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Generalization of Single-Span Beam for Ponding

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The ponding of single span with various end conditions is generalized in the present paper. The normal mode shape for vibration of corresponding beams with similar end conditions is utilized.

Closed form for deflections, moments, shears and slopes as well as ponding factor expressions are derived. Several cases of pre-ponding deflection are utilized in the numerical examples to demonstrate the methods and the significance of the ponding factor is explained.

Nomenclature

A,B,C,D	Constants of integration
A	Cross sectional area
a	$=\sqrt{EI/m}$
C_n	Factor
Dx	Length of element
E	Modulus of elasticity
F_p	Ponding Factor
f	Natural frequency
I	Moment of inertia of cross section about its centroidal axis
I_x	Moment of inertia of cross section about x axis of the cross section.
k	$=(\frac{\gamma}{EI})^{1/4}$
L	Span of beam
M	Bending moment
MF	Magnification factor
m	Mass per unit length
T(t)	Time dependent function
$(\dot{\ })$	$=\frac{d(\ )}{dt}$
w	Uniformly distributed load
w_p	Actual ponding load
w(x)	Loading function
x	Horizontal coordinate
X(x)	Location dependent function
y	Deflection
y_c	Complementary solution of differential equation
y_i	Initial deflection
y_2	Additional deflection
$y_{,i}$	$=\frac{d^iy}{dx^i}$
β^4	$=\frac{\omega^2}{a^2}$
γ	Unit weight of liquid times beam's spacing
Δ	Maximum deflection
Δ_i,Δ_o	Amplitude of the first sine term for initially imperfect beam
φ_m,φ_n	$d^2\varphi/dx^2$ Normal Mode Shape
ω,ω_n	Natural Circular Frequency

Introduction

This is the third paper in a series of articles dealing with Ponding of Liquid on Flat Roofs. The previous papers treated various cases of loading of single spans and/or initial imperfection combined with ponding. Several end conditions have also been considered.

A generalization of the ponding problem of a single span with various end conditions and different types of loading is the subject of the present paper. Series representing the normal mode shapes of a vibrating beam with the prescribed end conditions is used to express the deflection of the preponding shape. Closed form solution of the deflections and the critical ponding factors are developed. Numerical examples for various pre-ponding deflections are utilized to demonstrate the method.

General Approach

The problem of ponding of a single-span beam having any of several end conditions can be handled by using infinite series analogous to Fourier series. The deflection due to imperfections, applied loads, and liquid loads above support level can be expressed as a series of the form.

$$y_i = \sum_{1}^{\infty} A_n \varphi_n \qquad (1)$$

Specifically, the φ_n functions are the normal mode shapes which the beam with the prescribed end conditions takes on in free vibration.

Free Vibration of Beam

The solution to the problem of free vibration of a prismatic beam can be found in most books on vibrations in engineering. The differential equation of motion is

$$EI\frac{\partial^4 y}{\partial x^4}dx = (mdx)\left(-\frac{\partial^2 y}{\partial t^2}\right) \qquad (2)$$

This can be put in the form

$$\frac{EI}{m}\frac{\partial^4 y}{\partial x^4} = -\frac{\partial^2 y}{\partial t^2} \quad \text{or} \quad a^2\frac{\partial^4 y}{\partial x^4} = -\frac{\partial^2 y}{\partial t^2} \qquad (3)$$

in which $a^2 = \frac{EI}{m}$

By separation of variables the following two differential equations can be obtained

$$\ddot{T} + \omega^2 T = 0 \text{ and}$$
$$\chi^{IV} \frac{\omega^2}{a^2}\chi = 0 \qquad (4)$$

in which ω^2 is a positive constant.

The second part of equation (4) can be written in the form

$$\frac{d^4 y}{dx^4} - \beta^4 y = 0 \qquad (5)$$

in which

$$\beta^4 = \frac{\omega^2}{a^2} = \text{constant}$$

It is seen that equation (5) is identical in form to the drived ordinary differential equation (1) for the weightless, initially-straight beam subject to ponding [1]. Since the boundary conditions are identical, the deflection of the ponded beam is the fundamental normal mode shape of the vibrating beam.

The solution is

$$y = A\sin\beta x + B\cos\beta x + C\,\text{Sinh}\beta x + D\,\text{Cosh}\beta x \qquad (6)$$

The solutions exist only for certain discrete values of β, as illustrated below for the case of the fixed-ended beam.

