AXIALLY MOVING CAMS WITH TRANSLATING FOLLOWERS

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الكامات ذات الحركة المحورية بتوابع لها حركات ترددية

في هذا البحث استخدم جبر المتهجات لتحديد الانحناءات الأساسية لسطح الكامات ذات الحركة المحورية . كما استنبطت معادلات تمثلها . ووضع نمط تحليلي لايجاد أصغر حجم للكامة مع تحديد كمية انحراف التابع . وقدمت فكرة الكامه ذات ثباتين – واستخدم الحاسب الالكتروني لرسمها . كما عرضت بعض الأفكار لتبيان فعالية هذه الكامات .

The principal curvatures for the surfaces of axially moving cams are obtained by using vector algebra. The parametric equations representing the surface of the cam are determined. Also, an algorithm for determining the minimum cam size and the associated eccentricity is outlined.

A complete analysis for a cam with two continuous dwells was made. The shape of the surface was plotted by the computer.

Finally, some ideas to demonstrate the versatility of axially moving cams are presented.

Nomenclature

$\vec{I}, \vec{J}, \vec{K}$	unit vectors along fixed coordinates.
x,y,z,	coordinate system fixed with the cam.
$\vec{i}, \vec{j}, \vec{k}$	unit vector along x,y, and z, repectively, with $\vec{j}$ along $\vec{J}$.
h	eccentricity of the follower along $\vec{K}$.
R	distance of the center of the spherical tip along $\vec{I}$.
θ	cam rotational angle.
ω	angular velocity of the cam.
$\vec{r}$	position vector of the tip center relative to x,y, and z coordinates.
a	radius of the spherical tip.
$\vec{N}$	vector normal to the cam surface.
$\vec{n}$	unit vector along $\vec{N}$; $\vec{n} = \frac{\vec{N}}{\lvert\vec{N}\rvert}$

Introduction

Cam mechanisms are very important in practice. Design considerations for plane cams with translating and oscillating followers are presernted, for example, by Rothbart [4]. A synthesis for two bodies in contact was made by Fawcett [5]. The analysis, generally, is for space motion, but more details is concerned with planer motion. Reven[6] derived certain geometrical quantities for plane cams and a particular type of three dimensional cams.

Axially moving cams are classes of plane cams which offer variable characteristics for the motion of the follower. For any axial position the cam-follower mechanism performs exactly as a disk cam system. However, any axial displacement alters the nature of motion of the follower. This change can be specified according to the design requirement. In this case, the motion of the follower is specified in terms of the cam rotational angle and the axial displacement. Thus, an axially moving cam can be regarded as a combination of infinite disk cams with variable characteristics. However, it posses the advantage over a system of plane cams in that the variation in the follower motion is smoother and can be achieved while the cam is in operation.

The cam has a surface whose characteristics depends upon the type of motion of the follower. Obviously, the follower must have a spherical tip in order to follow the surface. The parametric equations of this surface can be easily determined. In order to determine the contact stresses it is necessary to obtain the principal curvatures of the surface. The center of the spherical tip is considered to be in contact

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with a pitch surface. At the point of contact, the principal curvature, and hence the normal curvature are determined for a space curve on the pitch surface which is represented by some parameters. The principal curvatures are the maximum and minimum of the normal curvature.

Among the factors affecting the design of cam is the pressure angle. Its maximum value should be limited [4]. Hence, a criteria is used for determining the minimum cam size and the associated accentricity.

Principal curvatures for the cam surface:

An axially moving cam with an offset spherical follower is illustrated in Fig. 1. The center line of the follower is parallel to $\vec{I}$. The motion of the follower is function of the cam rotational angle, and the axial displacement, i.e.

$$R = R_o + f(\theta, y) \qquad (1)$$

Where R_o is the minimum distance along $\vec{I}$. The position vector of the center of the spherical tip relative to the moving system is

$$\vec{r}=x\,\vec{i}+y\,\vec{j}+z\,\vec{k}$$

When $\vec{i}$ and $\vec{k}$ are replaced by $\vec{I}$, and $\vec{K}$, the position vector takes the form

$$\vec{r}=R\,\vec{I}+y\,\vec{j}+h\,\vec{K} \qquad (2)$$

The relation between these vectors are given by

$$\vec{I}=\cos\theta\,\vec{i}+\sin\theta\,\vec{k} \qquad (3)$$
$$\vec{K}=-\sin\theta\,\vec{i}+\cos\theta\,\vec{k}$$

