An Exposition of the Modern Syllogistic Method of Propositional Logic

Ali M. Rushdi* **and Omar M. Barukab****
** Department of Electrical and Computer Engineering, King Abdulaziz University,Jeddah, Saudi Arabia*
*** College of Telecommunications & Electronics, Jeddah, Saudi Arabia*

استعراض للطريقة الاستدلالية الحديثة للمنطق الإخباري

تستخدم قواعد الإحلال والاستنتاج في المنطق الإخباري التقليدي لتحديد ما إذا كان صدق عدد من المقدمات يؤدي إلى صدق نتيجة معينة. تصف ورقة البحث هذه أسلوباً أقوى من أسلوب هذه القواعد يعرف باسم الطريقة الاستدلالية الحديثة. يتم إثبات أن هذه الطريقة تستخلص من مجموعة المعطيات كل ما يمكن استنتاجه منها مع صياغتها للاستنتاجات في أبسط صورة متضامّة. نلاحظ تضمين قواعد الإحلال صراحة في الخطوات الذاتية للطريقة الاستدلالية الحديثة ونبرهن على أن جميع قواعد الاستنتاج ما هي إلا حالات خاصة محدودة من هذه الطريقة. يعني هذا أن الطريقة الاستدلالية الحديثة هي طريقة كاملة للاستنتاج المنطقي. نوضح أيضاً كيفية استخدام الطريقة الاستدلالية الحديثة لتحديد ما إذا كانت ثمة تناقضات ضمن مجموعة معطاة من المقدمات، وأيضاً للكشف عن المغالطات المنطقية الصورية. نوضح إمكانيات تطبيق هذه الطريقة في مجالات متنوعة عديدة باستخدام عدد كبير من الأمثلة التي تبين التفصيلات الرياضية للطريقة، كما تظهر طبيعة الاستنتاجات التي تنجم عنها حيث يتبين من الأمثلة إمكانية استخراج استنتاجات تبدو مستغربة بل ومذهلة أحياناً. كما يتضح منها أيضاً كيف يمكن إساءة استخدام المنطق وكيفية تفادي أو اكتشاف مثل هذه الإساءة أو اكتشافها.

Abstract
In traditional propositional logic, many replacement and inference rules are involved to ascertain if the truth of several antecedents implies the truth of a particular consequent. This paper describes a more powerful technique called the modern syllogistic method. This method is shown to ferret out from a set of premises all that can be concluded from it, with the resulting conclusions cast in the simplest compact form. We observe that all replacement rules are explicitly and inherently integrated within the modern syllogistic method, and prove that all inference rules are simply limited special cases of it. This means that the modern syllogistic method constitutes a complete method of logic deduction. We also show how to use the modern syllogistic method in determining whether inconsistencies exist within a given set of premises and also in detecting formal logical fallacies. We demonstrate the applicability of the method in many diverse fields via a large number of examples that illustrate its mathematical details and exhibit the nature of conclusions it can come up with. In fact, these examples demonstrate the possibility of extracting deductions that are not so obvious and even surprising. The examples also show how logic can be misused, and how logic misuse can be avoided or detected.

Key words: logic deduction, modern syllogistic method, completeness, inconsistency, fallacies.

INTRODUCTION

Propositional logic (also called sentential logic) has a long history of more than 2000 years (Al-Maidani 1993; Kamel 2004). It can be viewed as a grammar for exploring the construction of complex propositions from atomic statements using logical connectives such as "and" "or," and "not.". The fundamental inference problem in propositional logic is to ascertain if the truth of several propositions (called antecedents) implies the truth of a particular proposition of interest (called a consequent).

The traditional (symbolic) approach to propositional logic is based on a clear separation of the syntactic and semantic functions. The syntactic deals with the laws that govern the construction of logical formulas from the atomic propositions and with the structure of proof. Semantics, on the other hand, is concerned with the interpretation and meaning associated with the syntactic objects.

Propositional calculus is based on purely syntactic and mechanical transformation of formulas leading to inference. In traditional logic, deduction is carried out by invoking a number of rules of replacement or inference; these rules announce that certain conclusions follow from certain sets of premises. Some logic-texts list hundreds of such rules, while others make good efforts to summarize and classify them (see, e.g., Klenk 2007; Copi and Cohen 2002). Tables 1 and 2 include a concise summary of these rules.

