach – flow nonuniformities on laminar boundary layer flow has been studied for many years. However, most of the previous investigators were concerned with a free – stream velocity varying linearly in the direction normal to the plate [1 – 5]. Very little work has been reported on the case where the nonuniformities in the approach flow are nonlinear. Sparrow *et al.* [6] analysed the laminar flow and heat transfer on a flat plate situated in the laminar wake of an upstream plate. They found that the effect of nonuniformities is to reduce the wall shear and heat transfer on the downstream plate relative to their values for a uniform flow. Reductions up to fifty percent were encountered. The extent of reduction diminishes with increasing downstream distance, but non – negligible effects persist to a considerable length of the plate.

El – Taher [7] investigated the momentum laminar boundary layer over a plate in a wake or jet. The Karman – Pohlhausen integral method was used. The velocity profile in the central part of the wake or jet was approximated by a parabolic distribution of velocity. The boundary – layer displacement thickness, momentum thickness and wall shear stress were calculated for the plate at different lateral distances from the wake or jet centre – lines. It was found that the boundary layer characteristics depend strongly on the lateral shift of the plate from the wake or jet centre – line.

The present paper is concerned with laminar flow and heat transfer on an isothermal flat plate in a free stream which has a parabolic velocity and temperature distributions. The main concern of this study is to investigate the effect of the transverse gradient and second derivative of free stream velocity and temperature profiles at the plate leading edge on the plate momentum and thermal boundary layers. The Kármán – Pohlhausen integral method is employed to obtain the solutions. The results depend on the approaching velocity profile parameters A,B, the approaching temperature profile parameters A', B', and on the Prandtl number. The results are compared with the case of a flat plate in a uniform stream whose velocity and temperature are equal to the velocity and temperature at the plate leading edge in the non – uniform flow.

Analysis

Two – dimensional, incompressible, laminar and steady boundary – layer flow over a constant temperature flat plate situated in a parabolic shear flow is considered (Fig. 1). The approach – flow velocity and temperature are given by:

$$U_\infty / U_o = 1 + A \frac{y}{L} + B \frac{y^2}{L^2} \qquad \text{.....................(1)}$$

$$\frac{t_\infty - t_w}{t_o - t_w} = 1 + A' \frac{y}{L} + B' \frac{y^2}{L^2} \qquad \text{......... (2)}$$

where U_o, A, B, A', B' and t_o are constants, while t_w is the plate temperature and L is a characteristic length.

The boundary layer equations are:

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + \frac{1}{\rho} \frac{\partial p}{\partial x} = \nu \frac{\partial^2 u}{\partial y^2} \qquad \text{.......... (3)}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = O \qquad \text{.......... (4)}$$

$$u \frac{\partial t}{\partial x} + v \frac{\partial t}{\partial y} = \frac{k}{\rho c} \frac{\partial^2 t}{\partial y^2} \qquad \text{....... (5)}$$

In Eq. (5) the thermal conductivity k is assumed constant and the dissipation of viscous energy is neglected.

Let us define the momentum boundary – layer thickness δ (x) over the plate such that $u = U_\infty(\delta)$ at $y = \delta$ (x). Likewise, the thermal boundary – layer thickness δ_t (x) is defined such that $t = t_\infty(\delta_t)$ at $y = \delta_t$ (x). Integrating Eq. (3) in the y direction from y = O to $y = \delta$ the following momentum integral equation is obtained (7):

$$\frac{d}{dx} \int_o^\delta u^2 dy - U_\infty(\delta) \frac{d}{dx} \int_o^\delta u\, dy + 2 \nu U_o \frac{B\delta}{L^2} = \nu \left\{ \left(\frac{\partial U_\infty}{\partial y} \right)_{y=\delta} - \left(\frac{\partial u}{\partial y} \right)_{y=o} \right\} \qquad \text{........ (6)}$$

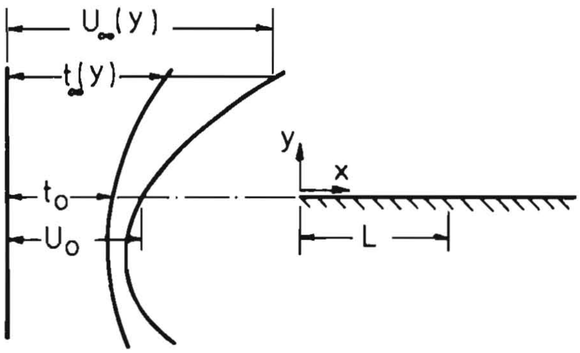

Fig. 1. Boundary-Layer Coordinates System.

The pressure outside the boundary – layer is related to the velocity by:

$$\frac{1}{\rho}\frac{\partial p}{\partial x} = \frac{\partial^2 U_\infty}{\partial y^2} = 2\nu \frac{U_o B}{L^2} \quad \text{.............. (7)}$$

Integrating Eq. (5) across the thermal boundary layer yields:

$$\frac{d}{dx}\int_0^{\delta_t}\{t_\infty(\delta_t) - t\}\, u\, dy - \frac{d}{dx}\, t_\infty(\delta_t)\int_0^{\delta_t} u\, dy$$

$$= \frac{k}{\rho c}\left\{\left(\frac{\partial t}{\partial y}\right)_{y=o} - \left(\frac{\partial t}{\partial y}\right)_{y=\delta_t}\right\} \quad \text{....... (8)}$$