The boundary conditions are:

$x=0,\ y=0$: $A(0)+B(1)+C(0)+D(1)=0$ (a)

$x=0,\ y,1=0$: $A(1)-B(0)+C(1)+D(0)=0$ (b)

$x=L,\ y=0$: $A\sin\beta L + B\cos\beta L + C\,\text{Sinh}\,\beta L + D\,\text{Cosh}\beta L = 0$ (c)

$x=L,\ y,1=0$: $A\cos\beta L - B\sin\beta L + C\,\text{Cosh}\,\beta L + D\,\text{Sinh}\beta L = 0$ (d)

From equation (a)

$$B = -D$$

And from equation (b)

$$A = -C$$

substituting the values of C & D in equations (c) and (d) and expressing B in terms of A,

Therefore

$$A(\cos^2\beta L - 2\cos\beta L\,\text{Cosh}\beta L + \text{Cosh}^2\beta L - \text{Sinh}\beta L + \sin^2\beta L) = 0, \text{ or}$$
$$A(2 - 2\cos\beta L\,\text{Cosh}\beta L) = 0 \qquad (e)$$

This can be satisfied if either A or the term in parentheses is equal to zero. If A equals zero, however, then B,C, and D, which are related to it are also zero, and this is a trivial solution.

Setting the second term equal to zero, one has a transcendental equation with an infinite number of discrete roots (eigen values)

Therefore

$$1-\cos\beta L\ \mathrm{Cosh}\beta L=0 \tag{7}$$

Such a relation has the following roots:

$\beta_1 L=4.730$, $\beta_2 L=7.853$, $\beta_3 L=10.996$, $\beta_4 L=14.137$

Substituting of these roots into equation (6) gives the various normal mode shapes (eigen functions) associated with the roots. For example, the fundamental mode shape is:

$$y=B_1[\mathrm{Cosh}4.730\frac{x}{L}-\cos4.730\frac{x}{L}-0.9825\mathrm{Sinh}4.730\frac{x}{L}] \tag{8}$$

B_1 is undetermined, and can have any value.

The various natural circular frequencies can be obtained from

$$\omega_n=(\beta_n L^2)\sqrt{\frac{EI}{mL^4}}.$$

The various natural frequencies can be found from

$$f_n=\frac{\omega_n}{2\pi}=\frac{(\beta_n L)^2}{2\pi}\sqrt{\frac{EI}{mL^4}} \tag{9}$$

The natural frequencies are thus seen to be dependent on E, I, and L of the beam and on the boundary conditions through the β_n values.

The normal mode shapes of a single span of a prismatic beam with any combined end conditions form a set of functions orthogonal over the interval described by the span of the beam. The deflection y_i, in the ponding problem can be expanded into an infinite series of these functions.

The condition of orthogonality means that

$$\int_a^b \varphi_m\varphi_n dx=0 \qquad \text{for } m=n \tag{10}$$

Where φ_m and φ_n are eigen functions of arbitrary amplitude, such amplitudes may be chosen to make

$$\int_a^b \varphi_n^2 dx=\text{any constant} \tag{11}$$

If this constant is taken as unity, the functions are said to be normalized. A well-known set of tables of normal mode shapes for the most commonly considered end conditions are those of Young and Felgar [2]. They chose the amplitudes such that

$$\int_0^L \varphi_n^2 dx=L \tag{12}$$

For these functions, a beam deflection can be expressed into an infinite series of the form

$$y_i=A_1\varphi_1+A_2\varphi_2+A_3\varphi_3+\ldots A_n\varphi_n=\sum_1^\infty A_n\varphi_n \tag{13}$$

Multiplying both sides of the equation by φ_m and integrating between the limits of 0 and L, noticing the value of each integral is zero except for the one term in which m=n. For this term, the value is L, and one obtains

$$\int_0^L y_i\varphi_n dx=A_n L \tag{14}$$

From which

$$A_n=\frac{1}{L}\int_0^L y_i\varphi_n dx \tag{15}$$

This is the equation for the coefficient of the general term of the series.

