تحليل لفقد القدرة الزائدة في المجاري المفتوحة المنحنية

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تقدم هذه الدراسة تحليلا نظريا لسرعة الجريان في المجاري المفتوحة المنحنية . لقد تبنى هذا التحليل لاعداد صيغة تمثل فقدان القدرة الزائدة في المجاري المنحنية . المعامل المستعمل في الصيغة المستنتجة قوم من عدد هائل من المعلومات التجريبية المأخوذة من عدد من المجاري المنحنية انحناء غير حاد و بانحناء قدره ١٨٠ درجة .

بالنسبة للمجاري المنحنية التي فيها النسبة (نصف قطر الانحناء المتوسط / عرض المجرى) أكبر من أو يساوي ٣,٥٦ المعامل احتفظ بقيمة لا تعتمد على الخصائص الشكلية أو الهيدروليكية للمجرى ، بينما للمجاري التي فيها النسبة (نصف قطر الانحناء المتوسط / عرض المجرى) أصغر من ٣,٥٦ وجد أن المعامل يعتمد على نسبة عمق المائع الى نصف قطر انحناء المجرى .

بالنسبة للمجاري المنحنية التي فيها النسبة (نصف قطر الانحناء المتوسط / عرض المجرى) تقع بين ١,٠ ، ٤,٦٣ استعملت تعبيرات (صيغ) لحساب فقدان القدرة الزائدة .

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ANALYSIS OF EXCESS ENERGY LOSS IN CURVED OPEN CHANNELS

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In the present work theoretical analysis of velocities of flow in curved open channels is introduced. This analysis is adopted to develop an expression for the excess energy loss in curved channels. The coefficient used in the expression obtained is evaluated from massive experimental data from different smooth curved channels with 180° bends. For curved channels with $(r_c/b) \geqslant 3.56$ the coefficient retained a value independent of the geometric and hydraulic characteristics of the channels. While for channels with $(r_c/b) < 3.56$ the coefficient is found to depend on (h/r). For curved channels with $1 < (r_c/b) \leq 4.63$ expressions are introduced to calculate the excess energy loss.

Nomenclature

Δh_b = excess head loss in curved channel
g = acceleration due to gravity
C = Chezy Coefficient
h = normal depth of flow
r_c = radius along the centreline of a bend
N = z/h
S_θ = Longitudinal slope of water surface
S_r = radial slope of water surface
S_{θ_1} = the loss of head per unit angle of bend
M = numerical value depends on Chezy coefficient
Δh_{b1} = excess energy loss due to curvature in the curved length of channel
K_b = $\Delta h_b/(\bar{u}_o/2g)$
β = Coefficient
m = hydraulic radius
λ = numerical coefficient
S = $\dfrac{K_b}{f}\dfrac{L_c}{h}\left[\dfrac{b^2}{r_c\left[F(r)\,{}_{r_i}^{r_o}\right]^2} - 1\right]$
r = radius of a bend
L_c = length of a bend
u_o = mean velocity of flow
u, v = longitudinal and radial velocities at any point in the cross section
μ_τ = dynamic viscosity in turbulent flow
ν_τ = μ_τ/ρ
$\bar{u}$ = mean velocity along the vertical
ζ = shear stress at any point in the flow
ζ_o = Shear stress along the bottom
K = Karman Universal Coefficient = 0.4
z = distance from a channel bed
Q = discharge over a channel cross section
b = width of channel
r_i = inner radius of bend
r_o = outer radius of bend
f = Darcy friction factor

1. Introduction

The energy loss in bends of open channels and pipes is a subject of a special interest for many engineers. Experiments conducted in these bends proved the existence of excess loss in the energy of flow above that of straight channels and pipes.

The excess energy loss in bends of open channels as a ratio of the kinetic energy of flow was the representation adopted by most of investigators. This ratio was studied in relation to the hydraulic and geometric conditions of the flow.

In open channels due to the difficulties encountered in the measurements of the energy loss as a result of the water surface configurations, the net loss of energy due to bends is not easily and accurately measured. This has led some investigators to declare that in subcritical flow there is no excess dissipation of the energy of flow around bends. Among them Muller [1] and Ippen and Drinker [2].

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In laboratories excess energy loss due to bends was measured by Shukry [3], Allen and Chee [4] Rosovskii [5] and Leopold and others [6].

Field measurements conducted by Allen [7] and Rosovskii [5] proved the existance of excess energy loss due to bends in natural channels.

The physical factors producing excess loss in the energy of flow round bends is not always discussed by investigators. The excess in energy dissipation as a result of the change of momentum between the upstream and downstream of a bend in process of adjusting and re-adjusting of velocities was proposed by Shukry [3]. Leopold and others [6] suggested that it is caused by deflection of flow, or part of it, away from its former direction, and by spill resistance similar to that which happens at the foot of weirs and spillways. Rosovskii [5] proposed it as a result of increased shear along the bottom and secondary circulation. He proposed the expression:

$$\Delta h_b = \left(\frac{24\sqrt{g}}{C} + \frac{60\,g}{C^2}\right)\left(\frac{h}{r_c}\right)^2 \left(\frac{Lc}{h}\right)\left(\frac{\bar{u}_o^2}{2g}\right) \quad (1)$$

According to Rosovskii this expression has not been checked experimentally and owing to the complexity of the problem, it certainly needs further theoretical and experimental investigation.