The fixed vectors $\vec{I}$, and $\vec{K}$ are considered to be rotating relative to $\vec{i}$, and $\vec{k}$ in the opposite sense. Differentiating Eq. (2) with respect to time, hence

$$\dot{\vec{r}}=(\dot{R}-h\omega)\,\vec{I}+\dot{y}\,\vec{j}+R\omega\,\vec{K} \qquad (4)$$
$$\ddot{\vec{r}}=(\ddot{R}-R\omega^2)\,\vec{I}+\ddot{y}\,\vec{j}+(2\dot{R}\omega-h\omega^2)\,\vec{K} \qquad (5)$$

Where

$\omega = \dfrac{d\theta}{dt} = \dot{\theta}$ and is considered constant.

The center of the spherical tip traces different curves on the pitch surface. These curves depend upon the relationship between y, and θ which is completely arbitrary. Consider now the family of curves represented by $y=\alpha\theta$; α is an arbitrary constant. For these curves

$$\dot{y}=\alpha\omega$$
$$\ddot{y}=0$$
$$\dot{R}=\omega(f_\theta+\alpha f_y) \qquad (6)$$

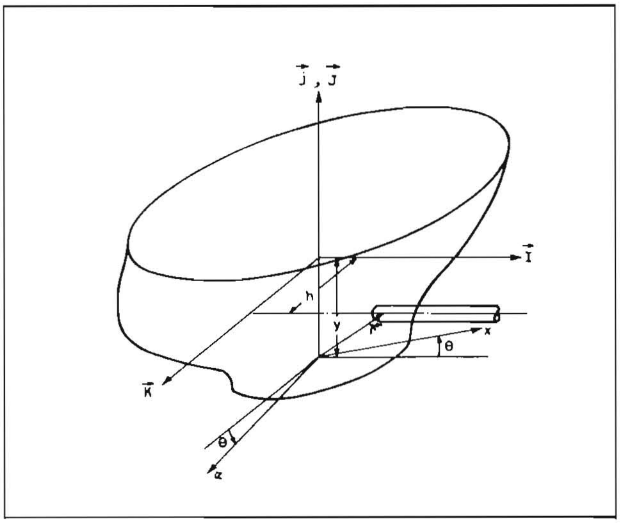

FIG. 1 COORDINATE SYSTEM FOR AN AXIALLY MOVING CAM.

Where the subscript means partial differentiation. Also

$$\ddot{R}=\omega^2(f_{\theta\theta}+2\alpha f_{\theta y}+\alpha^2 f_{yy}) \qquad (7)$$

Introducing

$$\dot{\vec{r}}=\omega\frac{d\vec{r}}{d\theta}=\omega\vec{r}',$$

$$\ddot{\vec{r}}=\omega^2\frac{d^2\vec{r}}{d\theta^2}=\omega^2\vec{r}'',$$

$$A = f_\theta + \alpha f_y$$

$$B = f_{\theta\theta}+2\alpha f_{\theta y}+\alpha^2 f_{yy}$$

and substituting into Eqs. (4), and (5), thus

$$\vec{r}' = (A-h)\,\vec{I} + \alpha\vec{j} + R\,\vec{K} \qquad (8)$$
$$\vec{r}'' = (B-R)\,\vec{I} + (2A-h)\,\vec{K} \qquad (9)$$

For a certain value of α, the curvature of the space curve is given by [3]

$$K_\alpha = \frac{\left|(\vec{r}' \times \vec{r}'').(\vec{r}' \times \vec{r}'')\right|^{\frac{1}{2}}}{(\vec{r}', \vec{r}')^{3/2}} \qquad (10)$$

From Eqs. (8), and (9), then

$$K_\alpha=\frac{[\alpha^2(2A-h)^2+(RB-R^2-2A^2+3Ah-h^2)^2+\alpha^2(R-B)^2]^{\frac{1}{2}}}{[(A-h)^2+\alpha^2+R^2]^{3/2}} \qquad (11)$$