The solution to the general case of the single span beam subjected to ponding can now be formulated. The additional deflection, y_2, due to ponding is expressed in the following differential equation

$$EI\frac{d^4y_2}{dx^4}=\gamma(y_i+y_2) \tag{16}$$

$$\frac{d^4y_2}{dx^4}-k^4y_2=k^4y_i=k^4\sum_1^\infty A_n\varphi_n \tag{17}$$

The particular solution will be an infinite series of φ_n. Each term in the series satisfies the boundary conditions since they are the same for both vibrating beam and ponded beam. The complementary solution therefore vanishes, and the complete solution consists of the particular solution alone, which is

$$y_2 = \sum_{n=1}^{\infty} C_n A_n \varphi_n \tag{18}$$

Where φ_n are solutions to the vibrating beam problem, so that

$$\frac{d^4\varphi_n}{dx^4} = \beta^4 \varphi_n. \tag{19}$$

y_2 is then found from

$$\sum_{n=1}^{\infty} C_n A_n \beta^4 \varphi_n - k^4 \sum_{n=1}^{\infty} C_n A_n \varphi_n = k^4 \sum_{n=1}^{\infty} A_n \varphi_n \tag{20}$$

Therefore

$$C_n = \frac{k^4}{\beta^4 - k^4} \tag{21}$$

The total deflection, y, is given by

$$y = y_i + y_2 = \sum_{n=1}^{\infty} A_n \varphi_n + \sum_{n=1}^{\infty} \frac{k^4}{\beta^4 - k^4} A_n \varphi_n \tag{22}$$

Therefore

$$y = \sum_{n=1}^{\infty} (y_i)n \frac{1}{1 - \frac{k^4}{\beta^4}} \tag{23}$$

Young and Felgar tabulate the values of $\beta_n L$ for the first five terms.

The deflection can now be expressed as,

$$y = \sum_{n=1}^{\infty} (y_i)_n \frac{1}{1 - \frac{k^4 L^4}{(\beta_n L)^4}} = \sum_{n=1}^{\infty} (y_i)_n \frac{1}{1 - \frac{\gamma L^4}{EI(\beta_n L)^4}} \tag{24}$$

The terms $1/\left\{1 - \left(\frac{K^4L^4}{\beta_n L}\right)^4\right\}$ are the deflection magnification factors of the various terms of the series.

The moment due to ponding may be obtained by differentiating equation (24) twice. Young and Felgar give values for the second derivative of φ_n with respect to $\beta_n x$ which can be used in this computation.

For many practical cases, only the first term of the series need be used. It is the only one which can physically become large, since all other terms require negative ordinates as well as positive ones.

The effect of ponding on a fixed-ended beam is illustrated in the examples presented at the end of this paper.

Significance of Critical Ponding Factors

The critical ponding factor is of little practical value as a limiting criterion to prevent ponding failures. It is a stability limit for a sustained storm situation, and as such, is analogous to the Euler load for a beam-column. If a beam has a greater-than-critical ponding factor, failure is certain. It it has one less than critical, failure may still occur because the beam may not be strong enough to carry the added load of the ponded liquid. Examples have been found in which failure occurred even though the ponding factor was only 0.3 of the critical.

Calculation of Critical Ponding Factors

The generalization of the single-span beam problem by expanding the pre-ponding deflection into an infinite series of normal mode shapes brings out the reason that the critical ponding factor is independent of load. The critical ponding factor can be defined as that one which will result in infinite deflections in a sustained storm with the beam depression always being fully filled. Any pre-ponding deflection can be expressed as an infinite series of normal mode shapes. Only the first term of the series can physically become large, however, for it is the only mode shape which has all positive ordinates. All other have both positive and negative, and a large negative ordinate would extend above the liquid surface.

An examination of the differential equations (19) reveals that γ in the ponding problem [1] is the counterpart of $m\omega^2$ in the vibrating beam problem. The critical ponding factor can then be evaluated as

$$\gamma = m\omega^2$$

The critical ponding factor is given in reference [1] as,

$$F_p = \frac{\gamma L^4}{\pi^4 EI}$$

Therefore $$F_p = \frac{\gamma L^4}{\pi^4 EI} = \frac{mL^4}{\pi^4 EI}\omega^2;$$

where $$\omega_n^2 = \beta_n^4 \frac{EI}{m},$$

where β_1 refers to the fundamental frequency

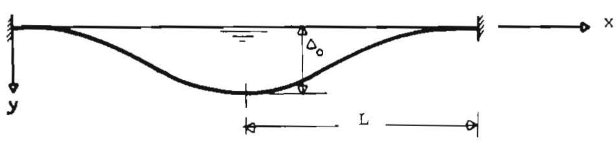
therefore

$$(F_p)_{cr} = \frac{(\beta_1 L)^4}{\pi^4} \qquad (25)$$

Moments in ponded beams can be easily calculated using the Young and Felgar tables for as many terms of the series as desired.

