THE BUCKLING OF SANDWICH BEAMS AND PANELS UNDER EDGEWISE COMPRESSIVE LOADS WITH FIXED ENDS

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The paper deals with the various modes of buckling of sandwich beams and panels under edgewise loads. A complete survey of all the existing theories is made. Sixty-one beams and panels made of two types of face materials, aluminum and rigid polyvinyle chloride, and two types of core materials, expanded P.V.C. and expanded polyurethene, were tested. The faces and the cores were of various thickness. It was found that each mode of buckling could be predicted from the properties of the sandwich beams and panels.

NOMENCLATURE

SYMBOLS	QUANTITY
A	cross sectional area
b	beam or panel width
c	coefficient of end fixity
d	core thickness
D	flexural rigidity of beam or panel, without the shear effect of the core
E_c	Young's modulus of core
E_f	Young's modulus of faces
$E_{xc}, E_{zc}\ E_{yc}$	is the tangent modulus at the origin of the core material in direction x,z,y, respectively
G_c	shear modulus of core
G_f	shear modulus of faces
h	overall depth of the beam or panel
l	beam length
L	$\frac{l}{\sqrt{c}}$
n	a constant = 1. 2
N	the amplitude of face dimpling or face wrinkling
N_o	initial amplitude of face dimpling or face wrinkling
P_{cr}	critical buckling load
P_e	Euler buckling load and is equal to $\frac{\pi^2 D}{L^2}$
t	face thickness
α	$\frac{1}{E_x}(1-\nu_{xy}\ \nu_{xy})$
μ	wave length
ν_c	Poisson's ratio of core
ν_f	Poisson's ratio of face
PP	sandwich beam or panel of P.V.C. -face and P.V.C.-core
PPU	sandwich beam or panel of P.V.C. -faces and Polyurethane core
ALP	sandwich beam or panel of aluminium faces and P.V.C.-core
ALPU	sandwich beam or panel of aluminium faces and polyurethane core

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INTRODUCTION

Since the late 1930's and the early 1940's up to the present time, many engineers and scientists have studied the problem of buckling of sandwich constructions, and many theories have been developed for different load arrangements and end conditions. In order to simplify the analysis of buckling theories here, only one type of loading and end fixity is going to be considered. This is the buckling of sandwich beams under edgewise compressive loads with fixed ends. This simplifies the experimental procedures and provides a direct comparison with most of the theories of buckling of sandwich constructions under the same loading arrangement which have been already established.

The buckling of sandwich beams and panels is a complementary work to that of bending under lateral loads[1]. If a sandwich construction is subjected to edgewise loads as in fig. (1), the faces act as elastically supported beams or columns and can, therefore, take a definite critical load. The deflections of such beams or panels occur at small loads and increase rapidly as the load approaches its critical value. The core restrains these deflections which results in tensile and shear stresses developing in it and in the bond between the core and the faces. These stresses increase very rapidly as the critical load is approached, and thus failure takes place in the core or bonds at loads less than the critical load. However failure of the composite beam could also be due to general instability of the sandwich construction arising mainly from situations where the applied loads cause the member under load to wrinkle, buckle, or collapse[2].

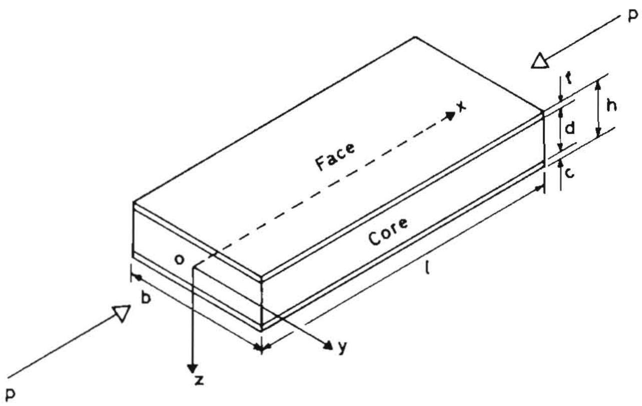

FIG.1_ SANDWICH PANEL UNDER COMPRESSIVE LOAD

To analyse the buckling effects on sandwich beams it is necessary to realise that different beams take different shapes under the influence of loads, depending on the beam size, face to core ratio, and the composition of the constituent materials. Each beam will take a definite pattern under load. This pattern is called the failing mode. In general four modes of failure can be identified in most of the buckling experiments of sandwich beams, as shown in fig. (2). They are as follows[3]:

1. General buckling mode
2. Face wrinkling mode
3. Shear crimping mode
4. Face dimpling mode.

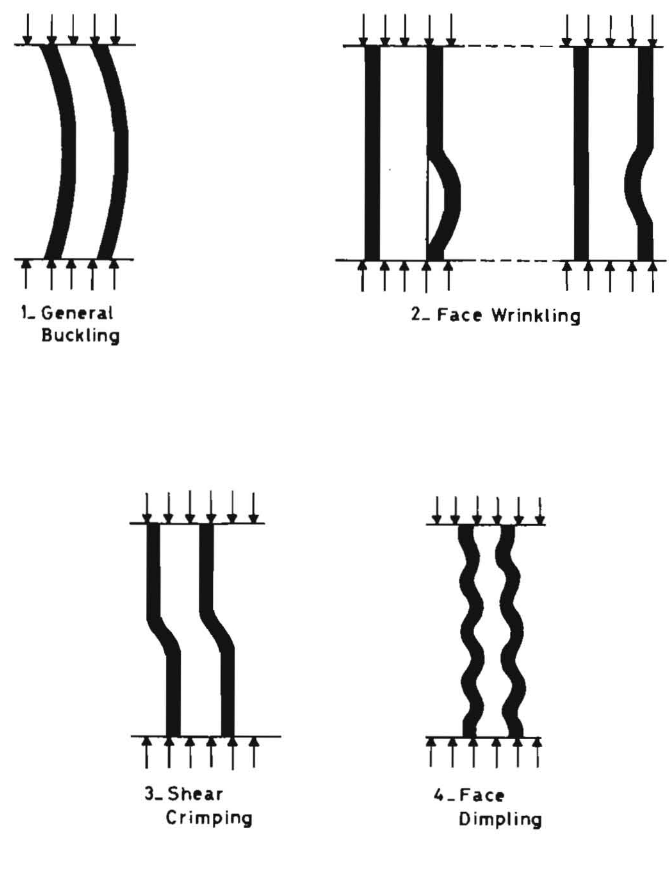

FIG. 2_ THE FOUR MODES OF FAILURE

1- GENERAL BUCKLING MODE

This mode of buckling is normally known as the quasi-Euler buckling mode, and occurs only among the medium and the long beams and it means that the buckling which occurs follows closely the lines of conventional Euler buckling. As the load on the beam is increased from zero, the lateral deflection at the centre increases very slowly, until the load is close to the buckling load, when this central deflection increases very rapidly with small increase in the end loads, until maximum load is reached, at which the beam fails. Expressions have been derived by several authors to cover this type of buckling. Starting with Timoshenko[4] and taking the shear deformation in the core into consideration, the following formula was derived

$$P_{cr} = \frac{P_e}{1+\dfrac{nP_e}{AG_c}} \qquad \ldots\ldots (1)$$

Feeding the proper constants[2], equation (1) becomes

$$P_{cr} = \frac{\dfrac{8\pi^2 bt}{3\alpha L^2}\left[3\left(\dfrac{d}{2}\right)^2 + 3\left(\dfrac{d}{2}\right)t + t^2\right]}{1+\dfrac{9.6\pi^2 t\left[3\left(\dfrac{d}{2}\right)^2 + 3\left(\dfrac{d}{2}\right)t + t^2\right]}{3\alpha L^2 d G_c}} \qquad \ldots\ldots (2)$$

Williams, Leggett and Hopkins[5] derived the following formula

$$P_{cr} = 2\left[\frac{G_c\dfrac{d}{2}b\,(P_1+P_2+P_c)+P_2+4P_c+\left(\dfrac{\frac{d}{2}}{\frac{d}{2}+t}\right)^2 P_1}{P_c+\left(\dfrac{\frac{d}{2}}{\frac{d}{2}+t}\right)^2 P_1+G_c\dfrac{d}{2}b}\right] \qquad \ldots\ldots (3)$$

where $P_1 = \dfrac{\pi^2}{2}\left(\dfrac{d}{2}+t\right)^2\dfrac{tbE_f}{L^2}$

and $P_2 = \dfrac{\pi^2}{6}\,\dfrac{t^3 bE_f}{L^2}$

and $P_c = \dfrac{\pi^2}{96}\,\dfrac{d^3 bE_c}{L^2}$

However in the work carried out by Van der Neut[6] and Hoff and Mautner[7], the core capacity to carry axial load was neglected. This leads to the equation

$$P_{cr} = \frac{P_1P_2+G_c\dfrac{d}{2}\,bp_1+G_c\dfrac{d}{2}\,b\,P_2}{P_1+G_c\dfrac{d}{2}\,b} \qquad \ldots\ldots (4)$$

If the bending rigidity of the faces is neglected, equation (4) will give the same result as equation (1).

Williams[8] used the split rigidity approach and derived the following formula

$$\frac{1}{P_{cr}} = \frac{1}{\frac{\pi^2 tb}{L^2}\left[2E_f\left(\frac{t^2}{3}+\frac{dt}{2}+\frac{d^2}{4}\right)+E_c\frac{d^3}{12t}\right]} + \frac{1}{bdG_c} \qquad \ldots\ldots (5)$$

and finally, Norris[9] derived the following formula[2]

$$P_{cr} = 2\left(\frac{\pi^2 BS}{SL^2+\pi^2 B}\right) \qquad \ldots\ldots (6)$$

where B = bending stiffness = $E_f \dfrac{(h^3-d^3)}{12\,(1-\nu_f^{\,2})}$

and S = shear stiffness = $\dfrac{(d+t)^2\, G_c}{d}$

From the previous equations it is clear that general buckling of sandwich beams and panels in the Euler fashion has been given a great deal of attention by many authors. This is partly due to its common occurance in practice, and partly due to the simplicity of deriving the formula for buckling loads. In general a sandwich beam will fail in a single mode if the elastic properties of its core are so chosen as to preclude the other three types of instability of the faces at lower values of the applied compressive stress than that corresponding to beam failure.

2- FACE WRINKLING MODE

This is often known as face buckling, face rippling or wrinkling type of instability of the face of a sandwich beam or panel, but in fact the problem is not one of instability, but rather one of progressive deformation due to initial irregularities and eccentricities in the facings. During the edgewise compression of the sandwich the irregularities of the faces increase gradually, thereby increasing the load on the adhesive layer until failure occurs, at which time rapid face deflection takes place to form the wrinkles.

The first study of the face wrinkling mode was in 1939, by Gough, Elan and De Bruyne[10]. For an infinite supporting medium they derived the following equation[2].

$$P_{cr} = 2bt\,\psi\,\sqrt[3]{E_c^2\,E_f} \qquad \ldots\ldots (7a)$$

where $\psi = \sqrt[3]{\dfrac{9}{4(1+\nu_c)\,(3-\nu_c)}}$

and the wave length is

$$\mu = 2\pi t\,\sqrt[3]{\frac{(1+\nu_c)\,(3-\nu_c)}{12}\left(\frac{E_f}{E_c}\right)} \qquad \ldots\ldots (7b)$$

Williams, Leggett and Hopkins[5], derived the equation

$$P_{cr} = 2bt\,(.85)\,(G_c\,E_{zc}\,E_f)^{\frac{1}{3}}\left(1+\frac{dE_{xc}}{2tE_f}\right) \qquad \ldots\ldots (8a)$$

$$\text{and } \mu = 2\pi t\,\sqrt[3]{\frac{2}{3\,(1-\nu_f^2)}}\left(\frac{E_f^2}{G_c\,E_{zc}}\right)^{\frac{1}{6}} \qquad \ldots\ldots (8b)$$

This form of failure can only be true if $l > \frac{1}{2}\mu$.