Following the Pohlhausen's approach it is assumed that the boundary layer velocity profile is adequately represented by a quartic series expansion,

$$u/U_\infty(\delta) = a\eta + b\,\eta^2 + c\,\eta^3 + d\,\eta^4 \quad \text{...... (9)}$$

where $\eta = y/\delta$ and a,b,c, and d are free constants to be determined from the following boundary conditions:

At y = O:

$u = v = o$

$$\frac{\partial^2 u}{\partial y^2} = \frac{1}{\rho}\frac{\partial p}{\partial x} = 2\nu\frac{U_o B}{L^2} \quad \text{........ (10)}$$

At $y = \delta$:

$$u = U_\infty(\delta),\ \frac{\partial u}{\partial y} = \frac{\partial U_\infty}{\partial y} \text{ and } \frac{\partial^2 u}{\partial y^2} = \frac{\partial^2 U_\infty}{\partial y^2} \quad \text{..... (11)}$$

Equation (11) implies that the first and second derivatives of velocity profile in the boundary layer should match with those of the stream flow at the edge of velocity boundary layer.

Applying these boundary conditions to Eq. (9), the velocity profile is determined as:

$$u/U_\infty(\delta) = (2\eta - 2\eta^3 + \eta^4) + K_1(-2\eta + \eta^2 + 2\eta^3 - \eta^4) + K_2(-\eta + 2\eta^3 - \eta^4) \quad \text{......... (12)}$$

$$\text{where } K_1 = \frac{A\,\delta/L}{1 + A\frac{\delta}{L} + B\frac{\delta^2}{L^2}}$$

$$K_2 = \frac{B\,\delta^2/L^2}{1 + A\frac{\delta}{L} + B\frac{\delta^2}{L^2}}$$

The wall shear stress is given by:

$$\tau = \mu \left.\frac{du}{dy}\right|_{y=O}$$

$$= \mu\frac{U_o}{\delta}\left(2 + A\frac{\delta}{L}\right) \quad \text{......... (13)}$$

Similarly, the boundary layer temperature profile is represented by:

$$\frac{t - t_w}{t_\infty(\delta_t) - t_w} = a'\eta' + b'\eta'^2 + c'\eta'^3 + d'\eta'^4 \quad \text{.... (14)}$$

where $\eta' = y/\delta_t$ and a', b', c' and d' are free constants. The appropriate boundary conditions are:

At y = 0

$$t = t_w\ ;\ \frac{\partial^2 t}{\partial y^2} = 0 \quad \text{........ (15)}$$

At $y = \delta_t$

$$t = t_\infty(\delta_t);\ \frac{\partial t}{\partial y} = \frac{\partial t_\infty}{\partial y} \text{ and } \frac{\partial^2 t}{\partial y^2} = \frac{\partial^2 t_\infty}{\partial y^2} \quad \text{..... (16)}$$

Equation (16) indicates that the first and second derivatives of the temperature profile in the boundary layer should match with those of the stream flow at the edge of the thermal boundary layer. The constants a',b',c', and d' can then be evaluated yielding:

$$\frac{t-t_w}{t_\infty(\delta_t)-t_w}=(2\eta'-2\eta'^3+\eta'^4)+K'_1(-\eta'+2\eta'^3-\eta'^4)$$

$$+K'_2\left(-\frac{5}{3}\eta'+3\eta'^3-\frac{4}{3}\eta'^4\right) \quad \text{......} (17)$$

where

$$K'_1=\frac{A\delta_t/L}{1+A'\frac{\delta T}{L}+B'\frac{\delta t^2}{L^2}}$$

$$K'_2=\frac{B'\delta_t^2/L^2}{1+A'\frac{\delta t}{L}+B'\frac{\delta t^2}{L^2}}$$

It should be noted that the boundary condition (10) for the velocity profile is different from the boundary condition (16) for the temperature profile. Therefore, the two profiles are not similar. The local heat flux is given by:

$$q=-k\left.\frac{\partial t}{\partial y}\right|_{y=0}$$

$$=\frac{2k(t_w-t_o)}{\delta t}\left(1+\frac{A'}{2}\delta_t+\frac{B'}{6}\delta_t^2\right) \quad \text{......} (18)$$

Substituting of u from Eq. (12) into Eq. (6) and normalizing leads to the following equation of momentum boundary layer thickness:

$$\frac{d\bar{\delta}^2}{d\bar{x}}\left(-0.11746+0.03334\,A\frac{\bar{\delta}}{R_L^{1/2}}-1.77143\,B\frac{\bar{\delta}^2}{R_L}\right.$$

$$\left.-2.57142\,AB\frac{\bar{\delta}^3}{R_L^{1.5}}-3.21428\,B^2\frac{\bar{\delta}^4}{R_L^2}\right)=-4 \quad \text{....}(19)$$

$$\text{where: } \bar{\delta}=\frac{\delta}{L}\sqrt{R_L};\ R_L=\frac{\rho U_o L}{\mu} \text{ and } \bar{x}=x/L$$

The integrals in Eq. (8) can be evaluated by using Eq. (17) for t, Eq. (12) for u (at $y<\delta$) and Eq. (1) for u (at $y>\delta$). This yields the following differential equations for r $(=\delta/\delta)$:

Case 1: r<1

$$\left[\frac{1}{15}\left(1+A\frac{\bar{\delta}}{\sqrt{R_L^{\frac{1}{2}}}}\right)r^3+\left(-\frac{A'}{6}\frac{\bar{\delta}}{\sqrt{R_L^{\frac{1}{2}}}}+\frac{B}{28}\frac{\bar{\delta}^2}{R_L}\right)r^4\right.$$

$$-\left(\frac{3}{280}+\frac{4}{15}B'\frac{\bar{\delta}^2}{R_L}+\frac{AB'}{90}\frac{\bar{\delta}^3}{R_L^{1.5}}\right)r^5$$

$$+\left(\frac{1}{360}+\frac{3}{20}A'\frac{\bar{\delta}}{\sqrt{R_L^{\frac{1}{2}}}}-\frac{BB'}{168}\frac{\bar{\delta}^4}{R_L^2}\right)r^6$$

$$\left.-\left(\frac{A'}{15}\frac{\bar{\delta}}{\sqrt{R_L^{\frac{1}{2}}}}+\frac{71}{280}B'\frac{\bar{\delta}^2}{R_L}\right)r^7-\left(\frac{11}{720}B'\frac{\bar{\delta}^2}{R_L}\right)r^8\right]\frac{d\bar{\delta}^2}{d\bar{x}}$$

$$+\left[\frac{1}{45}\left(4\bar{\delta}^2+2A\frac{\bar{\delta}^3}{\sqrt{R_L^{\frac{1}{2}}}}\right)+\frac{B}{42}\frac{\bar{\delta}^4}{R_L}r-\left(\frac{\bar{\delta}^2}{35}+\frac{2}{135}B'\frac{\bar{\delta}^4}{R_L}\right.\right.$$

$$+\frac{AB'}{135}\frac{\bar{\delta}^5}{R_L^{1.5}}\Big)r^2+\left(\frac{\bar{\delta}^2}{108}-\frac{BB'}{252}\frac{\bar{\delta}^6}{R_L2}\right)r^3+\frac{B'}{210}\frac{\bar{\delta}^4}{R_l}r^4$$

$$\left.-\frac{B'}{648}\frac{\bar{\delta}^4}{R_L}r^5\right]\frac{dr^3}{d\bar{x}}=\frac{2}{P_r}\left(1-\frac{5}{6}B'\frac{\bar{\delta}^2}{R_L}r^2\right) \quad ..(20)$$

Case 2: r>1

$$\left[\frac{1}{360}\frac{1}{r^3}-\frac{3}{280}\frac{1}{r^2}-\frac{B'}{360}\frac{\bar{\delta}^2}{R_L}\frac{1}{r}+\left(\frac{1}{15}+\frac{9}{560}B'\frac{\bar{\delta}^2}{R_L}\right)\right.$$

$$-\left(\frac{3}{20}-\frac{A'}{15}\frac{\bar{\delta}}{\sqrt{R_L^{\frac{1}{2}}}}\right)r+\left(\frac{3}{20}-\frac{3}{20}A'\frac{\bar{\delta}}{\sqrt{R_L^{\frac{1}{2}}}}+\frac{B'}{30}\frac{\bar{\delta}^2}{R_L}\right)r^2$$

$$+\left(\frac{A}{15}\frac{\bar{\delta}}{\sqrt{R_L^{\frac{1}{2}}}}-\frac{3}{20}B'\frac{\bar{\delta}^2}{R_L}\right)r^3+\frac{\bar{\delta}^2}{R_L}\left(\frac{B}{28}-\frac{B'}{40}\right)r^4-\frac{AB'}{90}\frac{\bar{\delta}^3}{R_L^{1.5}}r^5$$

$$\left.-\frac{BB'}{168}\frac{\bar{\delta}^4}{R_L^2}r^6\right]\frac{d\bar{\delta}^2}{dx}+\left[-\frac{\bar{\delta}^2}{135}\frac{1}{r^6}+\frac{3}{140}\frac{\bar{\delta}^2}{r^5}\right.$$

$$+\frac{B'}{810}\frac{\bar{\delta}^4}{R_L}\frac{1}{r^4}-\left(\frac{2}{45}\bar{\delta}^2+\frac{B'}{280}\frac{\bar{\delta}^4}{R_L}\right)\frac{1}{r^3}+\left(\frac{\bar{\delta}^2}{10}+\frac{B'}{135}\frac{\bar{\delta}^4}{R_L}\right)\frac{1}{r}$$

$$+\frac{2}{45}A\frac{\bar{\delta}^3}{\sqrt{R_L^{\frac{1}{2}}}}+\frac{\delta^4}{R_L}\left(\frac{B}{42}-\frac{B'}{60}\right)r-\frac{AB'}{135}\frac{\bar{\delta}^5}{R_L^{1.5}}r^2$$

$$-\frac{BB'}{252}\frac{\bar{\delta}^6}{R_L^2} r^3 \Bigg] \frac{dr^3}{d\bar{x}} = \frac{2}{Pr}\left(1-\frac{5}{6}B'\frac{\bar{\delta}^2}{R_L}r^2\right)$$
.... (21)

Eq. (19) was integrated numerically step by step giving the distribution of δ along the plate. Equations (20) and (21) can then be integrated for r using the same numerical technique. The local wall shear stress and local heat flux can be calculated using Eqs. (13) and (18) respectively. Results can be obtained for the drag D and over – all heat transfer rate Q defined as:

$$D = \int_0^X \bar{\tau}\, d x,\quad Q = \int_0^X q\, d x \quad ..... (22)$$

per unit span.