2. Theoretical Analysis of Velocities:

2.1 Distribution of Radial Velocity Along the Vertical :

The exceedingly complicated nature of the flow round bends of open channels does not enable investigators to arrive at a sound expression for estimating the excess energy loss due to bends. For wide open channels of very small depth to width ratio and of very gentle bends, Rosovskii [5] reduced the Navier-Stokes equations for incompressible flow to:

$$\frac{u\,v}{r} = g\,S_\theta + \frac{\partial}{\partial z}\left(\nu_\tau \frac{\partial u}{\partial z}\right) \quad (2)$$

$$\frac{-u^2}{r} = -\,g\,S_\tau + \frac{\partial}{\partial z}\left(\nu_\tau \frac{\partial v}{\partial z}\right) \quad (3)$$

$$\zeta = \mu_\tau \frac{\partial u}{\partial z} = \zeta_o\left(1 - \frac{z}{h}\right)$$

and substituting for (z/h) the value of N and for u its corresponding value in terms of the mean velocity along the vertical as given by Eq. (7).

Rosovskii expressed Eq. (3) in the form:

$$g\,S_r - \frac{\bar{u}^2}{r}\left[1 + \frac{\sqrt{g}}{KC}\,(1 + \log_e N)\right]^2 = \frac{1}{h^2}\,\frac{\partial}{\partial N}\left[\nu_\tau \frac{\partial V}{\partial N}\right] \quad (4)$$

Solution of Eq.(4) yields the distribution of the radial velocity component, of the spiral motion, along the vertical.

$$v = \frac{\bar{u}}{k^2}\,\frac{h}{r}\left[F_1(N) - \frac{\sqrt{g}}{KC}\,F_2(N)\right] \quad (5)$$

where $F_1(N) = \int \frac{2\log_e N}{N-1}\,dN$ and $F_2(N) = \int \frac{(\log_e N)^2}{N-1}\,dN$

As both $F_1(N)$ and $F_2(N)$ are not integrable functions, they are given graphically.

Investigation of Eq. (5) shows that the variation of v along the vertical only depends on N or (z/h).

The present author obtained values of $F_1(N)$ and $F_2(N)$ by interpolation from the graphs [5] and the distribution of v plotted in Fig.1 for both C=100 and 50. It was found that in the upper part of the channel the maximum change in v is up to 5 percent while near the bottom may rise to 15 percent. As this difference or part of it may be due to interpolation of $F_1(N)$ and $F_2(N)$, it could be suggested that in most of the channel section, the radial velocity distribution is slightly affected by the boundary roughness and the distribution of v could be approximated by the eqaution:

$$v = \left(\frac{\bar{u}}{k^2}\right)\left(\frac{h}{r}\right)(3\sqrt{N} - 2.0) \quad (6)$$

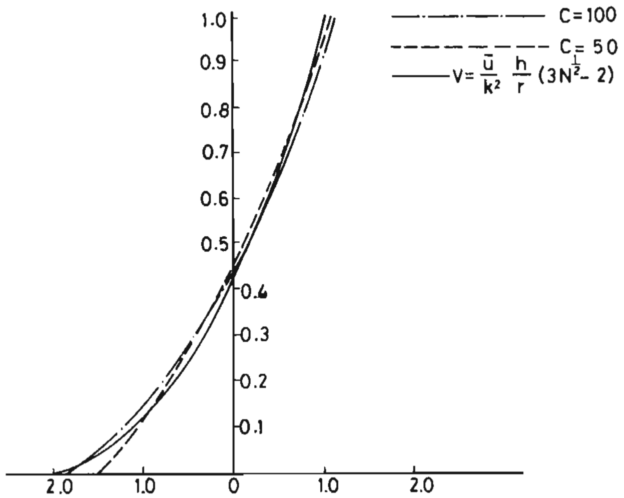


Fig. 1 RADIAL VELOCLTY DISTRIBUTION ACCORDING TO EQUATIONS 5 and 6

2.2 Distribution of u Over the Channel Width

Experiments conducted by investigators [3,5,6] have proved that at the entrance of a bend, there takes place a re-distribution of velocity over the channel width. Theoretically, if before entering the bend the velocities are uniformly distributed over the width then after entering the bend the greatest velocities will appear near the convex bank and the smallest velocities near the concave bank.

