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Synthesis of a Four-Bar Linkage for a Limited Variation in the Velocity Ratio

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In this work, a synthesis of the four-bar linkage for a prescribed maximum value of the velocity ratio, and a preassigned variation over a certain region is presented. The analysis yields a group of highly nonlinear equations which cannot be solved analytically. An algorithm which incorporates the double false method is suggested for solving these equations. A computer program written in Fortran -4 has been used.

Nomenclature

θ	rotational angle of the crank
φ	rotational angle of the rocker
E	length of the crank
C	length of the coupler
R	length of the rocker
S	distance between the crank and the rocker pivots
n	velocity ratio
n*	maximum velocity ratio

Introduction

Synthesis is related to specifying the dimensions of the links of a mechanism to yield a certain output requirement. It has been developed rapidly in the past few years due to its great practical importance.

Synthesis of the four-bar mechanism is presented in several text books, *e.g.* [1,2]. It is possible to design a four-bar linkage for different output requirements such as function generation which satisfies several position points, optimization of the transmission angle, development of coupler curves, and so forth. Several investigations were made for the kinematic synthesis of the four-bar linkage which involve output angular velocity conditions, *e.g.* [1,3,4,5].

Starting from an extreme position, the ratio of the angular velocity of the rocker to that of the driving link increases until it reaches a maximum value. It decreases back to zero at the other extreme position. It is possible to adjust the lengths of the links to obtain prescribed velocity ratios [3]. Also, the design of a four-bar linkage to give a prescribed maximum value of the velocity ratio is presented in [5]. There is a region within which the velocity ratio is almost uniform. It is true that the maximum length of the region, for a limited variation in the velocity ratio, includes the extreme value. The purpose of this work is to present a synthesis for a prescribed velocity ratio, and a predetermined velocity ratio variation over a certain region.

Kinematics of a four-bar linkage

For the four-bar linkage shown in Fig. 1, the relation between the rocker angle and the crank angle is obtained from the vector relation

$$\vec{E}+\vec{C}=\vec{S}+\vec{R} \tag{1}$$

When each vector is represented in terms of the components along and normal to S, Eq. (1) takes the form

$$E\cos\theta + C\cos\gamma + R\sin\varphi = S \tag{2}$$

$$E\sin\theta + C\sin\gamma = R\cos\varphi \tag{3}$$

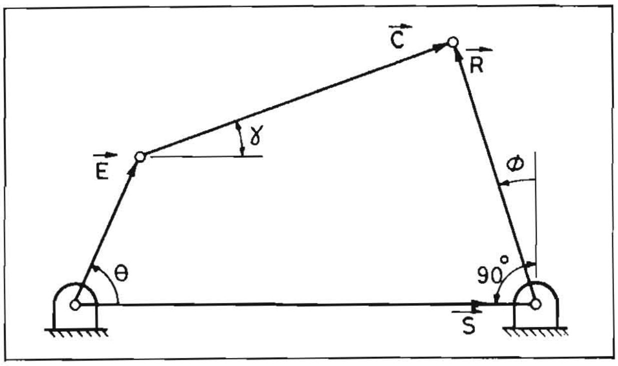

Fig. 1: The Four-Bar Linkage

Eliminating γ from Eqs. (2), and (3) and replacing the lengths of links by their ratios to S, one gets

$$\sin\varphi = \frac{ab \pm \sqrt{b^2+d^2-a^2}}{b^2+d^2} \tag{4}$$

Where

$$a = 1+e^2+r^2-c^2-2e\cos\theta$$

$$b = 2r(1-e\cos\theta) \tag{4a}$$

$$d = 2re\sin\theta$$

$$e = \frac{E}{S}$$

$$c = \frac{C}{S}$$

$$r = \frac{R}{S}$$

Equation (6) yields two values for φ. For the configuration shown in Fig. 1 it is clear that

$$\sin\varphi = \frac{ab-\sqrt{b^2+d^2-a^2}}{b^2+d^2} \tag{5}$$

The speed ratio is obtained by differentiating Eq. (5) with respect to θ. Noting that $n = \frac{d\varphi}{d\theta}$, and after simplifications, thus

$$n = \frac{e}{r}\left[\frac{r\cos(\theta-\varphi)-\sin\theta}{e\cos(\theta-\varphi)-\cos\varphi}\right] \tag{6}$$

Synthesis

The relation between n and θ is shown in Fig. 2. The maximum velocity ratio n^* occurs at θ_2. At θ_1, and θ_3 the velocity ratio, is λn^*, where λ is a factor less than one. In the region between θ_1 and θ_3 the velocity ratio varies within the range $(1-\lambda)n^*$.