The principal normal of a space curve is denoted by $\vec{N}_p$ and is given by [3]

$$\vec{N}_p = (\vec{r}' \times \vec{r}'') \times \vec{r}'$$

$$= \left[R(RB-R^2-2A^2+3Ah-h^2)-\alpha^2(R-B \right] \vec{I} + \left[(A-h)(R-B)-\alpha R(2A-h) \right] \vec{j} + \left[\alpha^2(2A-h)-(A-h)(RB-R^2-2A^2+3Ah-h^2) \right] \vec{K} \quad (12)$$

The tangent plane of the conjugate surface contains all the tangents represented by Eq. (8). Vector $\vec{N}$ is obtained by considering any two tangents with parameters, say, α_1, and α_2. $\vec{N}$ is proportional to

$$\left[(A_1-h)\vec{I}+\alpha_1\vec{j}+R\vec{K} \right] \times \left[(A_2-h)\vec{I}+\alpha_2\vec{j}+R\vec{K} \right]$$

The values of A_1, and A_2 are obtained from Eq. (6) for α_1, and α_2 respectively. Thus

$$\vec{N} = -R\vec{I}+Rf_y\vec{j}+(f_\theta-h)\vec{K} \quad (13)$$

The circles of curvature of all curves on a surface having a common tangent, at their point of intersection, lie on what is called "Meunsier's Sphere"[2] The circle of curvature is the intersection of the Meunsier's sphere with the osculating plane. The center of this sphere lies along $\vec{N}$. Its curvature is denoted as the normal curvature "$K_{\alpha n}$" and is given by

$$K_{\alpha n} = K_\alpha \cos\psi_\alpha \quad (14)$$

Where ψ_α is the angle between $\vec{N}$, and $\vec{N_p}$ and is given by

$$\cos\psi_\alpha = \frac{\vec{N_p}\cdot\vec{N}}{|\vec{N_p}|\,|\vec{N}|}$$

The normal curvature is given by

$$K_{\alpha n} = \left[\alpha^2(2A-h)^2+(RB-R^2-2A^2+3Ah-h^2)^2+\alpha^2(R-B)^2 \right]^{\frac{1}{2}} \Big[-R^2(RB-R^2-2A^2+3Ah-h^2)^2 +\alpha^2R(R-B)+\alpha Rf_y(A-h)(R-B)-\alpha R^2f_y(2A-h) +\alpha^2(f_\theta-h)(2A-h)-(f_\theta-h)(A-h)(RB-R^2-2A^2 +3Ah-h^2) \Big] \Big/ \left[(A-h)^2+\alpha^2+R^2 \right]^{3/2} \left[R^2+R^2f^2y +(f_\theta-h)^2 \right]^{\frac{1}{2}} \Big\{ \left[R(RB-R^2-2A^2+3Ah-h^2) -\alpha^2(R-B) \right]^2 + \left[\alpha(A-h)(R-B)-\alpha R(2A-h) \right]^2 +\left[\alpha^2(2A-h)-(A-h)(RB-R^2-2A^2+3Ah-h^2) \right]^2 \Big\}^{\frac{1}{2}} \quad (15)$$

The principal curvatures of the surface are the maximum and minimum values of Eq. (15). Their analytical determination is tedeous. However, satisfactory results are achieved by plotting the relation between $K_{\alpha n}$ and α This is easily done specially with the help of digital computers. Again, it is not practical to assign large values for α. Thus for $-1 \leqq \alpha \leqq 1$, Eq. (15) is used. For values outside this region it is better to introduce the parameter $\beta = \frac{1}{\alpha}$ which reduces the range to $-1 \leqq \beta \leqq 1$. In this case, Eq. (15) becomes:

$$K_{\beta n} = \Big[\beta^2(2c-\beta h)^2+\beta^2(Rd-\beta^2R^2-2c^2+3\beta ch- \beta^2h^2)^2+(\beta^2R-d)^2) \Big]^{\frac{1}{2}} \Big[-\beta^2R^2(Rd-\beta^2R^2- 2c^2+3\beta ch-\beta^2h^2)+R(\beta^2R-d)+Rf_y(c- \beta h)(\beta^2R-d)-\beta^2R^2f_y(2c-\beta h)+\beta(f_\theta-h)(2c-\beta h) -\beta(f_\theta-h)(c-\beta h)(Rd-\beta^2R^2- 2c^2+3\beta ch-\beta^2h^2 \Big] \Big/ \Big[(c-\beta h)^2+1+ \beta^2R^2 \Big]^{3/2} \Big[(R^2+R^2f_y^2+(f_\theta-h)^2 \Big]^{\frac{1}{2}} \Big\{ \Big[R\beta^2(Rd- \beta^2R^2-2c^2+3Bch-\beta^2h^2)-(\beta^2R-d) \Big]^2 + \Big[(c- \beta h)(\beta^2R-d)-\beta^2R(2c-\beta h) \Big]^2 + \Big[(\beta(2c- \beta h)-\beta(c-\beta h)(Rd-\beta^2R^2-2c^2+3\beta ch-\beta^2h^2) \Big]^2 \Big\}^{\frac{1}{2}} \quad (16)$$

Where

$$c = \frac{A}{\alpha} = \beta f_\theta + f_y$$

$$d = \frac{B}{\alpha^2} = \beta^2 f_{\theta\theta} + 2\beta f_{\theta_y} + f_{yy}$$

Equation of the surface of the cam

The position vector of a point on the pitch surface of the cam relative of the fixed coordinates is given by Eq. (2) which is in the form.

$$\vec{r} = R\vec{I} + y\,j + h\vec{K}$$

The position vector "$\vec{r_s}$" of a point on the surface of the cam itself is given by:

$$\vec{r_s} = \vec{r} + a\vec{n} \quad (17)$$

From Eqs. (16) and (20)

$$\vec{r}_s=(R-\frac{aR}{N})\vec{I}+(y+\frac{aRf_y}{N})\vec{j}+\left[h+\frac{a(f_\theta-h)}{N}\right]\vec{K} \quad (18)$$

where

$$N=|\vec{N}|=\sqrt{R^2+R^2f_y^2+(f_\theta-h)^2} \quad (19)$$

In order to obtain r_s in terms of the coordinate system fixed with the cam, one uses the relations given by Eqs. (3).

Thus

$$\vec{r}_s=\left[R-\frac{aR}{N})\cos\theta-<h+\frac{a(f_\theta-h)}{N}>\sin\theta\right]\vec{i}+(y+\frac{aRf_y}{N})\vec{j}+\left[(R-\frac{aR}{N})\sin\theta+<h+\frac{a(f_\theta-h)}{N}>\cos\theta\right]\vec{k}$$

The parametric equations of the surface are given by

$$x=(R-\frac{aR}{N})\cos\theta-\left[h+\frac{a(f_\theta-h)}{N}\right]\sin\theta$$

$$y_s=y+\frac{aRf_y}{N} \quad (20)$$

$$z=(R-\frac{aR}{N})\sin\theta+\left[h+\frac{a(f_\theta-h)}{N}\right]\cos\theta$$

Pressure angle and the minimum cam size

The pressure angle "Φ" is the angle between the center line of the follower and the normal to the surface. Thus.

$$\cos\Phi=\vec{I}.\vec{n}$$

or

$$\cos\Phi=\frac{R}{N} \quad (21)$$

It is clear that the value of Φ depends upon R_o, h, and the type of follower motion. The maximum value of Φ, denoted by ϕ^*, should be limited to some value otherwise the follower jams in its guides [1]. The value of ϕ^* occurs when [3]

$$\frac{\partial\cos\Phi}{\partial\theta}=0$$

$$\frac{\partial\cos\Phi}{\partial y}=0 \quad (22)$$

When the values of R_o, and h are known the values of θ and y where Φ^* occurs at the position of maximum pressure angle (denoted by θ^*, and y^*) are obtained from Eqs. (21), and (22)

$$\left.\begin{aligned}\frac{R}{N}&=\frac{f_\theta}{N_\theta}\\ \frac{R}{N}&=\frac{fy}{Ny}\end{aligned}\right\} \quad (23)$$