To study the correlation between the mean velocity along the vertical and the radius of curvature of currents, as a first approximation, the present author followed Rosovskii approach i.e. very gentle bends of small depth to width ratio. The longitudinal velocity u is given by:

$$U - u = \frac{U_*}{K}\log_e\frac{h}{z} = -\frac{U_*}{k}\log_e(z/h)$$

Therefore

$$u = U + \frac{U_*}{k}\log_e N \qquad (7\ A)$$

The mean velocity along the vertical can be expressed as

$$\bar{U} = \int_{N=0}^{N=1} u\,dN = \int_{N=0}^{N=1}\left(U + \frac{U_*}{k}\log_e N\right)dN$$

After integration, we get

$$\bar{u} = U - \frac{U_*}{k} = U - \frac{\bar{u}\sqrt{g}}{k_e} \qquad (7\ B)$$

In Eq. (7B) $U_* = \sqrt{\tau_o/e} = \sqrt{gRs}$ is written

in the form of $U_* = \bar{u}\frac{\sqrt{g}}{c}$ (7 C)

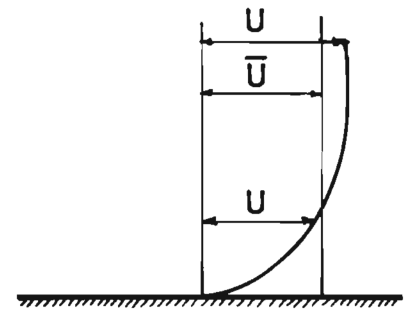
with the help of Chezy's equation $\bar{u} = C\sqrt{Rs}$ where R is hydraulic radius S is the slope of energy line and C is the Chezy's coefficient. Now putting values of U from Eq(7B) and U_* from Eq. (7C) in Eq.(7A) we can get the expression of u in terms of mean velocity along the vertical

$$u = \bar{u} + \bar{u}\frac{\sqrt{g}}{Kc} + \frac{\bar{u}\sqrt{g}}{Kc}\log_e N$$

$$= \bar{u}\left[1 + \frac{\sqrt{g}}{kc}(1 + \log_e N)\right] \qquad (7)$$

But $\frac{\zeta}{\rho} = \nu_\tau \frac{\partial u}{\partial z}$ and $S_\theta = \frac{\Delta H}{r\Delta\theta} = \frac{S_{\theta 1}}{r}$

where $S_{\theta 1}$ is the loss of head per unit angle of bend. Therefore Eq. (2) may be expressed in the form:



$$\frac{u\,v}{r} = \frac{g}{r}S_{\theta 1} + \frac{\partial}{\partial z}\left(\frac{\zeta}{\rho}\right) \qquad (8)$$

Substituting for v, u their values as given by Eqs. (6) and (7) respectively, then Eq. (8) may be written in the form:

$$\frac{\bar{u}^2}{k^2}\frac{h^2}{r^2}(3\sqrt{N} - 2)\left[1 + \frac{\sqrt{g}}{kC}(1 + \log_e N)\right]$$

$$-\frac{h}{r}g\,S_{\theta 1} = \frac{1}{\rho}\frac{\partial\zeta}{\partial N}$$

Solving this equation and considering $\zeta = 0$ at $N=1$ and $\zeta = \zeta_o$ at $N=0$, we obtain:

$$\frac{\zeta_o}{\rho} + \frac{\bar{u}^2}{k^2}\,\frac{h^2}{r^2}\left(\frac{2}{3}\,\frac{\sqrt{g}}{kC}\right) - \frac{h}{r}\,g\,S_{\theta 1} = 0 \qquad (9)$$

As another approximation let the mean velocity along the vertical be connected to the bottom shear ζ_o by the relation:

$$\frac{\zeta_o}{\rho} = \frac{\bar{u}^2\,g}{C^2}$$

Therefore Eq. (9) assumes the form:

$$\frac{\bar{u}^2\,g}{C^2}\left(1 + M\frac{h^2}{r^2}\right) = \frac{h}{r}\,g\,S_{\theta 1}$$

where $M = \dfrac{2}{3k^3}\,\dfrac{C}{\sqrt{g}}$

From the above equation the distribution of the mean velocity along the vertical ($\bar{u}$) over the channel width is given by:

$$\bar{u} = c\sqrt{S_{\theta 1}\left[\frac{r}{h}\,\frac{1}{\left(\frac{r^2}{h^2} + M\right)}\right]} \qquad (10)$$

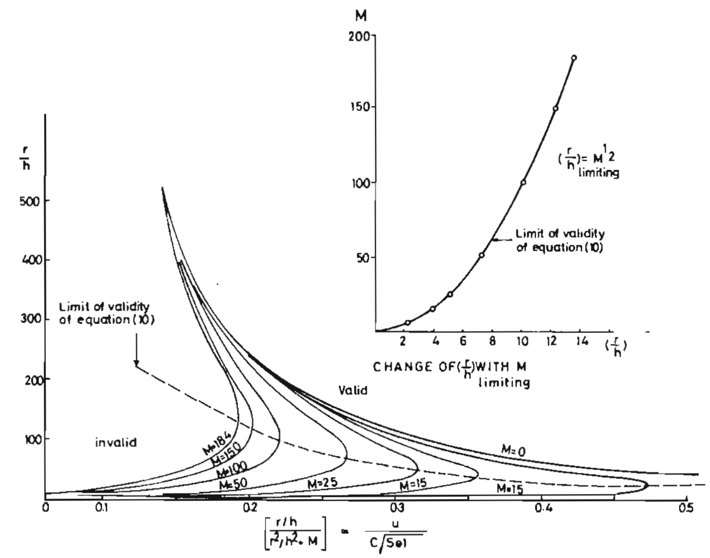
It should be asserted that in deriving Eq. (10) it is assumed that the ratio of water depth to the radius of curvature of the channel is small.