The value of θ_2 is obtained by differentiating Eq. (6) with respect to θ and equating to zero. This condition yields

$$[e\cos(\theta-\varphi)-\cos\varphi][r(1-n)\sin(\theta-\varphi)+\cos\theta]$$

$$-[r\cos(\theta-\varphi)-\sin\theta][e(1-n)\sin(\theta-\varphi)-n\sin\varphi]=0 \tag{7}$$

From Eqs. (6), and (7)

$$n^* = \frac{e}{r}\left[\frac{r(1-n^*)\sin(\theta_2-\varphi_2)+\cos\theta_2}{e(1-n^*)\sin(\theta_2-\varphi_2)-n^*\sin\varphi_2}\right] \tag{8}$$

Where φ_2 is the rocker angle corresponding to θ_2 and is obtained by Eq. (5); *i.e.*

$$\sin\varphi_2 = \frac{a_2b_2-\sqrt{b_2^2+d_2^2-a_2^2}}{b_2^2+d_2^2} \tag{9}$$

where a_i, b_i, and d_i are given by Eq. (4-a) for θ equal to θ_i. Also, θ_2, φ_2, and n^* satisfy Eq. (6). That is

$$n^* = \frac{e}{r}\left[\frac{r\cos(\theta_2-\varphi_2)-\sin\theta_2}{e\cos(\theta_2-\varphi_2)-\cos\varphi_2}\right] \tag{10}$$

Similarly

$$\lambda n^* = \frac{e}{r}\left[\frac{r\cos(\theta_1-\varphi_1)-\sin\theta_1}{e\cos(\theta_1-\varphi_1)-\cos\varphi_1}\right] \tag{11}$$

$$\lambda n^* = \frac{e}{r}\left[\frac{r\cos(\theta_3-\varphi_3)-\sin\theta_3}{e\cos(\theta_3-\varphi_3)-\sin\varphi_3}\right] \tag{12}$$

The values of θ_1, and θ_3 may be specified in terms of θ_2 such that

$$\theta_1 = \theta_2 - \psi' \tag{13}$$

$$\theta_3 = \theta_2 + \psi'' \tag{14}$$

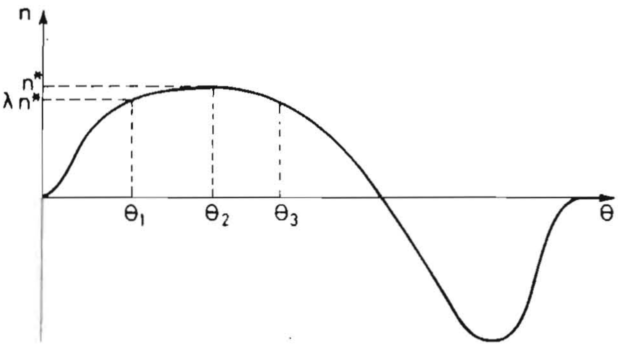

Fig. 2: The relation between n and Θ

The corresponding values of φ_1, and φ_3 are obtained from Eq. (5). Hence

$$\sin\varphi_1 = \frac{a_1 b_1 - \sqrt{b_1^2 + d_1^2 - a_1^2}}{b_1^2 + d_1^2} \tag{15}$$

$$\sin\varphi_3 = \frac{a_3 b_3 - \sqrt{b_3^2 + d_3^2 - a_3^2}}{b_3^2 + d_3^2} \tag{16}$$

Equations (8) through (16) are nine simultaneous equations which may be solved for nine unknowns. If the values of n^*, λ, ψ', and ψ'' are specified, these unknowns are $e, c, r, \theta_1, \theta_2, \theta_3, \varphi_1, \varphi_2$ and φ_3.

The analysis may be simplified by dropping the equations concerning θ_3 since they will be automatically satisfied except ψ'' which is to be determined after the synthesis. Also, it is advantageous not to use all the parameters of the mechanism, but leave some freedom which can be used to fulfil space requirements [2]. Hence, there are six equations which can be solved for $r, c, \theta_1, \theta_2, \varphi_1$, and φ_2. The value of e is selected according to design considerations.

Even with six equations an analytical solution cannot be obtained due to their high nonlinearity. Instead, a numerical scheme is suggested which yields satisfactory results.