On the other hand, when the value of Φ^* is assigned, θ^*, and y^* are determined by substituting Eqs. (23) into Eq. (22). Thus

$$\cos\phi^*=\frac{f_\theta}{N_\theta}$$

$$\cos\phi^*=\frac{f}{N_y} \quad (24)$$

Let R^*, and N^* be the values of R, and N at θ^*, and y^*. Hence

$$\cos\Phi^*=\frac{R^*}{N^*} \quad (25)$$

The values of R_o, and h in Eq. (24) are unknown. Consider the motion of the follower during the rise and return strokes. The maximum pressure angles are denoted by Φ_r^*, and Φ_t^* (the subscripts r, and t here belongs to the rise and return strokes respectively). From Eqs. (1). and (22) the values of R_r^*, N_r^*, R_t^*, and N_t^* are obtained Hence

$$\left.\begin{aligned}\cos\Phi_r^*&=\frac{R_r^*}{N_r^*}\\ \cos\Phi_t^*&=\frac{R_t^*}{N_t^*}\end{aligned}\right\} \quad (26)$$

Equations (26) are solved for R_o, and h.

Some particular cases

Example 1

Cams with continuous double dwells

The surface of the cam is divided into three portions. The first is cylindrical with a radius equal to the minimum radius of the cam. The second portion is developed to give the follower a reciprocating motion with a variable lift, at the same time the minimum size of the cam increases. The third portion is, again, cylindrical, no motion is imparted to the follower. For such cams the variation of the minimum size and the lift can be of any nature to insure smooth operation. When the variation is harmonic the displacement of the follower is represented by

$$R=R_o+\frac{S}{2}(1-\cos\frac{\pi y}{a_c})+\frac{L}{4}(1-\cos\frac{2\pi y}{a_c})(1-\cos\frac{\pi\theta}{\beta})$$