In this work, we deal with a more general inference problem. We do not ask simply: "Can a given proposition be inferred?" but we ask "What propositions relevant to a given question can be inferred?". This more general problem is called a problem of "logical projection" by Chandru and Hooker (1999), and is solved herein by a very powerful technique, which we call "the modern syllogistic method." An early but incomplete attempt to produce such a method appeared in a text on applied logic by Lynch (1980). The first popular correct description for the method is given by Brown (1990). Other presentations of the method followed (Gregg 1998; Rushdi and Al-Shehri 2002; Rushdi and BaRukab 2007; Rushdi and BaRukab 2008). The great advantage of the method is that it ferrets out from a given set of premises all that can be concluded from it, and it casts these conclusions in the simplest or most compact form. The core step in the modern syllogistic method is dual to the resolution principle in predicate logic (Robinson 1965; Haken 1985; Chang and Lee 1997). This principle is used as a basis for automated reasoning employing non-procedural logic programming languages such as PROLOG (Russel and Norvig 2002).

The remainder of this paper is organized as follows. Section II describes the modern syllogistic method and explains the various techniques of switching algebra (two-valued Boolean algebra) needed for its implementation. Section III shows that the modern syllogistic method is a complete method of logic deduction since it includes all rules (and hence all conventional methods) of propositional logic as special cases of it. Section IV shows that the modern syllogistic method has a built-in capability of deducting the existence of inconsistency within a given set of premises, and of demonstrating that inconsistent premises validly yield any conclusion whatsoever, no matter how irrelevant. Section V illustrates the use of the modern syllogistic method to invalidate formal fallacies. Section VI presents a large number of examples to illustrate the mathematical details of the method, demonstrate its applicability in many diverse fields, and exhibit the nature of conclusions it can come up with.

DESCRIPTION OF THE MODERN SYLLOGISTIC METHOD

Information is conveyed in conventional real algebra by equations. Boole (1847, 1854) and other logicians of the past two centuries therefore found it natural to write logical statements as equations. Such equations are usually reduced to a single equivalent equation of the form:

$$f(\boldsymbol{X}) = 0\ , \tag{1}$$

where f is a Boolean function while $\mathbf{X} = [X_1\ X_2\ ...\ X_n]^T$ is an n-tuple of symbols which represent classes of objects or propositions. Brown (1974, 1975, 1990) and Wheeler (1981) point out the existence of an axiom peculiar to the calculus of propositions, which is called the principle of assertion, and may be stated as:

$$[X_i = 1] = X_i\ . \tag{2}$$

Equation (2) states or asserts that: "To say that a proposition X_i is true is to state the proposition itself". It is therefore possible in the calculus of propositions to dispense entirely with equations. If $f(\mathbf{X})$ is a propositional (i.e., two-valued) function, then equation (1) may be stated equivalently by the proposition:

$$\overline{f}\,(\mathbf{X}) \tag{3}$$

Due to this principle of assertion, most contemporary logicians have abandoned equations in the formulation of propositional logic. The modern syllogistic method, however, symbolizes a revival or renaissance, of the older or classical equation-based approach.

The modern syllogistic method has the following steps:

1. Each of the premises is converted into the form of a formula equated to 0 (which we call an equational form), and then the resulting equational forms are combined together into a single equation of the form $f = 0$. If we have n equivalence relations of the form:

$$T_i = Q_i\ , \qquad 1 \leq i \leq n\ , \tag{4}$$

they are set in the equational form:

$$T_i\,\overline{Q}_i \vee \overline{T}_i\,Q_i = 0\ , \qquad 1 \leq i \leq n. \tag{5}$$

We may also have $(m-n)$ logical implication (logical inclusion) relations of the form:

$$T_i \rightarrow Q_i \,, \quad (n+1) \leq \text{i} \leq m \,. \tag{6}$$

These relations symbolize the statements "*If* T_i *then* Q_i" or equivalently "T_i *if only* Q_i". Condition (6) can be set into the equational form:

$$T_i \overline{Q}_i = 0 \qquad (n+1) \leq \text{i} \leq m. \tag{7}$$

The totality of m premises in (4) and (6) finally reduce to the single equation $f = 0$ (Rushdi 2001(a)), where f is given by:

$$f = \bigvee_{i=1}^{n} (T_i \overline{Q}_\text{i} \vee \overline{T}_i \, Q_i) \;\; \vee \bigvee_{i=(n+1)}^{m} T_i \overline{Q}_i. \tag{8}$$

Equations (4) and (7) represent the dominant forms premises can take. Other less important forms are discussed by Klir and Marin (1969) and can be added to (8) when necessary.

2. The function f in (8) is rewritten as a complete sum (Black canonical form), i.e., as a disjunction of all the prime implicants of f. There are many manual and computer algorithms for developing the complete sum of a switching function f (see, e.g., Muroga 1979; Brown 1990; and Rushdi 2001(b); Rushdi & Al-Yahya 2001).