Results and Discussions

To investigate the effect of nonuniformity of free stream velocity and temperature on plate skin friction and heat flux, three different cases were considered: a) plate in a free stream of parabolic velocity distribution and uniform temperature b) plate in a free stream of uniform velocity and parabolic temperature distribution and c) plate in a free stream of parabolic velocity distribution and parabolic temperature distribution. In all cases, the Reynolds number $R_L = U_o L/\nu$ (where L is a characteristic plate length) was taken 10^5. Boundary – layer calculations were extended up to x = 5L where the drag D and overall heat flux Q were calculated. The results were compared to those for a conventional flat plate boundary – layer where the velocity and temperature in the approach flow are uniform and equal to U_o and t_o respectively. Equations (19), (20) and (21) are solved for the special case of approach – flow with uniform velocity and temperature (*i.e.* A = B = A' = B' = 0), their solution yield:

$$\delta_o/x = 5.8356\left(\frac{\nu}{U_o x}\right)^{\frac{1}{2}}$$
$$\tau_o/\rho U_o^2 = 0.3427\left(\frac{\nu}{U_o x}\right)^{\frac{1}{2}}$$
$$r_o = \alpha/P_r^{1/3} \quad ......$$
$$q_o x / k(t_w - t_0) = \frac{0.3427}{\alpha} Pr^{1/3}\left(\frac{U_o X}{\nu}\right)^{\frac{1}{2}} \quad ...(23)$$

where α is a constant which depends on the value of Pr as shown in Table (1).

Table (1): Variation of the coefficient a with Pr

Pr	0.001	0.01	0.7	1.0	10.0	100.0	1000.0
a	2.0282	1.4525	1.0120	1.0000	0.9680	0.9607	0.9591

a. Approach – Flow of Parabolic Velocity and Uniform Temperature

Two different types of parabolic velocity distributions were considered: 1) Wake – type parabolic velocity distribution. 2) Jet type parabolic velocity distribution. The first type corresponds to positive values of B and the second type corresponds to negative values of B.

The ratios of δ/δ_o, δ_t/δ_{to}, τ/τ_o and q/q_o are plotted in Fig. (2) as a function of the dimensionless streamwise coordinate x/L for a wake – type approach – flow (B = 10) with positive local stream shear (A = 5). Fig. (3) shows a plot of the same ratios for a plate in wake – type approach flow (B = 10) but the local stream shear is negative (A = – 5).

Comparison of Figs. (2) and (3) indicates that the effect of positive value of local stream shear (A = 5) is to increase the wall shear and heat flux relative to their values for a uniform flow. The ratios τ/τ_o and q/q_o are equal to unity just downstream of the leading edge and they increase with increasing the downstream distance. The opposite is true for negative local stream shear (A = – 5): the ratio τ/τ_o and q/q_o are equal to unity just downstream of the leading edge but they decrease with increasing downstream distance.

Figs. (4) and (5) show the variation of δ/δ_o, δ/δ_{to} τ/τ_o and q/q_o along a plate in a jet – type parabolic approach shear flow (B = – 10) The approach – flow temperature is constant. Fig. (4) corresponds to positive local stream shear (A = 5) and Fig. (5) corresponds to a negative local stream shear (A = – 5). It is remarkable that in the first case (A = 5, B = – 10) the effect of stream nonuniformity on the ratios τ/τ_o and q/q_o is relatively small. On the other hand, the effect of stream nonuniformity in the second case (A=–5, B=–10) is more pronounced.

Figs. (2) to (5) indicate that the effect of the nonuniformity of approach – flow velocity on the ratios q/q_o and τ/τ_o is large when the stream shear A and its derivative B are of the same sign. This effect is relatively small when A and B are of opposite sign. This can be clearly noticed in Fig. (6) which shows a plot for D/D_o and Q/Q_o as a function of B and the

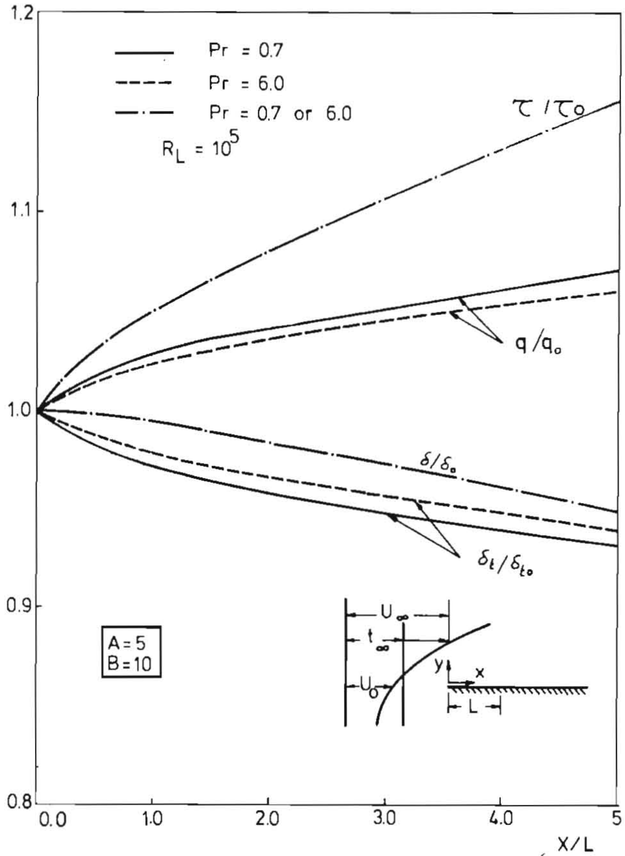

Fig. 2. Variation of Boundary-Layer Thicknesses, Local Wall Shear and Local Heat Flux Along a Plate in a Waketype Parabolic Shear Flow. (A=5, B=10 and t_∞ = Const.)