Example 1

Initial imperfection having same shape as first characteristic mode. For the first mode, the deflection, y_i will be



$$y_i(x) = A\varphi_1(x),$$

$$\varphi_1(x) = \mathrm{Cosh}\beta_1 x - \cos\beta_1 x - \alpha_1(\mathrm{Sinh}\beta_1 x - \sin\beta_1 x).$$

$$\beta_1 L = 4.73,\ (\beta_1 L)^4 = 500.5639,\ \alpha_1 = 0.9825,$$

For $\quad x = L/2,\ y_i = \Delta_o$
$\quad y_i = A(\varphi_1)_x = L/2;$

$$A = \frac{\Delta_o}{1.58815}.$$

The final deflection value, y, will be

$$y = y_i \times MF$$

$$MF = \left[\frac{1}{1-(\frac{KL}{\beta_1 L})^4}\right] = \left[\frac{1}{1-(\frac{KL}{4.73})^4}\right]$$

$$y(x) = \left(\frac{\varphi_1\Delta_o}{1.58815}\right)\left[\frac{1}{1-(\frac{KL}{4.73})^4}\right]$$

At $\quad x = L/2,\ y = \Delta_{max}$

$$\Delta_{max} = \Delta_o \times MF$$

or

$$\Delta_{max} = \frac{\Delta_o}{1-(\frac{kL}{4.73})^4}$$

$$\Delta_{max} \to \infty \qquad \text{as } kL \to 4.73$$

Calculation of the Moment M

$$y(x) = \frac{\Delta_o\varphi_1}{1.58815} \times MF$$

$$\frac{d^2y_2}{dx^2} = \left(\frac{\frac{d^2\varphi}{dx^2}}{1.58815}\right) \times MF$$

Tabular $\quad \varphi_{1,2} = \frac{1}{\beta_1^2}\frac{d^2\varphi_1}{dx^2}$

Since y_2 is the additional deflection due to ponding,

$$\frac{d^2y_2}{dx^2} = \frac{\beta_1^2\varphi_{1,2}}{1.58815} \times MF$$

$$M(x) = -EI\frac{d^2y_2}{dx^2} = -\left(\frac{EI\varphi_{1,2}}{1.58815}\right)\left[\frac{\Delta_o}{(\frac{4.73}{kL})^4 - 1}\right]$$

$$= -(\frac{EI}{1.58815}) \times (\frac{4.73}{L})^2 \times \varphi_{,2}(\beta x)\left[\frac{\Delta_o}{(\frac{4.73}{kL})^4 - 1}\right]$$

At $x = 0$, $\quad \varphi_{,2}(0) = 2$

$$M(0) = -28.17\frac{EI}{L^2}\left[\frac{\Delta_o}{(\frac{4.73}{kL})^4 - 1}\right]$$

At $x = \frac{L}{2}$, $\varphi_{,2}(L/2) = -1.21565$

Therefore,

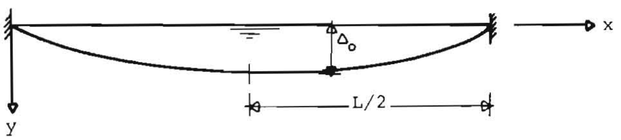
$$M(L/2) = +17.125\Delta_o(\frac{EI}{L^2})\left[\frac{1}{(\frac{4.73}{kL})^4 - 1}\right]$$

As $\ kL \to 4.73, \quad M \to \infty$

Example 2

Initial inperfection in shape of half sine wave. This example was worked in Ref. [3] using Fourier series, but as was pointed out, it yielded an awkward solution.

$$y_i(x)=\Delta_o\sin\frac{\pi x}{L}$$



$$\varphi_n=\mathrm{Cosh}\beta_n x-\cos\beta_n x-\alpha_n(\mathrm{Sinh}\beta_n x-\sin\beta_n x)$$

For the first mode, $n=1$

$$\varphi_1(x)=\mathrm{Cosh}\beta_1 x-\cos\beta_1 x-\alpha_1(\mathrm{Sinh}\beta_1 x-\sin\beta_1 x)$$

$$A_n=\frac{1}{L}\int_0^L y_i\varphi_n dx$$

$$A_1=\frac{1}{L}\int_0^L\left(\Delta_i\sin\frac{\pi x}{L}\right)[\mathrm{Cosh}\beta_1 x-\cos\beta_1 x-\alpha_1(\sinh\beta_1 x-\sin\beta_1 x)]dx$$