Most of these algorithms depend on two logical operations: (a) Consensus generation (or equivalently multiplying a product-of-sums into a sum-of- products), and (b) absorption.

3. Suppose the complete sum of f takes the form:

$$f = \bigvee_{i=1}^{\ell} P_i = 0 \,, \tag{9}$$

where P_i is the i the prime implicant of f. Equation (9) is equivalent to the set of equations:

$$P_i = 0 \,, \qquad 1 \leq i \leq \ell. \tag{10}$$

Equation (10) states in the simplest equational form all that can be concluded from the original premises. The conclusions in (10) can also be cast into implication form. Suppose P_i is given a conjunction of uncomplemented literals X_{ij} and complemented literals $\overline{Y}_{ij}$, i.e.

$$P_i = \bigwedge_{j=1}^{r} X_{ij} \;\wedge\; \bigwedge_{j=1}^{s} \overline{Y}_{ij} \,, \qquad 1 \leq i \leq \ell \,, \tag{11}$$

Then,

$$\bigwedge_{j=1}^{r} X_{ij} \rightarrow \overline{(\bigwedge_{j=1}^{s} \overline{Y}_{ij})}, \quad 1 \le i \le \ell, \tag{12}$$

or as

$$\bigwedge_{j=1}^{r} X_{ij} \rightarrow \bigvee_{j=1}^{s} Y_{ij}, \quad 1 \le i \le \ell. \tag{13}$$

We reiterate that the modern syllogistic method produces all possible consequents (since CS(*f*) is a disjunction of all the prime implicants of *f*, and that it casts these consequents in the most compact form (since all the implicants in CS(*f*) are prime). If any implicant (whether it is prime or not) of *f* is equated to 0, then the result is a true consequent (albeit not necessarily in the most compact form). To test the truth of any claimed consequent based on a given set of premises, one just needs to cast this consequent in the form of a term equated to 0, and check to see if this term subsumes (at least) one of the prime implicants in CS(*f*) derived for the set of premises.

COMPLETENESS OF THE MODERN SYLLOGISTIC METHOD

To demonstrate the power and completeness of the modern syllogistic method, we introduce Table 3 which shows how this method can be used to derive each of the rules of inference in Table 2. This amounts to a novel proof that each of these rules is a special case of the modern syllogistic method. Note that the set of consequents of the syllogistic method, being a complete set of conclusions, is usually a strict superset of the set of conclusions any rule of inference produces. The consequents of the syllogistic method include all the premises of a certain rule, possibly simplified or rephrased, plus several new conclusions, of which only one is pointed out by the rule. For example, the syllogistic method can handle the three premises of the rule of constructive dilemma to produce six conclusions, of which three are simply echoes of the original premises, and two are intermediate conclusions "ignored" by the rule, while the sixth is the ultimate conclusion of the rule. Table 3 is a major contribution of this paper, since it demonstrates definitely that the modern syllogistic method encompasses a complete set of inference rules. Winnie (1970) and Copi (1979) showed that the list of 10 replacement rules in Table 1 together with the top 9 inference rules in Table 2 constitute a complete system of truth-functional logic in the sense that it permits the construction of a formal proof of validity for any valid truth-functional argument. In fact these 19 rules are somewhat redundant, in the sense that they constitute more than a bare minimum which would suffice for the construction of formal proofs of validity for extended arguments (Copi and Cohen 2002).

HANDLING INCONSISTENCIES

In this section, we show that the modern syllogistic method has a built-in capability of detecting the existence of inconsistency within a given set of premises, and of explicitly demonstrating the ramifications of such inconsistency. If a set of premises is inconsistent, then their conjunction should be false, which means that the function f in their collective equational representation (1) should be equal to 1. However, this fact is usually not obvious, but it can be brought to light easily through the modern syllogistic method, which computes CS(*f*) naturally in its usual procedure for any set of premises and finds it to be equal to 1 if and only if such a set is inconsistent. The result

$$\mathrm{CS}(f) = 1, \tag{14}$$

has two important aspects:

I. Inconsistent premises (even when their inconsistency is highly concealed) lead to the self-evident contradiction 1=0, indicating that the conjunction of the premises is self contradictory, i.e. no truth functional assignment can make all the premises true simultaneously.

II. Inconsistent premises mean every consequent is true, since every term subsumes the term 1. Therefore, inconsistent premises can be used to assert the truth of any consequent to which the premises are totally irrelevant, and to assert simultaneously the truth of any statement and its denial or contradictory statement.