Yusuff[11,12] derived a thorough solution to the problem of wrinkling. Referring to fig. (3), three solutions were obtained.

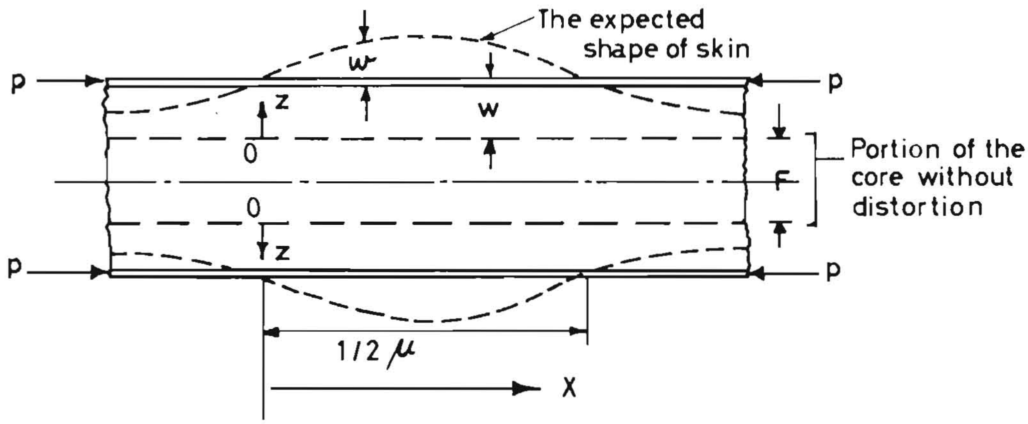

FIG. 3 – CONFIGURATION OF FACE – WRINKLING MODE

W = The depth of the distorted zone in the core measured from the skin.

ϖ = The depth of the distorted skin at x distance from its initial position.

d = 2W + F

$$P_{cr} = 2bt\,(.961)\,(E_f\,E_c\,G_c)^{\frac{1}{3}} \quad \text{for } w < \frac{d}{2} \qquad \ldots\ldots (9a)$$

$$\text{and } P_{cr} = 2bt\,(.68)\,(E_f\,E_c\,G_c)^{\frac{1}{3}} \quad \text{for } w > \frac{d}{2} \qquad \ldots\ldots (9b)$$

$$\text{and } P_{cr} = 2bt\,(.82)\,(E_f\,E_c\,G_c)^{\frac{1}{3}} \quad \text{for } w = \frac{d}{2} \qquad \ldots\ldots (9c)$$

where w is the distorted shape of the core and z is the vertical axis along which w is measured.

$$\text{and} \quad \mu = 2.614t\left(\frac{E_f^2}{E_c\,G_c}\right)^{\frac{1}{6}} \qquad \ldots\ldots (9d)$$

Also Cox's[13] approach led to the equation

$$P_{cr} = 1.2tb\,\left(E_c\,E_{zc}^3\,E_f^2\right)^{\frac{1}{6}} \qquad \ldots\ldots (10)$$

and finally a rough design formula was derived by Nieuwenhuizen[14]

$$P_{cr} = .5\,(E_f\,E_c\,G_c)^{\frac{1}{3}}\,2tb \qquad \ldots\ldots (11)$$

A rough formula is proposed[2] in the form

$$P_{cr} = 2tb\,(E_f\,E_c\,G_c)^{\frac{1}{3}} \qquad \ldots\ldots (12)$$

and equations (11) and (12) can be drawn in fig. (4) to represent the lower limit and the upper limit of failing loads in wrinkling mode. If the constant outside the bracket is referred to by (J), then $\sigma_{cr} = J(E_c E_c G_c)^{\frac{1}{3}}$ where J is one of the following, .961, .68, .82, .60, .50, or 1.0 depending on the formula used.

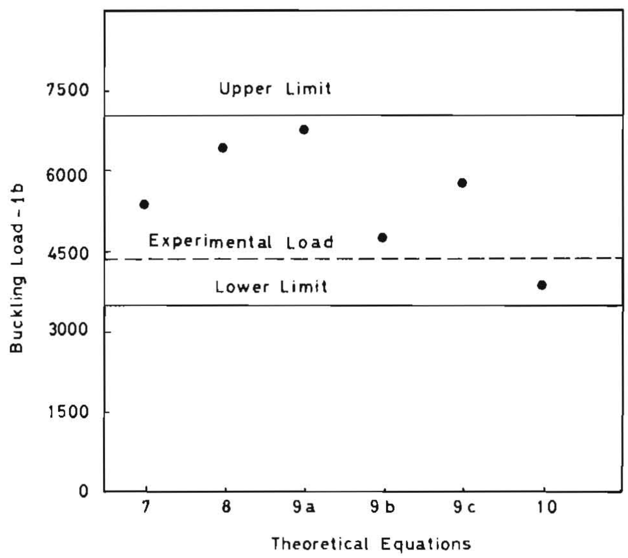

FIG. 4_UPPER AND LOWER LIMIT OF FACE_WRINKLING LOAD FOR No. 6 SANDWICH BEAM OF TABLE 2

3- SHEAR CRIMPING MODE

This mode of failure happens only in the core material. When the face of a sandwich beam or panel wrinkles under end load it will provide a shear instability in the core. If the core is weak then the sandwich beam will collapse in shearing of the core rather than face wrinkling failure. Therefore, the shear instability in a sandwich construction is only a result of its face wrinkling. Norris[15] shows that the critical buckling load of a sandwich beam when the whole load is taken by the skins is

$$P_{cr} = dbGc \qquad \ldots. (13)$$

This is clearly independent of the skin thickness or skin type of material.