Algorithm for the Numerical Solution

The system of equations to be solved are Eqs. (8), (9), (10), (11), (13), and (15). Equations (8), (10), and (11) are written in the forms

$$F_1 = er(1-n^*)^2\sin(\theta_2 - \varphi_2) + e\cos\theta_2 + rn^{*2}\sin\varphi_2 \tag{17}$$

$$F_2 = er(1-n^*)\cos(\theta_2 - \varphi_2) - e\sin\theta_2 + rn^*\cos\varphi_2 \tag{18}$$

$$F_3 = er(1-n^*)\cos(\theta_1 - \varphi_1) - e\sin\theta_1 + rn^*\cos\varphi_1 \tag{19}$$

The proper values of c and r make $F_1 = F_2 = F_3 = 0$.

An iterative scheme using the double false method [6] is described as follows:

1. A value for $r < 1 + e$ is assumed. A good starting value is $r = e/n^*$.
2. The allowable range of c is

   $1 + e - r \leqslant c \leqslant 1 - e + r$

   Thus, a starting value for c is taken equal to $1 - e + r - \varepsilon$; ε is a small quantity.
3. A starting value for θ_2 is assumed.
4. The corresponding value of θ_2 is computed from Eq. (9).
5. F_1 is calculated from Eq. (17).
6. Other values for θ_2 are assumed. Steps 4, and 5 are repeated until the sign of F_1 is changed.
7. The double false method is applied until the magnitude of F_1 is less than a permissible amount of error, say in the order of 10^{-6}.
8. Equation (21) is used to evaluate F_2.
9. The value of c is changed. Steps 3 through 8 are repeated until the sign of F_2 is changed.
10. The double false method is applied until the value of F_2 is less than 10^{-6}. The values of θ_2, φ_2, and c obtained thus far satisfy Eqs (8), (9), and (10).
11. Steps 2 through 7 are repeated.
12. The value of θ_1 is obtained from Eq. (13).
13. Equation (19) is used to compute F_3.
14. The value of c is changed, and steps 3,4,5,6,7,12, and 13 are repeated until the sign of F_3 is changed.
15. The double false method is applied until the value of F_3 is less than 10^{-6}. The values of θ_2, θ_1, φ_1, and c obtained satisfy Eqs. (13), (15), (17), and (19).
16. Several values for r are assumed according to the restriction in step 1. For each, the values of c from steps 10, and 16 are determined. A plot will yield two curves representing $F_2 \simeq 0$, and $F_3 \simeq 0$ which intersect at a point, Fig. 3 The values of r, and c at this point are the proper values. It is clear that if the difference between the values of c are plotted

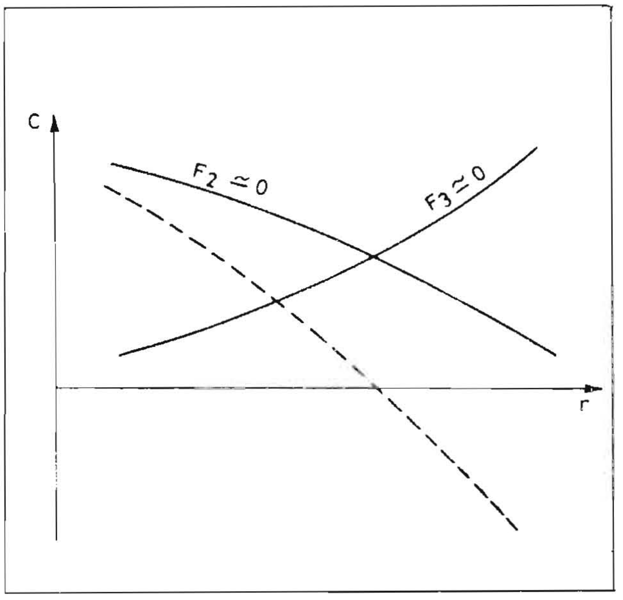

Fig. 3: Variation of C with r

versus r, a single curve is obtained (indicated by the dotted line, Fig. 3). In this case, the double false method can be applied to determine the proper values of c and r.

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تصميم آلية رباعية الأذرع ذات تغير محدود في نسبة السرع

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يقدم هذا البحث تركيبا لآلية رباعية الأذرع ذات قيمة عظمى لنسبة السرع محددة مسبقا مع افتراض تغير محدود لهذه النسبة عبر نطاق معين. ويعطى التحليل الرياضي مجموعة معادلات لاخطية لا يمكن حلها نظريا. ويقترح البحث أسلوبا لحل هذه المعادلات على الحاسب الآلي مستخدما طريقة الخطأ المزدوج العددية. وقد تم برمجة هذا الحل العددي بلغة فورتران ٤.