The above discussion shows that a user of the modern syllogistic method is immune against falling into the trap of using a set of inconsistent premises to derive any conclusion. The method will alert its user to the concealed inconsistencies by producing CS(*f*) =1. Here the user should refrain from making any conclusion, and should revise his set of premises to change it into a consistent one. The above discussion also demonstrates a possible way for the notorious misuse of logic. To prove any conclusion whatsoever, all one needs is to support it by a set of inconsistent premises, preferably (but not necessarily) with the inconsistency concealed as much as possible, and with some apparent or fictious relevance of the premises to the desired conclusion.

INVALIDATING FORMAL FALLACIES

In this section, we illustrate how the modern syllogistic method can be used in detecting and invalidating certain purported arguments or formal fallacies. One of the inference rules in Table 2 is Modus Ponens, which asserts that premises ($A \to B$) and A lead to consequent B. A similar purported "rule" claims that premises ($A \to B$) and B leads to consequent A. For this purported "rule", we can combine the premises in the single-equation form:

$$f = A\overline{B} \vee \overline{B} = 0, \tag{15}$$

from which we conclude that:

$$CS(f) = \overline{B} = 0. \tag{16}$$

Therefore, the claimed consequent ($\overline{A} = 0$) is not asserted by the premises. This purported "rule" is an invalid argument and is called the converse fallacy (Anderson 2001) or the fallacy of affirming the consequent (Copi and Cohen 2002).

Another purported "rule" (whose shape is somewhat like that of Modus Tollens) claims that premises ($\overline{A} \rightarrow \overline{B}$) and A lead to consequent B. Again, we can combine the premises of this purported "rule", to obtain the single equation:

$$f = \overline{A}B \vee \overline{A} = 0, \tag{17}$$

from which we obtain:

$$CS(f) = \overline{A} = 0. \tag{18}$$

The only consequent of the given premises is ($\overline{A} = 0$) which is irrelevant to the claimed consequent ($\overline{B} = 0$). This purported "rule" is again an invalid argument and is called the inverse fallacy (Anderson 2001), or the fallacy of denying the antecedent (Copi and Cohen 2002).

EXAMPLES

Example 1

The intelligence of a certain small country m is warning its leadership against an imminent severe attack from a neighboring wicked enemy k whose forces significantly outnumber those of the country m. A massive war seems unavoidable and the leadership of m is seeking assistance from its historical allies which we label a, b, c, and d. Unfortunately, internal conflicts between the regimes of these four countries set the following restrictions about their possible participation at the side of m in the upcoming war.

1. If *a* goes to the war, *b* will not go and c will.
2. If *b* and d go, then either *a* or *c* (but not both) will go.
3. If *c* goes and *b* does not, then d will go but *a* will not.

Let us define A to be the proposition "a will go to the war", B to be "b will go to the war", etc. Then statements 1 through 3 above may be translated into symbolic forms as follows:

Conditional form	Equational form
$A \rightarrow \overline{B}\,C$	$A(B \vee \overline{C}) = 0$
$BD \rightarrow \overline{A}\,C \vee A\,\overline{C}$	$BD(\overline{A}\,\overline{C} \vee A\,C) = 0$
$\overline{B}\,C \rightarrow \overline{A}\,D$	$\overline{B}\,C(A \vee \overline{D}) = 0$

The given data are therefore equivalent to the propositional equation $f = 0$, where f is given by:

$$f = A(B \vee \overline{C}) \vee B\,D(\overline{A}\,\overline{C} \vee A\,C) \vee \overline{B}\,C(A \vee \overline{D}) \tag{19.a}$$

$$= A\,B \vee A\,\overline{C} \vee \overline{A}\,B\,\overline{C}\,D \vee A\,B\,C\,D \vee A\,\overline{B}\,C \vee \overline{B}\,C\,\overline{D}. \qquad (19.b)$$

The complete sum for f (the Blake canonical form for f) is obtained by the improved Tison method (Rushdi and Al-Yahya (2001)) as shown in Fig. 1 in which consensi are formed with respect to each of the four variables A, B, C and D respectively. Each step of consensus generation is followed by a step of absorption in which a term is absorbed by another if the former subsumes the latter (i.e., if the set of literals for the term is a superset of the literals for the absorbing term). In Fig. 1, encircled terms are those absorbed, while those surviving absorption are set in bold. The formula expressing f gradually evolved as:

$$f = A\,B \vee A\,\overline{C} \vee \overline{A}\,B\,\overline{C}\,D \vee A\,B\,C\,D \vee A\,\overline{B}\,C \vee \overline{B}\,C\,\overline{D}, \qquad (20.a)$$

$$= A\,B \vee A\,\overline{C} \vee A\,\overline{B}\,C \vee B\,\overline{C}\,D \vee \overline{B}\,C\,\overline{D}, \qquad (20.b)$$