4- FACE DIMPLING MODE

This type of sandwich beam and panel buckling is normally known as intercellular buckling. It is similar to that of face wrinkling mode, that is, it is not a question of instability, but results from a progressive deformation due to initial irregularities and eccentricities in the facings. These irregularities increase gradually and take definite shape which depends on the stiffness of the core material and its cell size. Finally a failure of the core near the skins or the adhesive layer will occur. It is very difficult to distinguish between the face wrinkling mode and this mode of failure up to the buckling load, while it would be easier to differentiate between the two types of buckling modes when the beams or panels are loaded to the collapse due to face failure while in the face dimpling mode the beam will fail due to core failure near the faces or below the adhesive layer. It is even more difficult in practice to distinguish between those two modes of failure when the core materials are the cellular foam type, which have very small and irregular cell structures. However it is debatable whether shear failure of the core due to initial waviness of the face can be put under the intercellular failure of the sandwich beam and panel, rather than under the shear crimping mode.

So far there is no sound theoretical analysis for face dimpling mode. An empirical approach has been put forward by Norris and Kommers[16] which is based on the reduced Young's modulus (E_r) and the cell size of the core, but it is only applicable to honeycomb core materials.

$$P_{cr} = 2tb \left(\frac{E_r}{3}\right) \left(\frac{t}{R}\right)^{\frac{2}{3}} \qquad \ldots. (14)$$

where $E_r = \dfrac{4E_f E_{ft}}{(\sqrt{E_f} + \sqrt{E_{ft}})^2}$

where E_{ft}= tangent modulus of the face material at the origin

and R = cell radius.

Yusuff[11] taking into consideration the initial waviness of the faces, derives the following equations.

$$N = \frac{N_o}{\dfrac{\sigma_{cr}}{\sigma} - 1}$$

where σ_{cr}= critical stress or failing stress of the sandwich beam or panel.

and σ = the failing stress in the core material. Yusuff discusses two cases,

1) When the failure of the core is due to tension or compression, that is, when $d < 2W$, see fig. (3), and if T_c = the ultimate tensile or compressive strength of the core, then

$$N = \frac{T_c\, d}{2E_c} \text{ and therefore}$$

$$\frac{\sigma_{cr}}{\sigma} = \frac{2N_o E_c}{T_c\, d} + 1$$

and using $\sigma_{cr} = .68\,(E_f E_c G_c)^{\frac{1}{3}}$

therefore

$$P_{cr} = 2tb\left[\frac{.68 T_c\, d\,(E_f E_c G_c)^{\frac{1}{3}}}{2N_o E_c + T_c\, d}\right] \quad \ldots (15)$$

2) When the failure of the core is due to shear, that is, the core is assumed fairly thick, $d > 2W$ and if ζ_c = the ultimate shear strength of the core then

$$P_{cr} = 2tb\left[\frac{.96\,\zeta_c t\sqrt{E_f}\,(E_c G_c)^{\frac{2}{3}}}{\zeta_c t\left(\frac{E_f}{E_c G_c}\right)^{\frac{1}{6}} + 2.4 G_c N_o}\right] \quad \ldots (16)$$

In the inelastic region E_f must be replaced by E_{re} in equations (15) and (16), where E_{re} is another reduced modulus

$$E_{re} = \frac{2E_f E_{ft}}{E_f + E_{ft}}$$

EXPERIMENTAL RESULTS

Sixty-one sandwich beams and panels were prepared[2] and tested as shown in fig. (5). Figures (6) to(11) show some of the results plotted for various material combinations of sandwich beams and panels, while table (1) shows the comparison between the theoretical equations of general buckling mode(1–6) and the experimental values of critical loads.