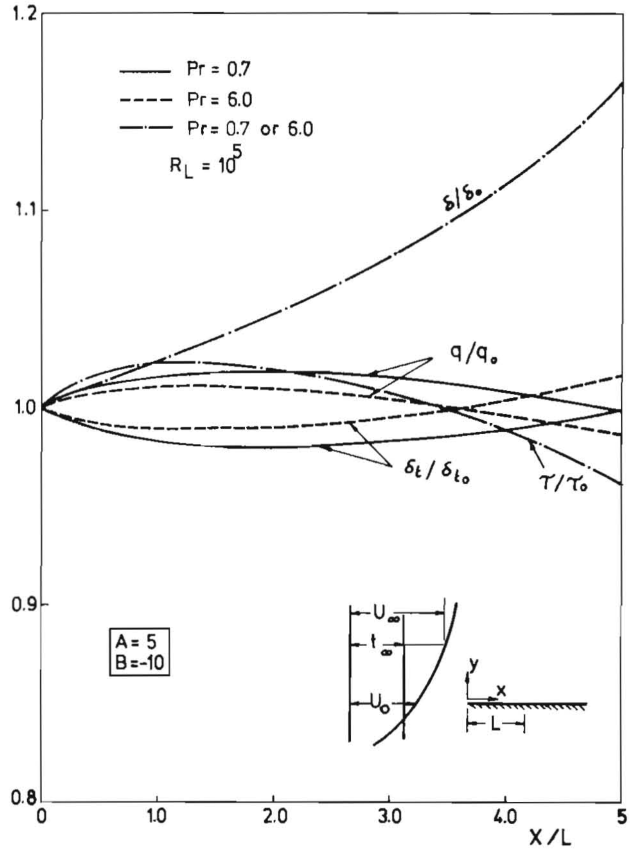

Fig. 4. Variation of Boundary-Layer Thicknesses, Local Wall Shear and Local Heat Flux Along a Plate in a jet-type Parabolic Shear Flow (A=5, B=−10 and t_∞ = Const.)

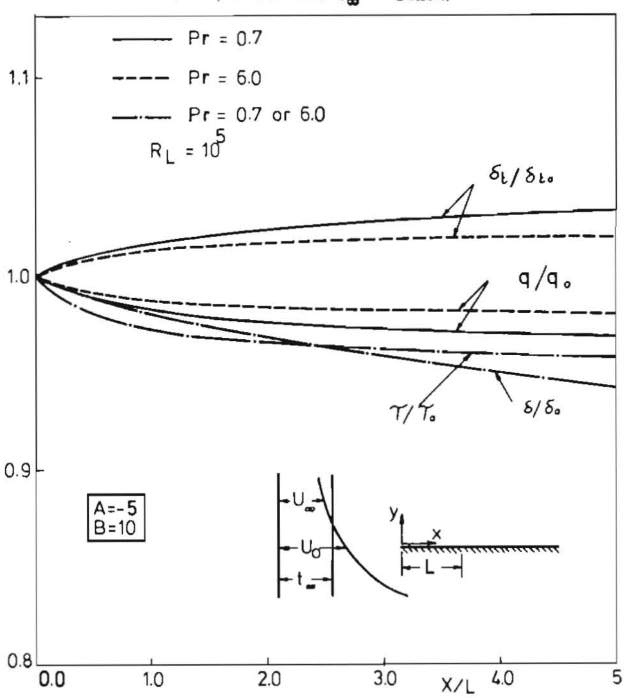

Fig. 3. Variation of Boundary-Layer Thcknesses, local Wall Shear and Local Heat Flux Along a Plate in a Waketype Parabolic Shear Flow (A=−5, B=10 and t_∞ = Const.)

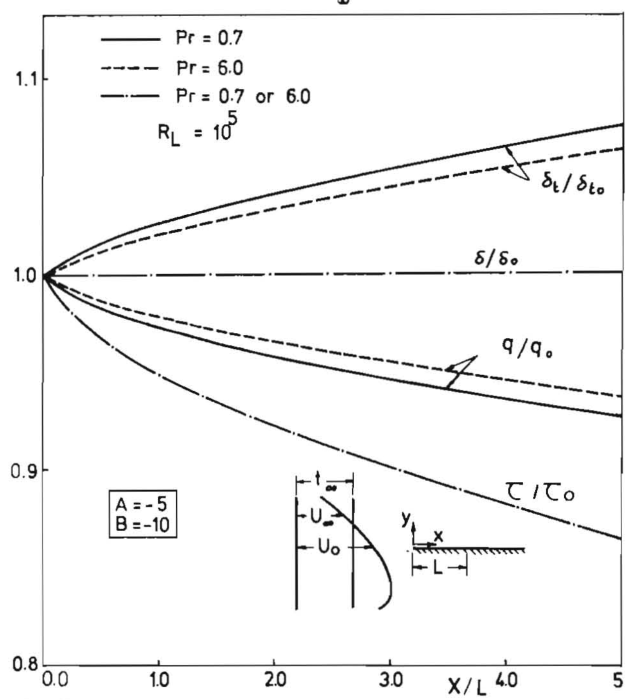

Fig. 5. Variation of Boundary-Layer Thicknesses, Local Wall Shear and Local Heat Flux Along a Plate in a jet-Type Parabolic Shear Flow. (A=−5, B=−10 and t_∞ = Const.)

curves are parameterised by A. It is seen from the figure that D/D_0 and Q/Q_0 increase with increase of the stream shear derivative B. Likewise, D/D_0 and Q/Q_0 increase with increase of the stream shear A. However, it is noteworthy that the effect of variation in A is larger than the effect of variation in B.