$$= A\,B \vee B\,\overline{C}\,D \vee A\,\overline{C} \vee A\,C \vee \overline{B}\,C\,\overline{D}, \qquad (20.c)$$

$$= \overline{B}\,C\,\overline{D} \vee B\,\overline{C}\,D \vee A, \qquad (20.e)$$

where the last formula stands for CS(f), i.e. it is a disjunction of all the prime implicants of f. Equation (14.f) is equivalent to

$$B\,\overline{C}\,D = 0, \qquad (21.a)$$

$$\overline{B}\,C\,\overline{D} = 0, \qquad (21.b)$$

$$A = 0. \qquad (21.c)$$

The prime clauses of the possible war scenarios are, therefore, as follows:

$BD \to C$	"If b and d go to the war, then c will go."
$C \to B \vee D$	"If c goes to the war, then either b or d will go."
$A \to 0$	"Country a will not go to the war."

This last not-very obvious conclusion leads the leadership of m to despair of any possible backing by the ally a. This state of affairs should be accepted only if all the given premises are guaranteed to be true.

Example 2

Discuss the possible consequents of the following premises:

An engineering student will find a job when he graduates only if he is well-prepared, and he will be well-prepared only if he can read and write extremely well and has a good technical education. He will read and write extremely well if and only if he takes many humanities courses, but if he takes many humanities courses he will not take many technical courses, and if he does not take many technical courses then he will not have a good technical education.

Let us define:

J = The student will find a job when he graduates,

P = He is well-prepared,

C = He can read and write extremely well,

E = He has a good technical education,

H = He takes a lot of humanities courses,

T = He takes many technical courses.

The statements above may be translated as follows:

Conditional form	Equational form
$J \to P$	$J\,\overline{P} = 0$
$P \to C\,E$	$P\,(\overline{C} \vee \overline{E}) = 0$
$C \equiv H$	$C\,\overline{H} \vee \overline{C}\,H = 0$
$H \to \overline{T}$	$H\,T = 0$
$\overline{T} \to \overline{E}$	$\overline{T}\,E = 0$

Then the given data are equivalent to the propositional equation $f = 0$, where f is given by

$$f = J\,\overline{P} \vee P\,\overline{C} \vee P\,\overline{E} \vee C\,\overline{H} \vee \overline{C}\,H \vee H\,T \vee \overline{T}\,E. \qquad (22)$$

The complete sum of f is

$$CS\,(f) = P \vee J \vee C\,\overline{H} \vee \overline{C}\,H \vee H\,T \vee C\,T \vee \overline{T}\,E \vee H\,E \vee C\,E = 0 \quad (23)$$

There are nine prime consequents, which include in particular

$P = 0$ {The student is not well prepared},

$J = 0$ {He will not find a job when he graduates}.

This example is a very good illustration of how logic can be easily misused. All the innocent-looking premises seem plausible when viewed separately, but taken together they combine to produce some totally unexpected (sometimes shocking) results. Historically, logic has been misused by being manipulated to give some sort of "proof" for false propositions. When one understands this, it is possible to identify the pitfall(s) within the whole process, which are sometimes hidden in not-so-thoroughly-investigated premises, but are occasionally due to the use of incorrect "rules" of inference (Nelson et al. 2003).

Example 3

What are the consequents hidden within the following premises?

1. If a student drinks too much coffee then he cannot sleep well and he does not study properly.
2. If he does not drink enough coffee he cannot stay awake and he does not study at all.
3. Either he drinks too much coffee or not enough.

Let us symbolize the pertinent propositions as

M = The student drinks too much coffee,

P = He studies properly,

A = He does not study at all,

N = He does not drink enough coffee.

The statements 1 through 3 above may be translated as follows:

Clausal form	Equational form
$M \to \overline{P}$	$M\,P = 0$
$N \to A$	$N\,\overline{A} = 0$
$M \vee N$	$\overline{M}\;\overline{N} = 0$

Then the given data are equivalent to a propositional equation $f = 0$, namely:

$$f = M\,P \vee N\,\overline{A} \vee \overline{M}\;\overline{N} = 0 . \qquad (24)$$

The complete sum of f is

$$CS(f) = M\,P \vee N\,\overline{A} \vee \overline{M}\,\overline{N} \vee P\,\overline{N} \vee \overline{M}\,\overline{A} \vee P\,\overline{A} = 0\,. \tag{25}$$

In addition to consequents that are restatements of the original premises, the following "new" consequents emerge:

$P\,\overline{N} = 0$ $\{P \rightarrow N\}$ {If he studies properly then he drinks enough coffee}.