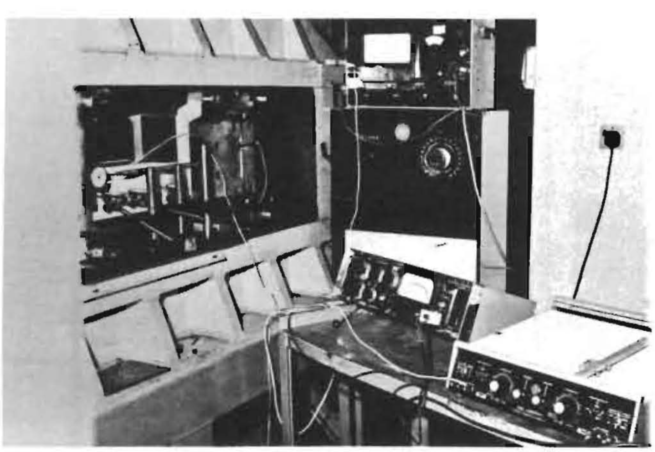

FIG.5_BUCKLING MACHINE WITH LOAD AND STRAIN RECORDERS

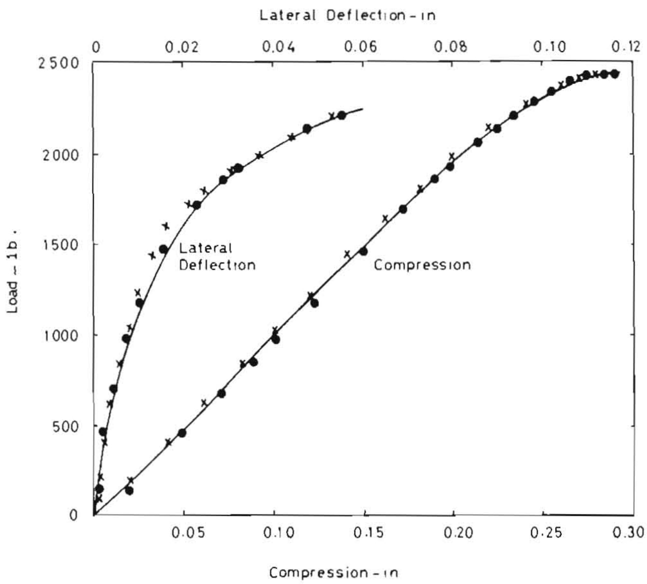

FIG.6_COMPRESSION AND LATERAL DEFLECTION VERSUS LOAD FOR PP_ SANDWICH BEAMS

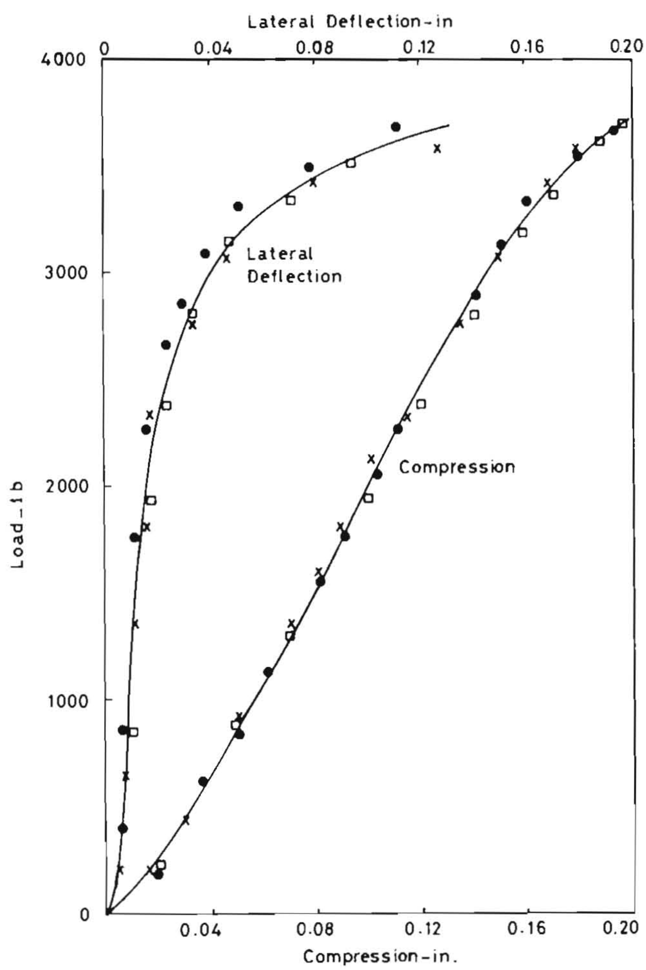

FIG.7_ COMPRESSION AND LATERAL DEFLECTION VERSUS LOAD FOR PPU_ SANDWICH BEAMS

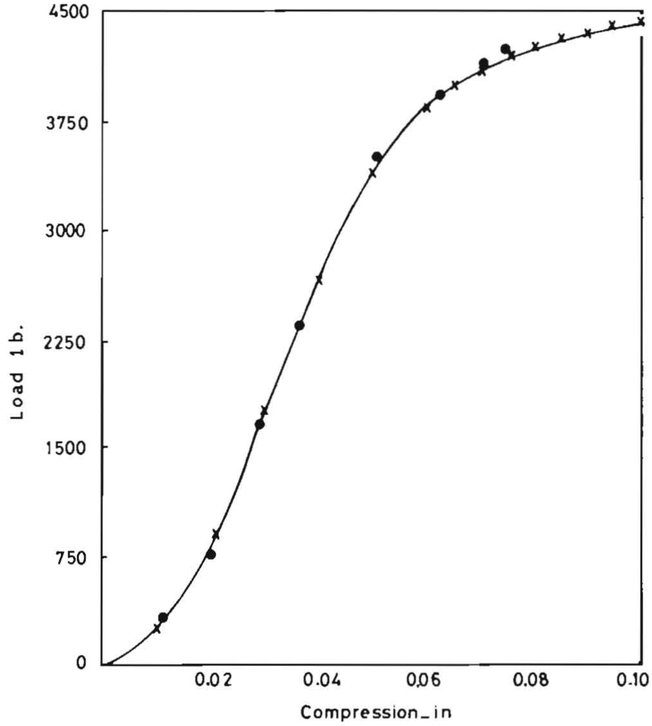

FIG.8_ COMPRESSION VERSUS LOAD FOR ALP_ SANDWICH BEAMS

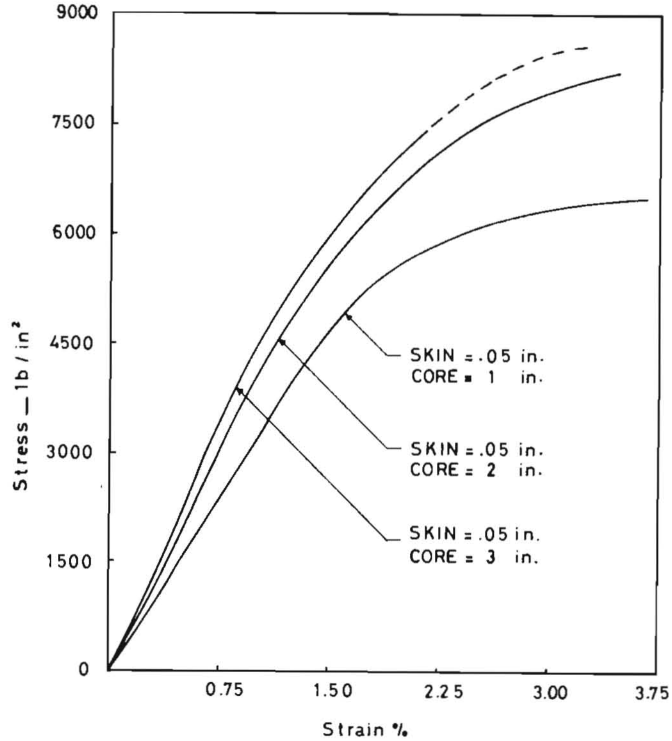

FIG.10_ THE EFFECT OF THE CORE THICKNESS IN THE STRESS VERSUS STRAIN CURVES FOR PP_ SANDWICH BEAMS.