Generally, Figs. (2) to (6) indicate that the effect of the nonuniformity of the approach – flow velocity on the plate shear stress t is larger than its effect on the plate heat flux q.

Fig. (7) shows a plot for Q/Q_0 against Pr for four different cases of approach – flow. It is noticed that the effect of nonuniformity of approach – flow velocity on total heat flux is more pronounced at the smaller values of Pr.

b. Approach—Flow of Uniform Velocity and Parabolic Temperature

Several cases have been calculated to investigate the effect of the second derivative B' of the approach – flow temperature on the plate heat transfer. Table (2) summarises the results of four of those cases. The first two cases correspond to zero local temperature gradient in the approach flow (A'=0). The second two cases correspond to a local temperature gradient A'=10.0.

It can be concluded from the results presented in Table 2 that the second derivative of the approach – flow temperature distribution B' has negligible effect on the plate heat flux. It can also be concluded that the first derivative A' of the approach-flow temperature distribution has a significant effect on thermal boundary – layer along the plate. In all results presented and discussed below, B' will be equal to zero.

Fig. (8) shows the variation of δ_t/δ_{to} and q/q_o along a plate subjected to an approach – flow of uniform velocity and linear distribution of temperature. Cases with different values of A' are shown. It is seen from the figure that positive values of A' increases the local heat flux q as compared to the case of a plate in a free stream of uniform velocity and uniform temperature. The opposite is true for negative values of A'. It is noticed that the ratios δ_t/δ_{to} and q/q_o are equat to unity just

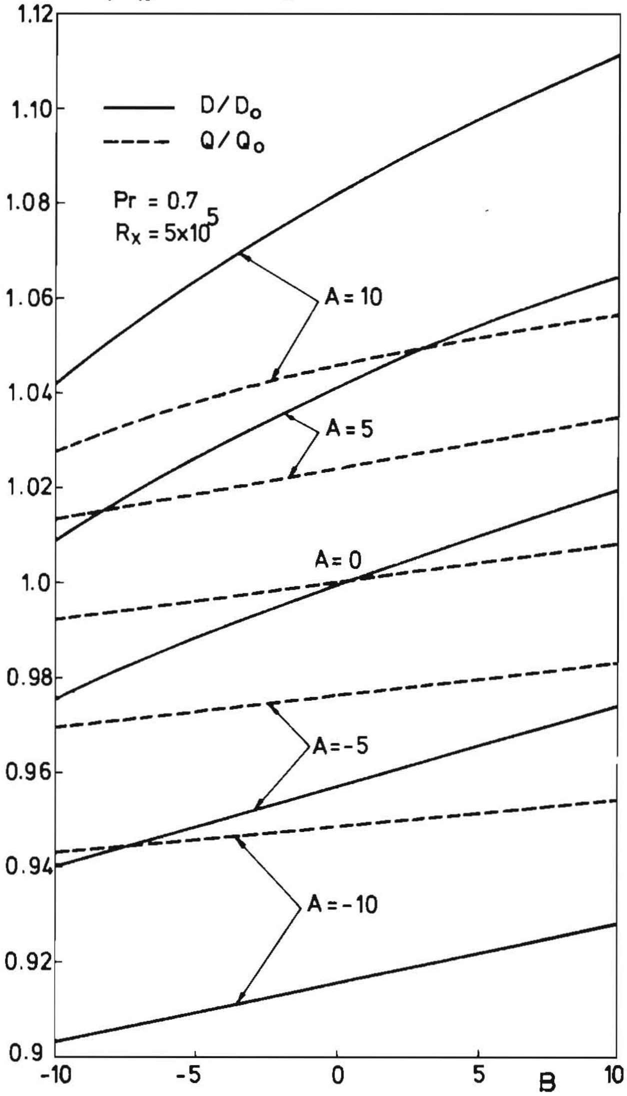

Fig. 6. Effect of Stream Shear A and Shear Derivative B on Drag Force and Overall Heat-Transfer Rate for a Plate in Parabolic Shear Flow (t_∞ = Const.)

Table 2: Variation of local heat flux along a plate with approach – flow of uniform velocity and parabolic temperature distribution (p_r=0.7 and R_L=10^5)

A'	B'	X/L	1.0	2.0	3.0	4.0	5.0	Q/Q_o
0.0	-10.0	q/q_o	1.0000	1.0000	1.0000	1.0001	1.0001	1.001
	10.0	q/q_o	0.9999	0.9999	0.9999	0.9999	0.9998	0.9999
10.0	-10.0	q/q_o	1.0473	1.0665	1.0812	1.0936	1.1046	1.0533
	10.0	q/q_o	1.0472	1.0662	1.0806	1.0928	1.1035	1.0530