Therefore

$$A_1=\frac{\Delta_o}{\pi}\left[\frac{1}{1+\left(\frac{\beta_1 L}{\pi}\right)^2}(\cosh\beta_1 L+1-\alpha_1\mathrm{Sinh}\beta_1 L)+\frac{1}{1+\left(\frac{\beta_1 L}{\pi}\right)^2}(\alpha_1\sin\beta_1 L-\cos\beta_1 L+1)\right]$$

$$y_1(x)=A_1\varphi_1(x)$$

$$y_i=\frac{\Delta_o}{\pi}\left[\frac{1}{1+\left(\frac{\beta_1 L}{\pi}\right)^2}(\mathrm{Cosh}\beta_1 L+1-\alpha_1\mathrm{Sinh}\beta_1 L)+\frac{1}{1-\left(\frac{\beta_1 L}{\pi}\right)^2}(\alpha_1\sin\beta_1 L-\cos\beta_1 L-1)\right]\varphi_n$$

But $$y_i(x)=\sum_{n=1}^{\infty}(y_i)_n\left[\frac{1}{1-\left(\frac{kL}{\beta_n L}\right)^4}\right]$$

For the first mode, $\beta_1 L=4.73$, $\alpha_1=0.9825$ then,

$$y=\left(\frac{\Delta_o\varphi_1}{\pi\left[1-\left(\frac{kL}{4.73}\right)^4\right]}\right)\left[\frac{1}{1+\left(\frac{4.73}{\pi}\right)^2}(\mathrm{Cosh}4.73+1-0.9825\mathrm{Sinh}4.73)+\left(\frac{1}{1-\left(\frac{4.73}{\pi}\right)^2}\right)(0.9825\sin4.73-\cos4.73-1)\right]$$

thus $$y=0.6973\Delta_o\varphi_1\left[\frac{1}{1-\left(\frac{kL}{4.73}\right)^4}\right]$$

at $x=L/2$, $\varphi_1\ (L/2)=1.58815$ and $y_1(L/2)=\Delta$

$$\Delta=(0.6973\times1.58815\Delta_o)\left[\frac{1}{1-\left(\frac{kL}{4.73}\right)^4}\right]$$

i.e.

$$\Delta=1.1074\Delta_o\left[\frac{1}{1-\left(\frac{kL}{4.73}\right)^4}\right]$$

Δ_o could be any real positive number other than zero. It is clear that

$$\text{as } kL\rightarrow4.73,\ \Delta\rightarrow\infty$$

Calculation of the moment, M

$$\frac{d^2y_2}{dx^2}=\frac{\Delta_o}{\pi}\left[\frac{1}{1+\left(\frac{\beta_1 L}{\pi}\right)^2}(\mathrm{Cosh}\beta_1 L+1-\alpha_1\mathrm{Sinh}\beta_1 L)+\frac{1}{1-\left(\frac{\beta_1 L}{\pi}\right)^2}(\alpha_1\sin\beta_1 L-\cos\beta_1 L-1)\right]\left(\frac{\frac{d^2\varphi}{dx^2}}{\left(\frac{\beta_1 L}{kL}\right)^4-1}\right)$$

Since y_2 is the additional deflection due to ponding,

$$M=-\left(\frac{0.6973EI}{\left(\frac{4.73}{kL}\right)^4-1}\right)\times\left(\frac{22.3729}{L^2}\right).\varphi_{,2}(\beta x)$$

Therefore

$$M = -15.6006\Delta_o \frac{EI}{L^2}\left[\frac{\varphi_{,2}}{\left(\frac{4.73}{kL}\right)^4 - 1}\right]$$

At $x=0,\ \varphi_{,2}(0)=2$

Therefore

$$M(0) = -31.202\Delta_o \frac{EI}{L^2}\left[\frac{1}{\left(\frac{4.73}{kL}\right)^4 - 1}\right]$$

At $x=L/2,\quad \varphi_{,2}(L/2)=1.21565.$

Therefore,

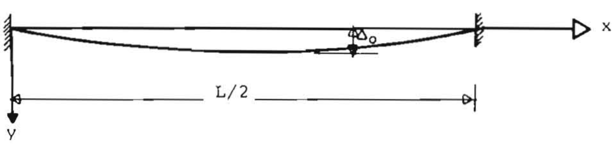
$$M(L/2) = 18.965\Delta_o \frac{EI}{L^2}\left[\frac{1}{\left(\frac{4.73}{kL}\right)^4 - 1}\right]$$

It should be noticed that for both expressions of the moments,

As $\quad kL \to 4.73,\ M \to \infty$

Example 3

Initial imperfection in shape of a parabola.