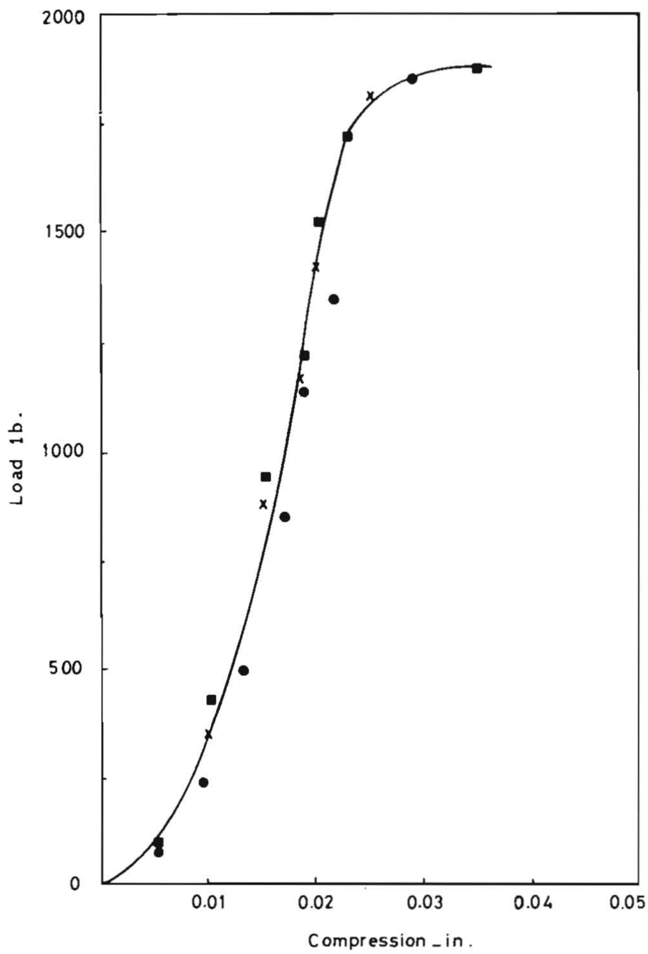

FIG.9_COMPRESSION VERSUS LOAD FOR ALPU_ SANDWICH BEAMS

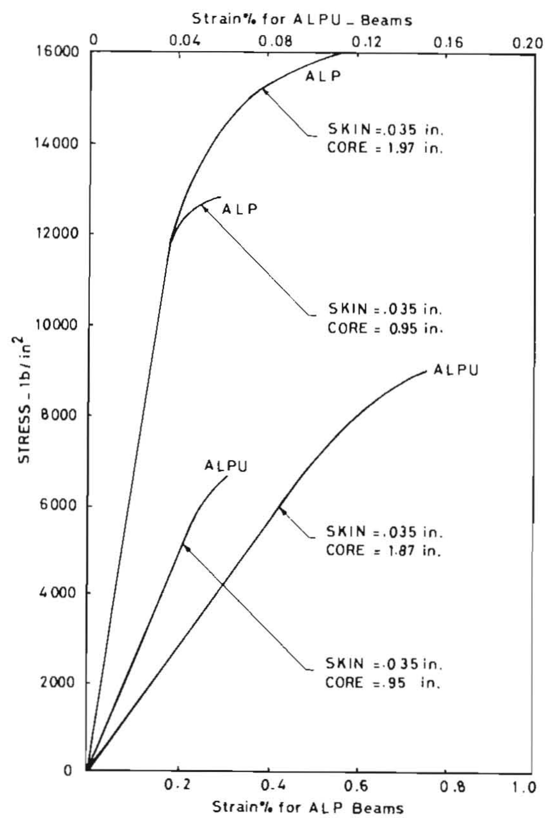

FIG.11_ STRESS VERSUS STRAIN FOR ALP AND ALPU SANDWICH BEAMS UNDER COMPRESSIVE LOADS WHERE THE EFFECT OF THE VARIOUS CORE THICKNESSES ON THE FAILING STRESS IS SHOWN

TABLE (1)

No.	Type	t in	d in	b in	l in	Load Axis	Theoretical P_{cr} lb. Equ 1	Equ 2	Equ 3	Equ 4	Equ 5	Equ 6	P_{cr} Exp.
1	pp	.050	1.00	3.8	19.0	XX	2966	2066	1959	2967	3668	1763	2440
2	pp	.065	1.00	3.0	14.2	XX	3392	3097	2762	3628	3249	2758	3020
3	ppu	.065	1.05	4.4	14.0	XX	627	877	697	737	445	555	825
4	ppu	.125	1.05	4.0	18.8	XX	843	1516	956	993	1943	619	1710
5	ppu	.125	1.78	4.0	17.0	XX	1459	2312	1692	1735	1269	994	2860

Some sandwich beams and panels failed in wrinkling mode, and fig. (12) and fig. (13) show some of them after failure.

Table (2) shows the critical load for some of these beams and panels which failed in wrinkling mode.

As for shear failure of the core, several test pieces failed in shear crimping mode, and fig. (14) shows some of these tests after failure. Table (3) shows the critical failing load of some sandwich beams and panels due to shear failure of the core.

Finally some sandwich beams and panels failed in face dimpling mode, as in fig. (15). Table (4) exhibits the critical load for some of these beams and panels. In this table two values of (0.01 and 0.005) for the initial waviness (N_o) were asumed. It is clear from fig. (16) that the initial waviness (N_o) greatly influences the buckling loads whether in tension or shear.

TABLE (2)

No.	Type	t in	d in	b in	l in	Load Axis	Theoretical P_{cr} lb. 7	8	9a	9b	9c	10	11	12	P_{cr} Exp.
1	PP	.050	1.00	3.8	19.0	XX	3177	3613	3944	2790	3367	2237	2052	4104	2440
2	PP	.050	2.00	5.0	14.75	XX	4180	4918	5190	3665	4428	2944	2700	5400	4210
3	PP	.065	2.00	4.9	13.8	XX	5325	6169	6612	4680	5642	3751	3440	6880	4480
4	PP	.065	1.93	3.0	14.6	XX	3260	3770	4048	2864	3454	2296	2106	4214	3500
5	PP	.050	3.00	4.4	18.8	XX	3678	4472	4567	3228	3897	2591	2376	4752	4060
6	PP	.065	3.00	5.0	11.4	XX	5435	6460	6747	4780	5764	3827	3510	7020	4360
7	ALP	.012	3.00	3.7	17.0	XX	2241	2501	2782	1970	2377	1578	1448	2896	1440
8	ALP	.035	3.00	3.0	14.6	XX	5299	5852	6579	4660	5614	3732	3432	6864	6730
9	ALP	.035	3.00	3.8	17.0	XX	6712	7412	8334	5900	7111	4727	4336	8672	7400
10	ALPU	.017	3.00	5.4	18.9	XX	1967	1661	1872	1330	1597	1374	974	1948	1965