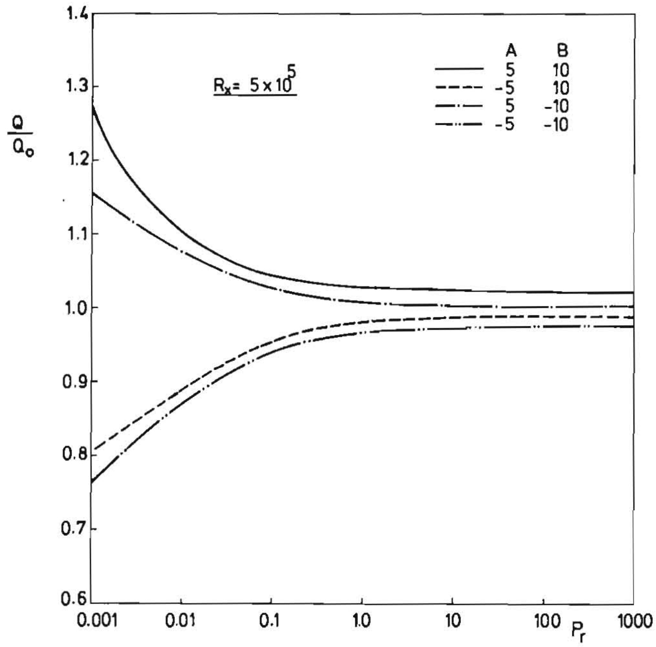

Fig. 7. Effect of Prandtl Number on Overall Heat-Transfer Rate for a Plate in Parabolic Shear Flow (t_∞ = Const.)

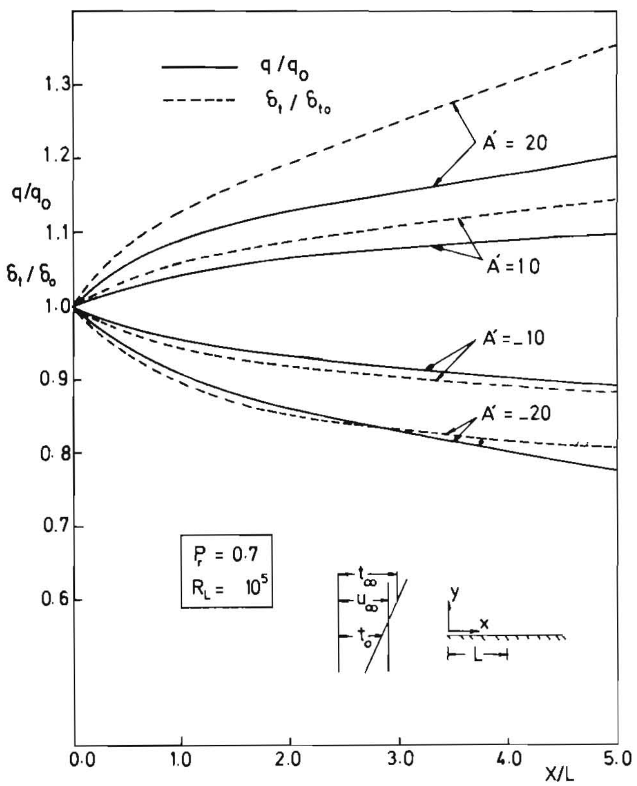

Fig. 8. Variation of Thermal Boundary-Layer Thickness and Local Heat Flux Along a Plate in a Free Stream of Uniform Velocity and a Linear Distribution of Temperature

downstream of the leading edge, and the effect of the temperature gradient A' on these ratios increases with increasing the downstream distance x/L.

Fig. (9) shows the effect of the approach – flow temperature gradient A' on the total heat flux Q/Q_o at different values of Pr. It is seen from the figure that at a certain value of Pr the value of Q/Q_o increases linearly with A'. The figure also shows the effect of nonuniformity of free stream temperature on Q/Q_o which increases with decreasing Pr.

c. Approach – Flow of Parabolic Velocity and Linear Temperature

The variation of the relative change in local heat flux $(q-q_o)/q_o$ along a plate in nonuniform approach – flow is shown in Fig. (10). Five different cases of different approach – flow velocity and temperature distributions are considered. Each case is characterized by specific values for the parameters A,B and A'. The table in the figure gives the relative change in the total heat flux $(Q-Q_o)/Q_o$ for the five cases. The figure shows that the response of heat transfer on a plate due to nonuniformities of both approach – flow velocity and temperature can be calculated to an acceptable degree of accuracy by superposition of effects of an approach – flow having the same nonuniform velocity but with uniform temperature and effects of an approach flow having the same nonuniform temperature but with uniform velocity. For example: for an approach – flow having A=5, B=10.0 and A'=0, $(Q-Q_o)/Q_o$ = 0.0336, and for an approach – flow having A=O, B=O and A'=5, $(Q-Q_o)/Q_o$ = 0.0276. For an approach – flow having A=5, B=10 and A' = 5 it is shown that $(Q-Q_o)/Q_o$ = 0.0618 which is approximately the sum of $(Q-Q_o)/Q_o$ for the first two cases.

Conclusions

The results of the present analysis have demonstrated the effect of parabolic nonuniformities of velocity and temperature in the approach – flow on the wall shear and heat transfer in a flat plate boundary – layer flow. The main conclusions are:

1. Positive approach – stream shear at the plate leading edge increases wall shear stress and heat flux relative to their values for uniform flow. Negative approach – stream shear has an opposite effect. Likewise, positive shear derivative of approach – flow increases wall shear stress and heat flux. However, the effect of stream shear is larger than the effect of shear derivative.