$\overline{M}\,\overline{A} = 0$ $\{M \vee A = 1\}$ {Either he drinks too much coffee or he does not study at all}.

$P\,\overline{A} = 0$ $\{\overline{P} \vee A = 1\}$ {Either he does not study properly or he does not study at all}.

These new consequents, and in particular the last one of them, were indeed not-very-obvious conclusions at the outset. Of course, the last consequent may drive the student to despair of being able to study properly or to study at all. It may be argued that the real culprit behind this state of affairs is premise 3 which asserts extreme coffee-drinking habits for the student. If the student strikes the right balance between these two extremes by drinking a "reasonable" amount of coffee, premise 3 ceases to be true, and the consequents obtained herein cannot be reached. However, we must stress that the undesirable consequent that the student cannot study properly or at all, arises not because of premise 3 alone, but it stems from the three premises combined together. Each of these premises, must be scrutinized thoroughly and individually if the derived consequents are to make any sense at all and if their truth is to be accepted.

Example 4

Ferret out all consequents hidden in the following premises:

1. If nuclear power becomes our chief source of energy, then either there will be a terrible accident or severe waste disposal problems.

2. If there are severe waste disposal problems and an increase in uranium costs, then people will cut their energy consumption.

3. There will be a terrible accident only if safeguards are inadequate.

4. Nuclear power will become our chief source of power.

5. Uranium costs will increase.

6. Safeguards are not inadequate.

Let us define:

N = Nuclear power becomes the chief source of energy,

A = There is a terrible accident,

W = There are severe waste disposal problems,

U = There is an increase in uranium cost,

C = People will cut their energy consumption,

Q = Safeguards are adequate.

The statements 1 through 6 above may be translated as follows:

Clausal form	Equational form
$N \to A \vee W$	$N\,\overline{A}\,\overline{W} = 0$
$W\,U \to C$	$W\,U\,\overline{C} = 0$
$A \to \overline{Q}$	$A\,Q = 0$
N	$\overline{N} = 0$
U	$\overline{U} = 0$
Q	$\overline{Q} = 0$

The given data are equivalent to a propositional equation $f = 0$, where f is given by

$$f = N\,\overline{A}\,\overline{W} \vee W\,U\,\overline{C} \vee A\,Q \vee \overline{N} \vee \overline{U} \vee \overline{Q} = 0\,. \tag{26}$$

The complete sum of f is

$$CS(f) = \overline{W} \vee \overline{C} \vee A \vee \overline{N} \vee \overline{U} \vee \overline{Q} = 0\,. \tag{27}$$

New consequents are

$\overline{W} = 0$	{There will be severe waste disposal problems}.
$\overline{C} = 0$	{People will cut their energy consumption}.
$A = 0$	{There will be no terrible accident}.

In our opinion, the premises and conclusions of this example seem to be educated forecasts of the future. However, some or all of the premises (and hence, the consequents) may easily be disputed by many people including experts. This is a clear indication of our shortcomings as human beings, especially when we attempt to extrapolate existing data or scenarios. One might enquire whether the given premises support a certain given conclusion. For example, one could ask: Is it possible to infer from the premises that $\{A \to W\}$? The answer is yes since the required conclusion is equivalent to the equation $A\overline{W} = 0$, and the term $A\overline{W}$ subsumes the prime implicant A (or the prime implicant $\overline{W}$) in (27). Is it possible to conclude from the premises that $\{A \equiv W\}$? The answer is no since the required conclusion is equivalent to the equation $A\overline{W} \vee \overline{A}W = 0$, and the term $\overline{A}W$ therein does not subsume any of the prime implicants in (27).

Example 5

Discuss what happens under the following conditions.

1. Pollution will increase if government restrictions are relaxed.
2. If pollution increases there will be a decline in the general health of the population.
3. If there is a decline in health in the population, productivity will fall.
4. The economy will remain healthy only if productivity does not fall.

Let us define:

P = Pollution will increase,

R = Government restrictions are relaxed,

D = There is a decline in general health of the population,

F = Productivity will fall,

E = The economy remains healthy.

The statements 1 through 4 above may be translated as follows:

Conditional form	Equation form
$R \to P$	$R\,\overline{P} = 0$
$P \to D$	$P\,\overline{D} = 0$
$D \to F$	$D\,\overline{F} = 0$
$E \to \overline{F}$	$E\,F = 0$

Then the given data are equivalent to the propositional equation $f = 0$, where f is given by

$$f = R\,\overline{P} \vee P\,\overline{D} \vee D\,\overline{F} \vee E\,F = 0 . \tag{28}$$

The complete sum of f is:

$$CS(f) = R\,\overline{P} \vee P\,\overline{D} \vee D\,\overline{F} \vee E\,F \vee R\,\overline{D} \vee D\,E \vee P\,\overline{F} \vee P\,E \vee R\,\overline{F} \vee R\,E = 0 \tag{29}$$

One of the 6 new consequents is:

$R\,E = 0$ { $R \rightarrow \overline{E}$, i.e. if government restrictions are relaxed, then the economy will not remain healthy}.