TABLE (3)

							P_{cr} lb.	
No	Type	t in	d in	b in	l in	Load Axis	Theo. Equ. 13	P_{cr} Exp.
1	PPU	.065	1.05	3.00	14.00	XX	794	825
2	PPU	.125	1.05	4.00	18.8	XX	1058	1200
3	PPU	.125	1.78	4.00	17.0	XX	1794	2800
4	PPU	.065	1.78	4.30	14.0	XX	1929	2410
5	PPU	.065	1.78	3.95	17.25	XX	1772	1760
6	PPU	.125	3.00	3.50	18.6	XX	2646	3570
7	PPU	.125	3.00	4.25	13.6	XX	3213	3930

TABLE (4)

							Theoretical P_{cr} lb. Equ 15 and Equ 16				
No	Type	t in	d in	b in	l in	Load Axis	N_o =.01	0.005	0.01	0.005	P_{cr} Exp.
1	PP	.125	1.00	5.0	14.0	XX	8556	10307	6394	8563	8350
2	PP	.065	3.00	4.0	18.6	XX	2710	3607	1812	2713	2300
3	PP	.065	3.00	5.0	8.8	XX	3387	4508	2265	3391	3840
4	ALP	.035	0.95	4.5	19.5	XX	6127	7557	4450	6132	4100
5	ALP	.035	1.97	4.3	19.5	XX	5855	7221	4252	5860	4430
6	ALPU	.035	1.78	4.0	19.3	XX	2307	2550	1684	2117	2280