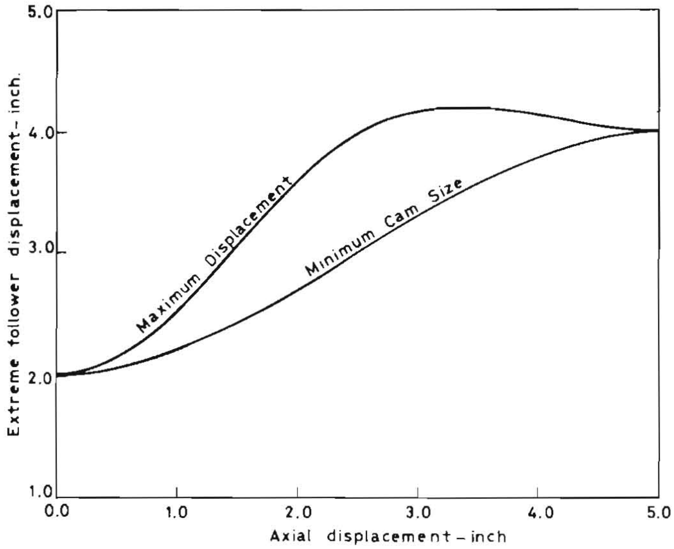

FIG. 2 CAMS WITH DOUBLE DWELLS

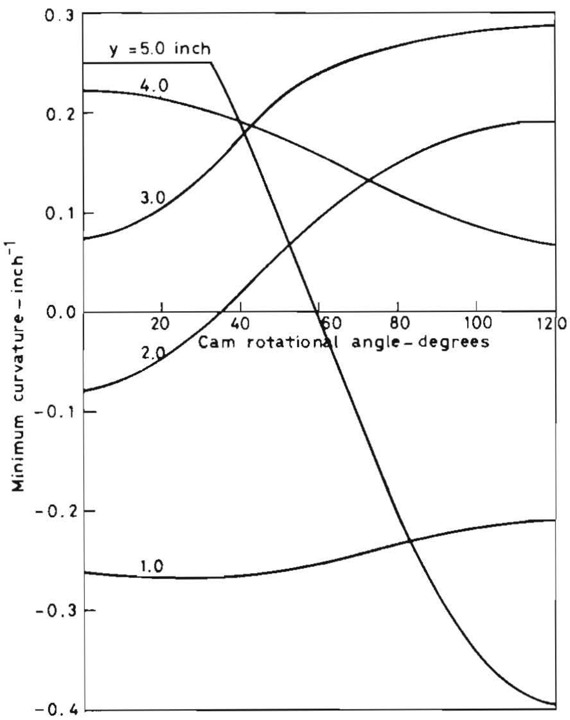

FIG. 4 DISTRIBUTION OF MINIMUM CURVATURE OF SURFACE

Where

S is the total increase in the minimum size of cam

L is the maximum lift of the follower

a_c is the length of the second portion

β is the rise angle

The amplitude of R is illustrated in Fig. 2.

The characteristics of the surface of the cam are determined from Eqs. (15), (16), (19), and (20) by using the following partial derivatives

$$f_\theta = \frac{L}{4}\left(\frac{\pi}{\beta}\right) \left(1-\cos \frac{2\pi y}{a_c}\right) \sin \frac{\pi\theta}{\beta}$$

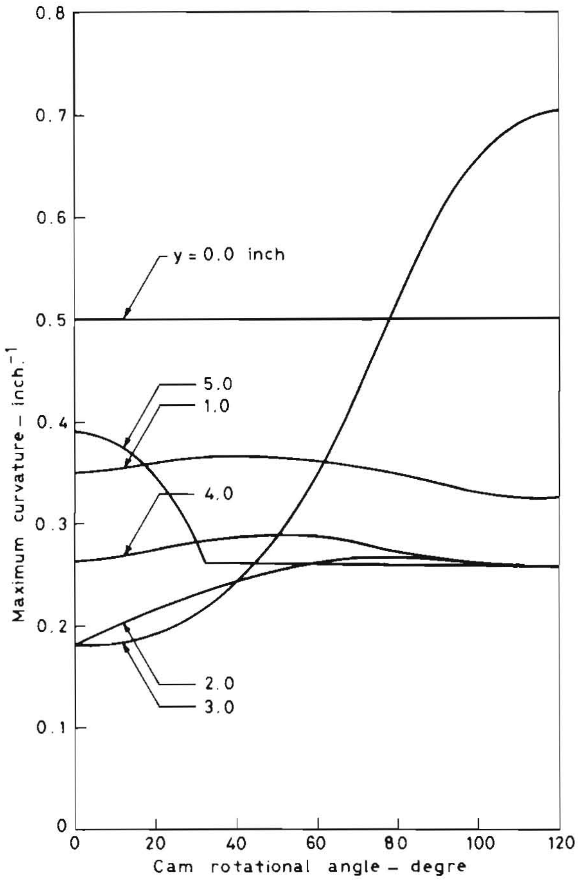

FIG. 3 DISTRIBUTION OF MAXIMUM CURVATURE OF SURFACE

$$f_{\theta\theta} = \frac{L}{4}\left(\frac{\pi}{\beta}\right)^2 \left(1 - \cos \frac{2\pi y}{a_c}\right) \cos \frac{\pi\theta}{\beta}$$

$$f_y = \frac{S}{2}\left(\frac{\pi}{a_c}\right) \sin \frac{\pi y}{a_c} + \frac{L}{2}\left(\frac{\pi}{a_c}\right) \sin \frac{2\pi y}{a_c} \left(1-\cos \frac{\pi\theta}{\beta}\right)$$

$$f_{yy} = \frac{S}{2}\left(\frac{\pi}{a_c}\right)^2 \cos \frac{\pi y}{a_c} + L\left(\frac{\pi}{a_c}\right)^2 \cos \frac{2\pi y}{a_c} \left(1-\cos \frac{\pi\theta}{\beta}\right)$$

$$f_{\theta y} = \frac{L}{4}\left(\frac{\pi}{\beta}\right)\left(\frac{2\pi}{a_c}\right) \sin \frac{2\pi y}{a_c} \sin \frac{\pi\theta}{\beta}$$

A computer program was made for determining the principal curvatures, the coordinates of the surface, and the pressure angle. Results were obtained for the following numerical results.

R_o = 2 in., S = 2 in., L = in.,
h = 0 ' a = .25 in, β = 120°',
a_c = 5 in.