$$y_i(x) = Ax^2 + Bx + C$$

At $x=0,\ y_i=0,\ C=0$

At $x=L/2,\ y_i=\Delta_o$. Therefore,

$$\Delta_o = \frac{AL^2}{4} + \frac{BL}{2} \qquad \text{(a)}$$

At $x=L,\quad y_i=0$. Therefore,

$$0 = AL^2 + BL \qquad \text{(b)}$$

Solution of (a) and (b) gives,

$$A = -\frac{4\Delta_o}{L^2},\quad B = \frac{4\Delta_o}{L}$$

Therefore

$$y_i(x) = \frac{4\Delta_o}{L}\left(x - \frac{x^2}{L}\right).$$

$$A_1 = \frac{4\Delta_o}{L^2}\int_0^L \left(x - \frac{x^2}{L}\right)\left[\cosh\beta_1 x - \cos\beta_1 x - \alpha_1(\sinh\beta_1 x - \sin\beta_1 x)\right]dx.$$

or

$$A_1 = \frac{4\Delta_o}{\beta_1^2 L^2}\left[(\cosh\beta_1 L + \cos\beta_1 L + 2) - \alpha_1(\sinh\beta_1 L + \sin\beta_1 L) + \frac{2}{\beta_1 L}\left\{\alpha_1(\cosh\beta_1 L - \cos\beta_1 L) - (\sinh\beta_1 L + \sin\beta_1 L)\right\}\right]$$

For $\quad \beta_1 L = 4.73$, one gets

$A_1 = 0.7168\ \Delta_o.$

But

$y_i(x) = A_1\varphi_1(x).\ = 0.7168\Delta_o\varphi_1(x)$

The deflection at any point will be,

$$y(x) = y_i \times MF = 0.7168\Delta_o\varphi_1(x)\left[\frac{1}{1 - \left(\frac{kL}{4.73}\right)^4}\right]$$

At $x=L/2,\qquad y_{max}=\Delta$

Therefore

$$\Delta = 1.138\Delta_o\left[\frac{1}{1 - \left(\frac{kL}{4.73}\right)^4}\right]$$

As $kL \to 4.73,\qquad \Delta \to \infty$

Calculation of the moment, M

$$y_{,2} = \frac{d^2 y_2}{dx^2} = 0.7168\Delta_o \frac{d^2\varphi}{dx^2}\left[\frac{1}{\left(\frac{4.73}{kL}\right)^4 - 1}\right]$$

Since y_2 is the additional deflection due to ponding, one obtains

$M(x) = -EIy_{,2}.\qquad$ Therefore,

$$M(x) = -EI(0.7168\Delta_o) \times \frac{22.3729}{L^2} \times \varphi_{,2}(\beta x)\left[\frac{1}{(\frac{4.73}{kL})^4 - 1}\right]$$

At $x = 0$, $\varphi_{,2} = 2.0$ Therefore,

$$M(0) = -32.074\Delta_o \frac{EI}{L^2}\left[\frac{1}{(\frac{4.73}{kL})^4 - 1}\right]$$

At $x = L/2$, $\varphi_{,2} = -1.21565$. Therefore

$$M(\frac{L}{2}) = +19.495\Delta_o \frac{EI}{L^2}\left[\frac{1}{(\frac{4.73}{kL})^4 - 1}\right]$$

As $kL \rightarrow 4.73$, $M \rightarrow \infty$

Conclusion

The problem of ponding of the single-span beam has been generalized by expressing the pre-ponding deflection as an infinite series of the normal mode shapes of a vibrating beam having the same end conditions. It is shown that the magnification factor for the general term of the series is $\frac{1}{1 - \frac{F_p}{n^4 (F_p)_{cr}}}$. It is pointed out that the significance of the critical ponding factor, $(F_p)_{cr}$ lies in its presence in the magnification factor rather than its being a solution to the stability problem.

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تعميم الكمرة ذات البحر الواحد بالنسبة لأحمال المياه (المتراكمة)

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يعمم هذا البحث الدراسة على أشكال تحمل الكمرة ذات البحر الواحد بالنسبة لتركيزات مختلفة.

وقد استخدم في البحث شكل الدالة الذاتية لنفس التركيزات.

وقد أمكن الوصول الى حلول متكاملة للانحناء والعزم وكذلك القص والانحدار.

وأخيراً عولجت حالات عديدة ذات انحناء سابق واستخدمت في الحل العددي لشرح تطبيق الطريقة وأهميتها فيما يتعلق بمعامل تحميل المياه.