Introduction of v and u as given by Eqs. (6) and (7) established the parameter M. Accordingly there may be a limiting value of (r/h), below which Eq. (10) remains invalid. For $C=100$ and $k=0.4$ then M in ft. units=184.0.

If $\sqrt{\dfrac{r}{h}\,\dfrac{1}{\left(\frac{r^2}{h^2} + M\right)}}$ is plotted against (r/h) for $M=184$, the limiting (r/h) can be defined as that value at which

$\sqrt{\dfrac{r}{h}\,\dfrac{1}{\left(\frac{r^2}{h^2} + M\right)}}$ starts to decrease

with decreasing (r/h) values. This plotting is shown in Fig. 2 where the limiting (r/h) complying with the above definition is shown to be between 13-14 for $M=184$. Accordingly with $M=184$ Eq. (10) starts its validity for values of (r/h) greater than 14. To study the relation between M and the limiting (r/h) values, different values for M less than the theoretical were assumed and



Fig(2) DISTRIBUTION OF u ALONG THE VERTICAL OVER THE CHANNEL WIDTH FOR VARIOUS VALUES OF M.

$\sqrt{\dfrac{r}{h}\,\dfrac{1}{\left(\frac{r^2}{h^2} + M\right)}}$ were plotted against (r/h)

for every assumed value of M. The values of (r/h) at which $\sqrt{\dfrac{r}{h}\,\dfrac{1}{\left(\frac{r^2}{h^2} + M\right)}}$

starts to decrease with further decrease in (r/h) are considered as the limiting values.

Studying the values of M and the corresponding limiting values as in Fig.2, it is easy to notice that

$$(r/h)_{\text{limiting}} = \sqrt{M} \qquad (11)$$

As (r/h) changes over the channel width then to apply the above theoretical conclusions to curved channels at all values of (r/h) the proper value of M should be taken as $(r_i/h_i)^2$ as (r_i/h_i) is minimum over the channel cross section, where r_i is the radius of curvature of the inside wall of the bend and h_i is the depth of flow at the inside wall. Then Eq. (10) assumes the form:

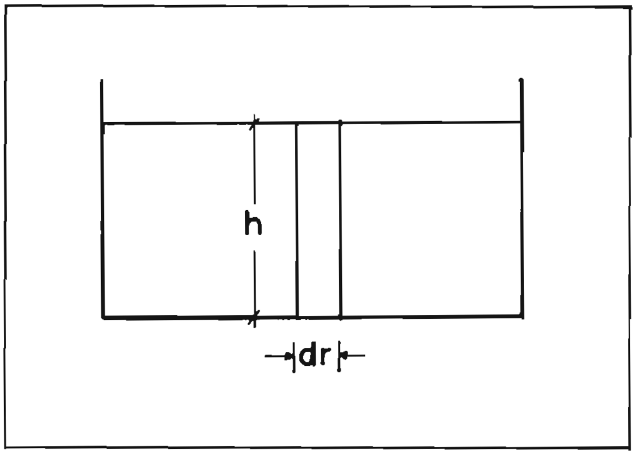
$$\bar{u} = C\sqrt{S_{\theta 1}\left[\frac{r}{h}\,\frac{1}{\frac{r^2}{h^2} + \frac{r_i^2}{h_i^2}}\right]} \qquad (12)$$

and Eq. (12) is assumed valid for all values of (r/h).

3. Excess Energy Loss

The excess energy loss in bends of open channels may be evaluated from Eq. (10) where:

$$\bar{u} = C\sqrt{S_{\theta 1}\left[\frac{r}{h}\,\frac{1}{\frac{r^2}{h^2}+M}\right]}$$



Considering an area-element of width dr and height h, the average discharge passing through this element may be given by:

$$dQ = \bar{u}\,h\,dr$$

$$= hC\sqrt{S_{\theta 1}\left[\frac{r}{h}\,\frac{1}{\left(\frac{r^2}{h^2}+M\right)}\right]}\;dr$$

To solve this equation, as a first approximation, neglect the effect of radial surface inclination.

$$Q = hC\sqrt{S_{\theta 1}h}\int_{r_i}^{r_o}\sqrt{\frac{r}{r^2+Mh^2}}\;dr$$

Substituting for $r = hy\sqrt{M}$

Then:

$$Q = C\sqrt{S_{\theta 1}}\,h^{3/2}\sqrt{h\sqrt{M}}\int_{r_i/\sqrt{M}h}^{r_o/\sqrt{M}h} \bar{y}^{1/2}\left(1+\frac{1}{y^2}\right)^{-1/2} dy \qquad (13)$$

$\left(1+\frac{1}{y^2}\right)^{-1/2}$ could be transformed into:

$$\left(1+\frac{1}{y^2}\right)^{-1/2} = \left[1+\left(-\frac{1}{2}\right)\left(\frac{1}{y^2}\right) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2\,!}\left(\frac{1}{y^2}\right)^2 + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{3\,!}\left(\frac{1}{y^2}\right)^3 + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)\left(-\frac{7}{2}\right)}{4\,!\qquad 4\,!}\left(\frac{1}{y^2}\right)^4 + \dots\dots\right]$$