The present argument can be used to support the case for a stronger regulatory role by the government.

Example 6

Consider the following premises:

1. If the quadrilateral *abcd* is cyclic (can be inscribed in a circle), then

   $m\angle a + m\angle c = p$. { $m\angle a$ means measure of angle a}

2. If the quadrilateral *abcd* is a parallelogram, then $m\angle a = m\angle c$.
3. If $m\angle a + m\angle c = p$, then $m\angle a = \frac{p}{2}$.
4. If quadrilateral *abcd* is a parallelogram and $m\angle a = \frac{p}{2}$, then *abcd* is a rectangle.
5. Quadrilateral *abcd* is both a parallelogram and cyclic.

What can be concluded?

Use the following switching variables to symbolize various propositions

A = quadrilateral *abcd* is cyclic,

B = quadrilateral *abcd* is a parallelogram,

C = angles a and c are complementary angles ($m\angle a + m\angle c = p$),

D = angles a and c are equal ($m\angle a = m\angle c$),

F = angle a is aright angle ($m\angle a = \frac{p}{2}$),
E = quadrilateral *abcd* is a rectangle.

The premises can be stated as:

Conditional form	Equation form
$A \rightarrow C$	$A\,\overline{C} = 0$
$B \rightarrow D$	$B\,\overline{D} = 0$
$C\,D \rightarrow F$	$C\,D\,\overline{F} = 0$
$B\,F \rightarrow E$	$B F\,\overline{E} = 0$
$A\,B$	$\overline{A} \vee \overline{B} = 0$

Then the given data are equivalent to the propositional equation $f = 0$, where f is given by:

$$f = A\,\overline{C} \vee B\,\overline{D} \vee C\,D\,\overline{F} \vee B\,F\,\overline{E} \vee \overline{A} \vee \overline{B} = 0. \tag{30}$$

The complete sum of f is:

$$CS(f) = \overline{C} \vee \overline{D} \vee \overline{F} \vee \overline{E} \vee \overline{A} \vee \overline{B} = 0. \tag{31}$$

The consequents are $\overline{\mathrm{C}} = \overline{\mathrm{D}} = \overline{\mathrm{F}} = \overline{\mathrm{E}} = \overline{\mathrm{A}} = \overline{\mathrm{B}} = 0$. Notable among these is $\overline{\mathrm{E}} = 0$ ($\mathrm{E} = 1$) which means that the quadrilateral *abcd* is a rectangle.

This example is a sample on how problems of Euclidean geometry can be handled by the syllogistic method. For 2000 years, these problems served as the best and most useful domain for applying the conventional method of logic. The premises of such problems are the axioms, postulates and theorems of geometry as well as the given information for a particular problem (Kamel 2004).

Example 7

The following problem is encountered in the study of automatic control systems (Kuo, 1995). For the transfer function

$$\frac{Y(s)}{U(s)} = K\,\frac{(s + q_1)(s + q_2)}{(s + p_1)(s + p_2)}, \tag{32}$$

a certain state decomposition is uncontrollable if and only if $K = 0$ or $q_1 = p_2$. This decomposition is unobservable if and only if $q_1 = p_1$ or $q_2 = p_1$ or $q_2 = p_2$. The transfer function has some zero-pole cancellations. What can be concluded from these premises?

Use the following switching variables to symbolize the various propositions:

C_{ij} = Zero q_i cancels pole p_j ($q_i = p_j$), $1 \le \mathrm{i}, \mathrm{j} \le 2$,

S = Decomposition is controllable,

V = Decomposition is observable,

G = The gain K is zero.