The distributions of maximum and minimum curvatures for the conjugate surface at various axial locations are shown in Figs. 3 and 4 respectively. The distribution of the minimum curvature, with a different scale, near the lower dwell zone is shown in Fig. 5. It was found, at this region, that the surface is concaved with a maximum absolute value of k equals 1.2 $inch^{-1}$.

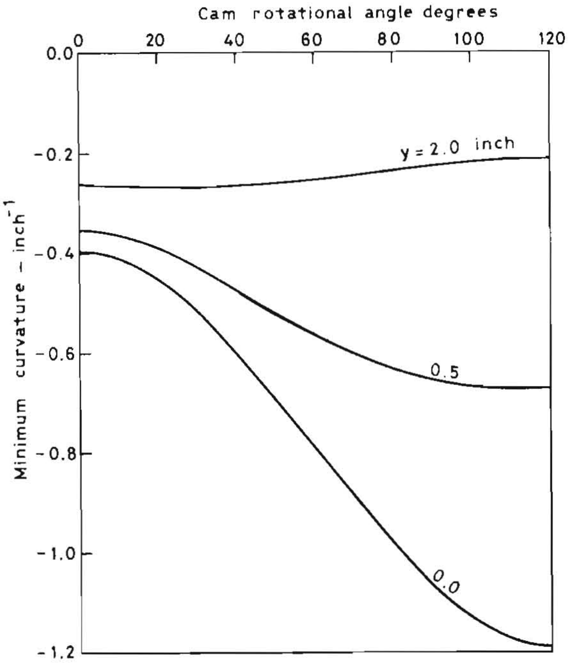

FIG. 5 DISTRIBUTION OF MINIMUM CURVATURE NEAR THE LOWER DWELL ZONE

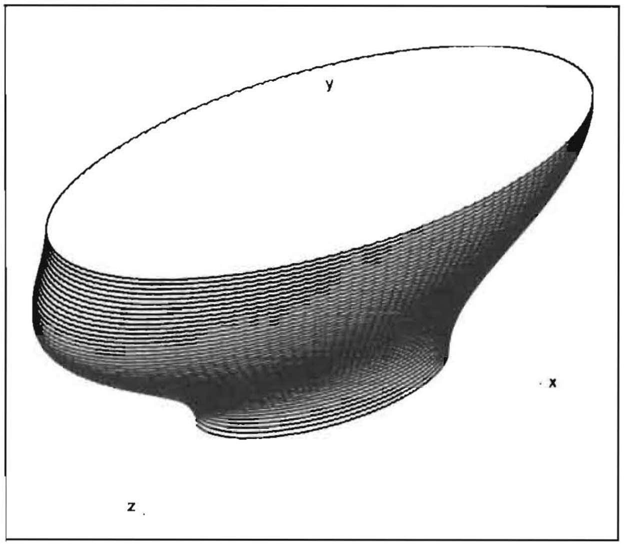

FIG. 6 A CAM WITH TWO CONTINUOUS DWELLS

This means that the sum of the radius of curvature of the surface and the radius of the follower should be less than 0.833. In other words, the radius of the spherical tip must be less than 0.416 in. For a tip with radius 0.25 in. the coordinates of the surface were obtained. An oblique view for the cam surface, plotted by the computer plotter, is illustrated in Fig. 6.

Example 2

Cams with variable lift

Another advantage for the use of axially moving a cam, actuating a reciprocating spherical follower, with variable lift. The change of the lift is arbitrary. However, for harmonic variation the displacement of the follower is represented by

$$R = R_o + \frac{1}{4}\left(1 - \cos\frac{\pi y}{a_c}\right)\left(1 - \cos\frac{\pi\theta}{\beta}\right)$$

Example 3

A third example which demonstrates the versatility of this types of cams is cams with variable rise angles. Suppose that β_1 and β_2 are the rise angles at the beginning and the end of a region of an axial length "a_c". For a linear variation in the rise angle the motion of the follower is represented by

$$R = R_o + \frac{L}{2}\left\{1 - \cos\frac{\pi\theta}{\beta_1 + \left(\frac{\beta_2 - \beta_1}{a_c}\right)Y}\right\}$$

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