Substituting into Eq. (13) we obtain:

$$Q = C\sqrt{S_{\theta 1}}\,h^{3/2}\sqrt{h\sqrt{M}}\int_{r_i/\sqrt{M}h}^{r_o/\sqrt{M}h}\left(\frac{1}{y^{1/2}} - \frac{1}{2y^{5/2}} + \frac{3}{8y^{9/2}} - \frac{15}{48y^{13/2}} + \frac{105}{384}\,\frac{1}{y^{17/2}} - \frac{945}{3840}\,\frac{1}{y^{21/2}} + \dots\dots\dots\right)dy$$

The above equation after integration assumes the form:

$$Q = C\sqrt{S_{\theta 1}}\,h^{3/2}\,F(r)\Big|_{r_i}^{r_o} \qquad (14)$$

where $F(r)\Big|_{r_i}^{r_o}$ is given by :

$$F(r)\Big|_{r_i}^{r_o} = \left[\sqrt{r}\left\{2 + 0.333\left(\frac{\sqrt{M}h}{r}\right)^2 - 0.107\left(\frac{\sqrt{M}h}{r}\right)^4 + 0.057\left(\frac{\sqrt{M}\,h}{r}\right)^6 - 0.0365\left(\frac{\sqrt{M}\,h}{r}\right)^8 + \dots\dots\dots\dots\right\}\right]_{r_i}^{r_o}$$

$$F(r)\Big|_{r_i}^{r_o} = \left[\sqrt{r}\,\{2 + 0.333\,(\frac{r_i}{r})^2 - 0.107\,(\frac{r_i}{r})^4 + 0.057\,(\frac{r_i}{r})^6 - 0.0365\,(\frac{r_i}{r})^8 + \ldots\ldots\}\right]_{r_i}^{r_o} \quad (15)$$

The total energy loss per unit length in the curved channel is given by:

$$S_\theta = S_t + S_c$$

S_t is the energy loss per unit length in the straight channel.

S_c is the energy loss per unit length due to curvature of the channels:

From Chezy Eq. $S_t = \dfrac{\bar{u}_o^2}{c^2 h}$

$$S_c = S_\theta - S_t$$

As shown before $S_\theta = \dfrac{S_{\theta 1}}{r_c}$, and putting the value of $S_{\theta 1}$ from Eq. (14), we get:

$$S_\theta = \frac{Q_2}{C^2 r_c h^3 \left[F(r)\Big|_{r_i}^{r_o}\right]^2} = \frac{\bar{u}_o^2 \cdot b^2}{C^2 r_c h \left[F(r)\Big|_{r_i}^{r_o}\right]^2}$$

Therefore $$S_c = \frac{\bar{u}_o^2}{C^2 h}\left[\frac{b^2}{r_c\left[F(r)\Big|_{r_i}^{r_o}\right]^2} - 1\right]$$

and the excess energy loss due to curvature of a channel is given by:

$$\Delta h_{b1} = \frac{\bar{u}_o^2}{C^2 h}\, L_c \left[\frac{b^2}{r_c\left[F(r)\Big|_{r_i}^{r_o}\right]^2} - 1\right] \quad (16)$$

where L_C is the curved length of the bend and u_o is the average velocity of flow in the bend.

Owing to the fact that the effect of a bend on the flow extends in the upstream and downstream channels, an additional energy loss will be taking place in both these channels. The resulting excess energy loss in the flow around bends will exceed the value given by Eq. (16).

Eq. (16) should be multiplied by a coefficient $\beta > 1$ to take into account the effect of the bend on the upstream and downstream channels.

Accordingly Eq. (16) may be expressed in the form:

$$\Delta h_b = \beta\,\frac{\bar{u}_o^2}{C^2 h}\, L_c \left[\frac{b^2}{r_c\left[F(r)\Big|_{r_i}^{r_o}\right]^2} - 1\right]$$

$$\frac{\Delta h_b}{\frac{u_o^2}{2g}} = \beta\,\frac{2g}{C^2 h}\, L_c \left[\frac{b^2}{r_c\left[F(r)\Big|_{r_i}^{r_o}\right]^2} - 1\right]$$

Expressing $$\frac{\Delta h_b}{\frac{u_o^2}{2g}} = K_b = \beta\,\frac{2g}{C^2 h}\, L_c \left[\frac{b^2}{r_c\left[F(r)\Big|_{r_i}^{r_o}\right]^2} - 1\right]$$

as $(2g/C^2) = f =$ friction Factor in Darcy friction formula

Then: $$K_b = \beta\,\frac{f}{h}\, L_c \left[\frac{b^2}{r_c\left[F(r)\Big|_{r_i}^{r_o}\right]^2} - 1\right] \quad (17)$$

As the term $\dfrac{b^2}{r_c\left[F(r)\Big|_{r_i}^{r_o}\right]^2}$ in Eq. (17) is only a function of the geometry of flow, then the coefficient of energy loss, K_b as given by Eq. (17) depends on the geometry of flow, the coefficient β and the friction factor f.

4. Comparison with Observation in Curved Channels

The coefficient β is calculated from observations on six curved perspex channels, of 180 degree bends, having the following characteristics (8).