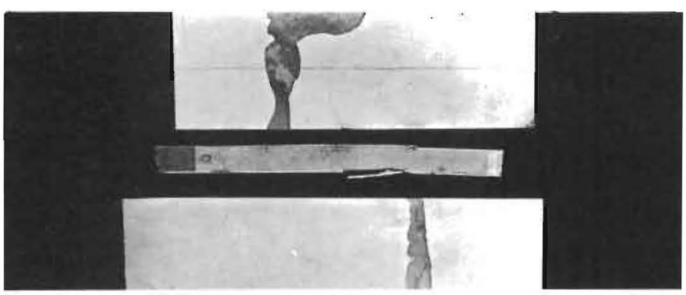

FIG. 12_ FACE FAILURE OF PP_ SANDWICH BEAMS AND PANELS IN FACE_WRINKLING MODE

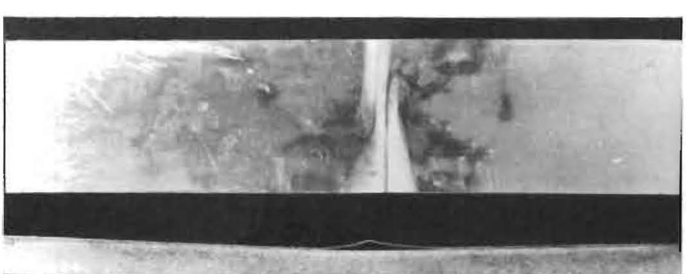

FIG. 13_ ALP_ SANDWICH BEAMS AND PANELS IN FACE_WRINKLING MODE

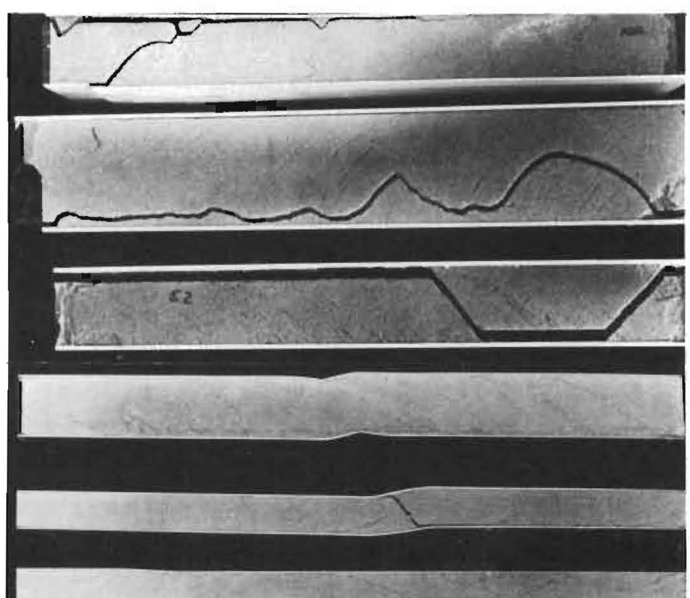

FIG. 14_ SANDWICH BEAMS AND PANELS IN SHEAR CRIMPING MODE

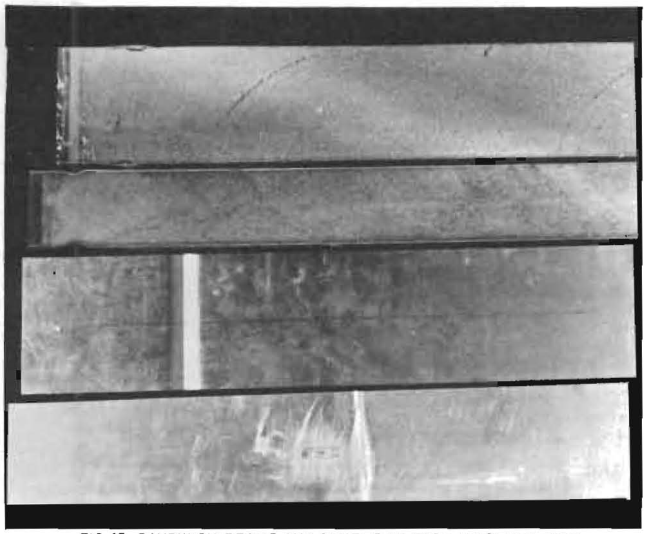

FIG.15_SANDWICH BEAMS AND PANELS IN FACE DIMPLING MODE

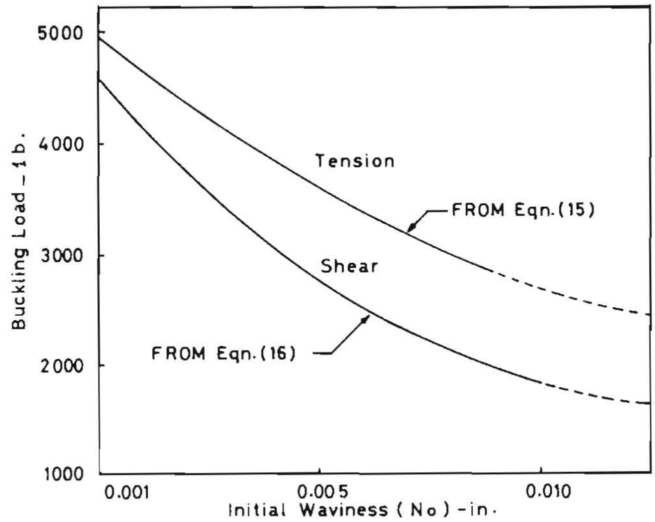

FIG.16_EFFECT OF INITIAL WAVINESS ON BUCKLING LOADS FOR BEAM (2) IN TABLE (4)

CONCLUSIONS

The failure of sandwich beams and panels in general buckling mode follows closely that predicted by Euler's formula. This is clearly indicated from the values of the theoretical failing loads and their comparison with those which were obtained experimentally. This reasonable agreement is shown in Table (1). For most of the plastic sandwich constructions under buckling loads it has been observed that as the load increased, the lateral deflection of the beam or panel under test increased only slightly, but as the load approached the buckling load, the lateral deflection increased rapidly with further small increase in load until failure occured. In most cases the failure resulted in the shattering of the most highly stressed part of the skins. This differs from the conventional pattern of failure for metal sandwich constructions, which follows the same pattern up to the buckling load, but after that the lateral deflection increases while the load decreases until failure is reached. The relationship between lateral deflections and loads is not quite hyperbolic as the case in metal sandwich constructions.This is because plastic materials under test are of visco elastic nature, and they can be strained to a higher limit than metals. Another reason is that the plastic materials have a greater elastic-plastic zone than metals.

Comparing the formulae for face wrinkling mode with one another, they are basically derived in a similar way and all those which include the shear effect of the core are of cubic nature and differ only in the coefficient (J) which varies from (0.5 to 1.0). This variation in (J) can only be attributed to the slightly different assumptions made in deriving each formula. However the empirical formulae of Nieuwenhuizen and the authors should be used in design as lower and upper limits of failing loads.

If the shear effect of the core is ignored, then most of these formulae can be reduced to a square root formula. It is important to know the ratio of $\left(\frac{W}{d}\right)$ which governs whether the shear effect is large or small If the depth of the distorted zone (W) is greater than half of the core thickness $\left(\frac{d}{2}\right)$ then the wrinkling stress is given by the square root formula, and if it is less than $\left(\frac{d}{2}\right)$ the wrinkling stress is given by the cubic root formula. In all the sandwich beams and panels which were tested, shear effect was prominent and this is clearly shown in Table (2) from the close agreement between theory, which considers the shear, and between the experimental values of critical loads.

In the case of a weak core material like polyurethane, the sandwich beams or panels failed due to shearing of the core material. Table (3) shows the comparison between the theoretical and experimental loads. The theory gave lower values than the experimental ones. This was partly due to the face failure which accompanied the shear failure of the core and cannot be separated from it, and partly due to the effect of the adhesive layers between the faces and the core which cause the stiffness of the core and hence a larger failing load is required.

Once more it is difficult to differentiate between the shear crimping and face dimpling modes, because of the existence of face wrinkling before buckling, but after failure the two modes can be easily recognised. In plastic sandwich constructions the face

dimpling occurs normally due to initial irregularities in the faces which produce initial waviness. This waviness grows as the load is increased and finally leads to the failure of the core in tension, compression or shear. In contrast in metal sandwich constructions with honeycomb core this irregularity leads to the cell buckling of the core across the member and then failure of the sandwich construction. In order to understand the face dimpling mode better, a close look at the sandwich beams and panels reveals some imperfections as a result of making the sandwich constructions. In particular initial waviness, even if very small, in the faces, can result in face dimpling if enlarged in the case of beams and panels under end loads. Considering Table (4), most of the theoretical values of critical buckling loads, where (N_o) is either 0.01 in. or 0.005 in., are in agreement with the experimental values. The agreement is fairly close if the sandwich constructions fail in tension with the large initial waviness, while if the initial waviness is small, then failure is by shear.
Therefore equation (16) is to be used, while if it is large then equation (15) must be used.

In studying the experimental results, one can deduce that for even, regular shaped sandwich beams and panels the factors governing the first three modes of failure are, l/d, t/d and E_f/E_c.

1. For first mode
   $l/d > 10$, $t/d > \frac{1}{100}$ and $E_f/E_c < 1000$.
2. For the second mode to exist
   $l/d < 10$, $t/d < \frac{1}{100}$ and $E_f/E_c > 1000$.
3. As for the 3rd mode
   $l/d < 15$, $t/d > \frac{1}{100}$ and $E_f/E_c > 1000$.

If there is an irregularity in the faces and condition (2) exists then the sandwich beam and panel will fail in face dimpling.

It is very difficult to predict the mode of buckling in plastic sandwich constructions accurately, because of the various factors which influence the pattern of buckling, however it is possible to use the conventional theories of buckling on plastic sandwich constructions and the results are fairly satisfactory. Therefore in the design world, the designer must construct his structure in such a way so as to avoid failure in the mode which gives the smallest load. This can only be done by examination of the four different modes, because a sandwich beam or panel can first show a general mode, but due to the face waviness can also show a wrinkling mode or dimpling mode, while probably fails in shearing of the core material.

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