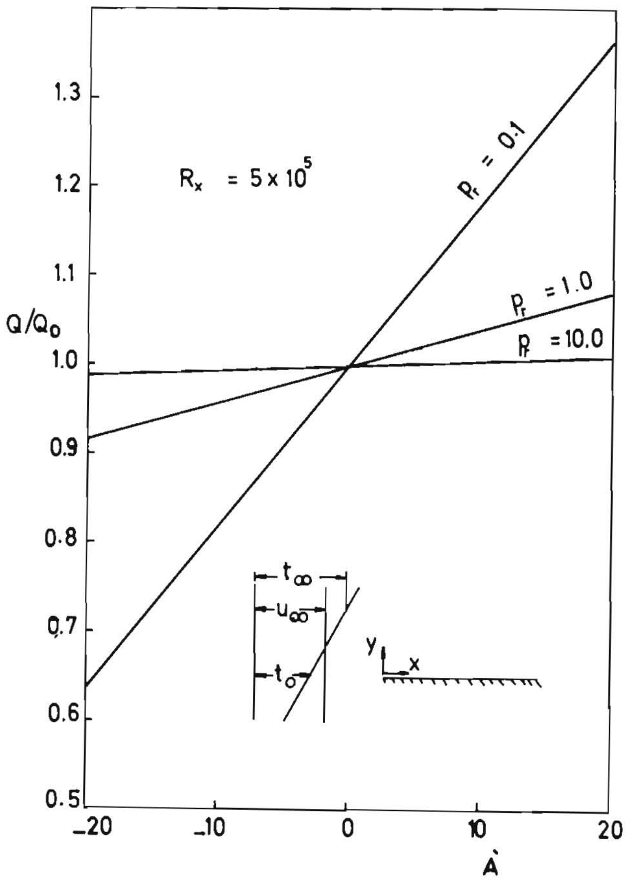

Fig. 9. Effect of Approach-Flow Temperature Gradient A' on Overall Heat-Transfer Rate for a Plate in a Free Stream of Uniform Velocity. (t_∞ = Const.)

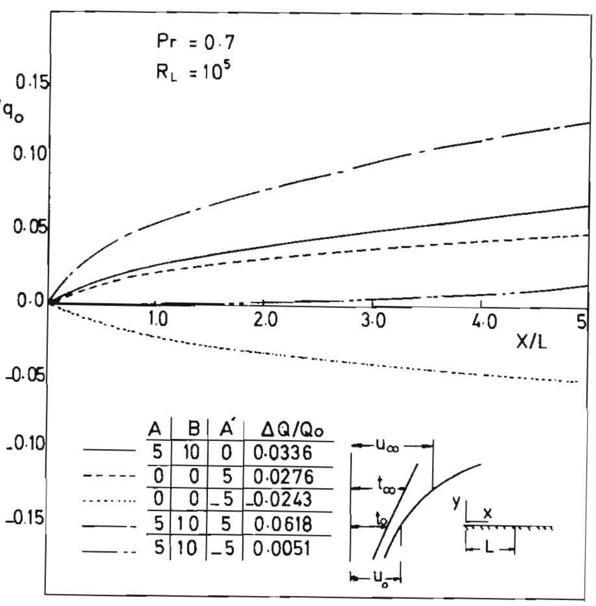

Fig. 10. Variation of Local Heat Flux Along a Plate in a Wake-Type Parabolic Shear Flow with a Linear Distribution of Temperature.

2. Positive temperature gradient in the approach – flow increases heat flux as compared to the case of approach – flow of uniform temperature. The opposite is true for negative temperature gradient. However, the second derivative of the temperature distribution in the approach – flow has a negligible effect on the plate heat flux.

3. The effect of nonuniformities of approach – flow velocity and temperature on wall shear stress and heat flux increase with increasing downstream distance along the plate.

4. The effect of nonuniformities in approach – flow velocity and temperature on plate heat – flux increases with decreasing the value of Prandtl number.

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السريان الانسيابي والانتقال الحراري على لوح مستوٍ في مجال سريان ذي توزيع سرعة وحرارة على شكل قطع مكافىء

رفعت الطاهر

أستاذ مشارك بقسم الطيران بكلية الهندسة جامعة القاهرة. وحاليا معار لجامعة الملك عبد العزيز بجدة، المملكة العربية السعودية.

يستخدم هذا البحث طريقة كارمن - بوهلاوزن التقريبية في دراسة السريان في الطبقة الجدارية على لوح مستو ذي درجة حرارة ثابتة موجود في مجال سريان حر له توزيع سرعات ودرجات حرارة على شكل قطع مكافىء. بينت الدراسة أن المشتقة الاولى الموجبة لتوزيع السرعة تؤدي الى زيادة الاجهاد القصي والانتقال الحراري على السطح بالمقارنة بحالة السريان المنتظم. كما ينتج أيضا عن المشتقة الثانية الموجبة لتوزيع السرعة زيادة في كل من الاجهاد القصي والانتقال الحراري على السطح ويحدث العكس في حالة المشتقة الأولى والثانية السالبتين. وقد ثبت من الدراسة أن المشتقة الأولى الموجبة لدرجة الحرارة تؤدي الى زيادة الانتقال الحراري على السطح بالمقارنة بحالة درجة الحرارة المنتظمة ولكن الدراسة أثبتت أن المشتقة الثانية لدرجة الحرارة ذات تأثير ضعيف يمكن إهماله.