Ch. No.	1	2	3	4	5	6
b	12.0"	5.875"	12"	5.875"	5.875"	5.875"
r_c/b	1	1.52	2	2.56	3.56	4.63

These curved channels were built to investigate the effect of Reynolds Number R_n and Froude Number F on K_b. For this reason the experimental program was devised to serve this purpose.

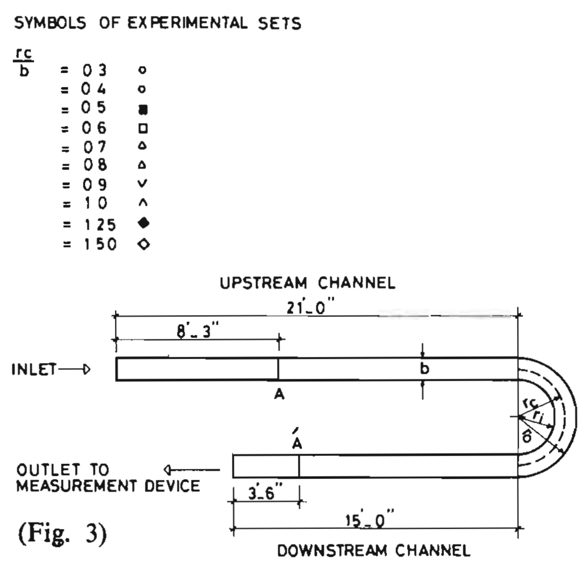
In every channel, at a fixed (h/b) ratio a set of 10 to 15 observations, at different kinetic energy were obtained and the corresponding K_b was calculated.

In channels 1 and 3, sets of measurements were conducted at (h/b) ratios of 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, and 1.0. In the rest of channels, sets of measurements were conducted at (h/b) ratios of 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.25 and 1.50.

The surface roughness of the perspex channels was studied in terms of Darcy friction factors f in the equation:

$$h_f = f \frac{L}{m} \frac{\bar{u}_o^2}{2g}$$

Values of f were obtained at different values of Reynolds Numbers in the straight channel, forming the upstream part of the curved channels before fixing the bends. The friction factors obtained in the straight channel were compared with the results obtained by different investigators in "perspex" and glass pipes [9] and channels [10] and found to be in full agreement.



(Fig. 3)

The total energy loss in the curved channels, was measured at established sections, A and A', in Fig. 3. Sections A and A' were established on the basis that, the hydraulic structure of the flow, at these sections is independent of the bend. Longitudinal velocity distributions have returned to normal, and no induced spiral motion by the bend appeared at these sections. This was verified by experiments.

The energy loss in a straight channel of the same length as the curved one between sections A and A' was calculated from the equation:

$$h_f = f \frac{L}{m} \frac{\bar{u}_o^2}{2g}$$

u_o and m were taken at the middle of the bend, and f was obtained from the established $f - R_n$ relation. L is the length of the curved channel between sections A and A'. The excess energy loss caused by a bend is defined as the difference between the total energy loss measured between the two definite sections A and A' and that in a straight channel of the same length L.

$$\Delta h_b = \Delta H - h_f$$

ΔH = The total energy loss between sections A and A' and the coefficient of energy loss in bends is given by :

$$K_b = \frac{\Delta h_b}{\frac{\bar{u}_o^2}{2g}}$$

As every set is conducted at a fixed (h/b) ratio, or fixed geometry of the flow at the middle of the bend, and K_b in Eq. (17) depends on the geometry of flow and β, then the average K_b value for each set could be considered as an accurate value for estimating β.

Values of $F(r)\,{}^{r_o}_{r_i}$ according to Eq. (15) was calculated for each channel and the corresponding β values were calculated according to Eq. (17) and the average K_b values obtained from each set.

For channels 5 and 6, β retained an average value of 2.75 for all the sets conducted in both channels. While in the rest of channels β varied with (h/b) ratio employed.

In Fig. 4 log β is plotted against $\log_e(r_c/h)$, where for the channels with gentle curvature log β is shown to have a constant value. For these two channels and perhaps for channels (with gentle bends) or with low depth of flow i.e. $(r_c/b) < 3.56$ Eq. (17) assumes the form:

$$K_b = 2.75 \frac{f}{h} L_c \left[\frac{b^2}{r_c \left[F(r) \frac{r_o}{r_i} \right]^2} - 1 \right] \quad (18)$$

The plotting of the observed K_b values against the parameters given by the above equation is shown in Fig. 5

For the rest of channels with $(r_c/b) < 3.56$ and according to the plotting in Fig. 4 β may be expressed by:

$$\beta = 53.0 \left(\frac{h}{r_c}\right)^{2/3} \qquad \text{Ch.1} \quad \frac{r_c}{b} = 1$$

$$= 15.8 \left(\frac{h}{r_c}\right)^{2/3} \qquad \text{Ch.2} \quad \frac{r_c}{b} = 1.56$$

$$= 10.6 \left(\frac{h}{r_c}\right)^{2/3} \qquad \text{Ch.3} \quad \frac{r_c}{b} = 2$$

$$= 8.5 \left(\frac{h}{r_c}\right)^{2/3} \qquad \text{Ch.4} \quad \frac{r_c}{b} = 2.56 \qquad (19)$$

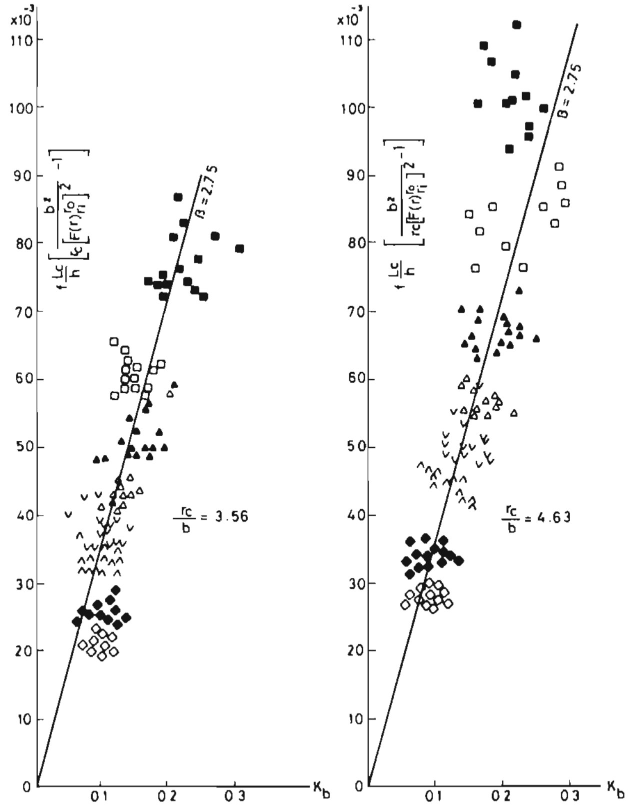
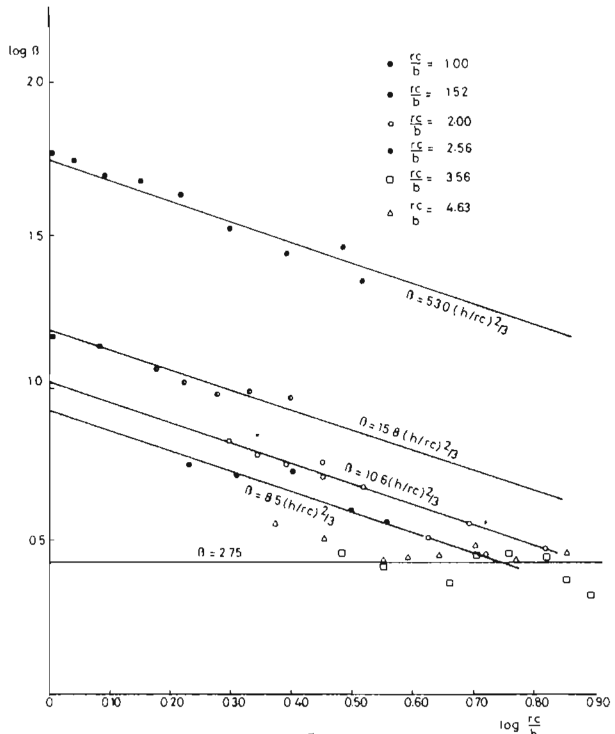


Fig.(5) OBSERVED VALUES OF K_b IN CHANNELS 5 and 6



Fig(4) CHANGE OF β WITH $\frac{r}{h}$ IN CHANNELS 1 to 6.

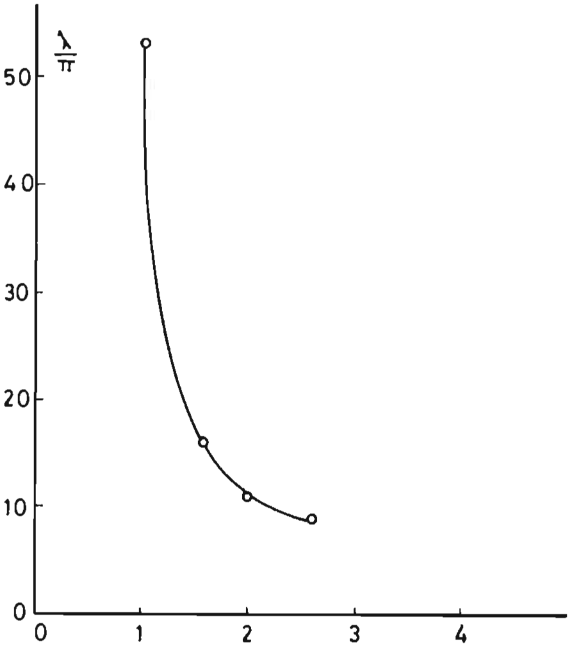


Fig (6) CHANGE OF $\frac{\lambda}{\pi}$ WITH $\frac{r_c}{b}$

For further analysis of β these equations may be written in the form:

$$\beta = \frac{\lambda}{\pi}\left(\frac{h}{r_c}\right)^{2/3} \qquad (20)$$

As shown by Eq. (19) and Fig. 6, (λ/π) increases with the curvature of the channels. The abrupt increase in (λ/π) and consequently in β indicates that different features had taken place in this channel and to a lesser extent in channels 2, 3 and 4. The most prominent of these features are the separation of the flow from its boundary and the increased turbulence in the flow due to the mixing action which takes place particularly in sharp bends.

Separation was indeed detected in channel 1, but observed in more gentle channels by many investigators [3,5.]

Accordingly for channels with $(r_c/h) < 3.56$ Eq. (17) may be expressed in the form:

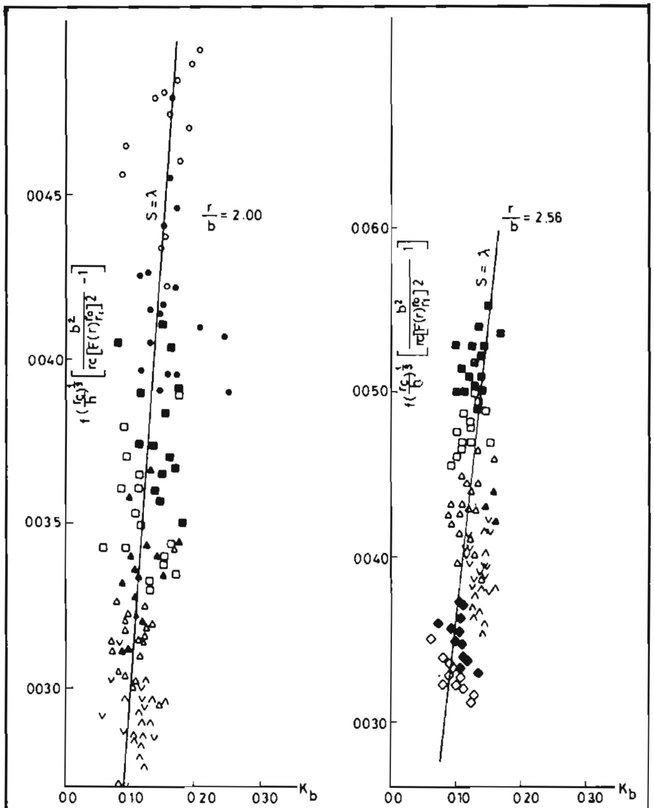
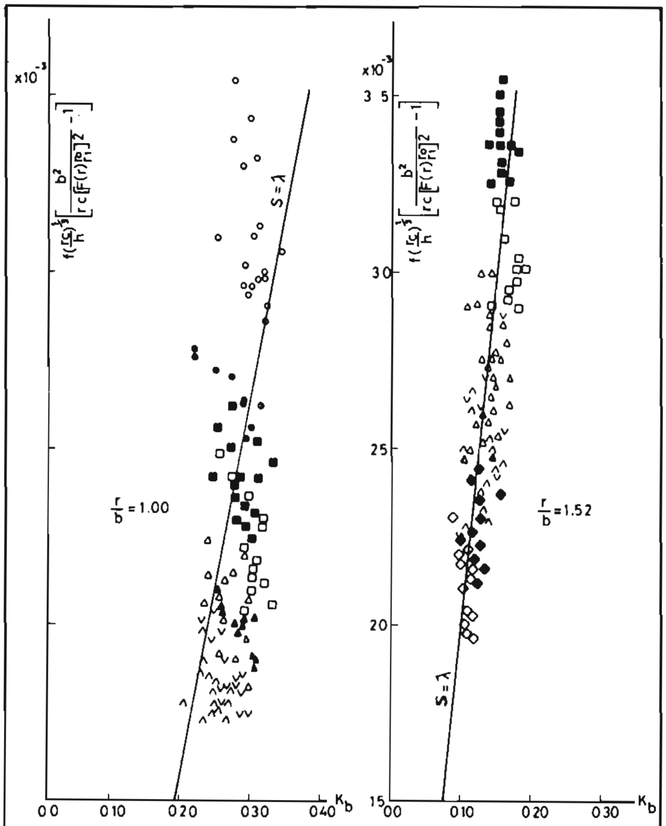


Fig.(8) OBSERVED VALUES OF K_b IN CHANNELS 3 and 4

$$K_b = \frac{\lambda}{\pi}\left[\left(\frac{r_c}{h}\right)^{1/3} \frac{b^2}{r_c\left[F(r)\,\frac{r_o}{r_i} \right]^2} - 1 \right] \qquad \dots (21)$$



Fig(7) OBSERVED VALUES OF K_b IN CHANNEL 1 and 2

In Fig.7 and 8 the observed K_b values are plotted against the parameters of Eq. (21). In these figures as well as in Fig. 5 the grouping of points is actually as a result of the original programing of the experimental work.

5. Conclusions:

1. The theoretical analysis of the excess energy loss in curved channels, based on the hydrodynamical concepts adopted in this work may be applied to curved channels with $(r_c/_b) > 3.56$. This excess energy loss could be evaluated from Eq. (18).

2. As $F(r)\,\frac{r_o}{r_i}$ in this analysis is a function of geometry of the channel, the excess energy loss in curved channels depends on the geometry of flow and the surface roughness of the channel.

3. The excess energy loss in curved channels with $(r_c/h) < 3.56$ could be evaluated from Eq. (21) where values of (λ/π) are obtained by interpolation from Fig. 6.

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