The premises can be stated as

Clausal Form	Equational form
$\overline{S} \equiv (G \vee C_{12})$	$S\,G \vee S\,C_{12} \vee \overline{S}\,\overline{G}\,\overline{C}_{12} = 0$
$\overline{V} \equiv (C_{11} \vee C_{21} \vee C_{22})$	$V\,C_{11} \vee V\,C_{21} \vee V\,C_{22} \vee \overline{V}\,\overline{C}_{11}\,\overline{C}_{21}\,\overline{C}_{22} = 0$
$C_{11} \vee C_{12} \vee C_{21} \vee C_{22}$	$\overline{C}_{11}\,\overline{C}_{12}\,\overline{C}_{21}\,\overline{C}_{22} = 0$

Then the given data are equivalent to the propositional equation $f = 0$, where f is given by:

$$f = S\,G \vee S\,C_{12} \vee \overline{S}\,\overline{G}\,\overline{C}_{12} \vee V\,C_{11} \vee V\,C_{21} \vee V\,C_{22} \vee \overline{V}\,\overline{C}_{11}\,\overline{C}_{21}\,\overline{C}_{22} \vee \overline{C}_{11}\,\overline{C}_{12}\,\overline{C}_{21}\overline{C}_{22} = 0\,. \tag{33}$$

The complete sum of f is:

$$CS(f) = S\,G \vee S\,C_{12} \vee \overline{S}\,\overline{G}\,\overline{C}_{12} \vee V\,C_{11} \vee V\,C_{21} \vee V\,C_{22} \vee \overline{V}\,\overline{C}_{11}\,\overline{C}_{21}\,\overline{C}_{22} \vee \overline{C}_{11}\,\overline{C}_{12}\,\overline{C}_{21}\,\overline{C}_{22} \vee S\,\overline{C}_{11}\,\overline{C}_{21}\,\overline{C}_{22} \vee V\,\overline{C}_{12} \vee S\,V = 0. \tag{34}$$

In addition to the old premises, we have three new conclusions:

(1) $S\,\overline{C}_{11}\,\overline{C}_{21}\,\overline{C}_{22} = 0$ or $\overline{S} \vee C_{11} \vee C_{21} \vee C_{22} = 1$ {Either the decomposition is uncontrollable , q_1 cancels p_1 , q_2 cancels p_1, or q_2 cancels p_2 (or any combination thereof)}.

(2) $V\,\overline{C}_{12} = 0$ or $\overline{V} \vee C_{12} = 1$ {Either the decomposition is unobservable or q_1 cancels p_2 (or both)}.

(3) $S\,V = 0$ or $\overline{S} \vee \overline{V} = 1$ {Either the decomposition is uncontrollable or it is unobservable or both}.

This example demonstrates a well known theorem in control theory (Kuo 1995) stating that any state decomposition for a transfer function having a pole-zero cancellation is either uncontrollable or unobservable or both.

Example 8
This example is adapted from the famous symbolic-logic text by Carroll (1955).
Here, we want to decide what can be concluded from the following premises:

1. When I work a logic example without grumbling, you may be sure it is one that I can understand.

2. These sorites are not arranged in regular order, like the examples I am used to.

3. An easy example never makes my head ache.

4. I cannot understand examples that are not arranged in regular order, like those I am used to.

5. I never grumble at an example, unless it gives me a headache.

We use the following switching variables to symbolize the following propositions

G = I grumble,

U = I understand the logic example I am working with.

A = These sorites are arranged in regular order like the examples I am used to.

E = The example is easy,

H = I have a headache.

The premises can be stated as:

Clausal form	Equational form
$\overline{G} \rightarrow U$	$\overline{G}\,\overline{U} = 0$
$\overline{A}$	$A = 0$
$E \rightarrow \overline{H}$	$E\,H = 0$
$\overline{A} \rightarrow \overline{U}$	$\overline{A}\,U = 0$
$\overline{H} \rightarrow \overline{G}$	$\overline{H}\,G = 0$

Then the given data are equivalent to the propositional equation $f = 0$, where f is given by

$$f = \overline{G}\,\overline{U} \vee A \vee E\,H \vee \overline{A}\,U \vee \overline{H}\,G = 0\,. \tag{35}$$

A syllogistic formula of f is:

$$\begin{aligned} f = {} & \overline{G}\,\overline{U} \vee A \vee E\,H \vee \overline{A}\,U \vee \overline{H}\,G \vee \overline{U}\,\overline{H} \vee \overline{G}\,\overline{A} \vee \overline{H}\,\overline{A} \\ & \vee U \vee \overline{G} \vee \overline{H} \vee E = 0\,, \end{aligned} \tag{36}$$

and hence its complete sum is:

$$CS(f) = A \vee U \vee \overline{G} \vee \overline{H} \vee E = 0 . \tag{37}$$

The conclusions are:

A = 0 {There sorites are not arranged in regular order like the examples I am used to}.

U = 0 { I cannot understand the example I am working with}.

$\overline{\mathrm{G}} = 0$ { I do grumble at this example}.

$\overline{\mathrm{H}} = 0$ {This example makes my head ache}.

$\mathrm{E} = 0$ {This example is not easy}.

Only 4 out of these 5 conclusions are new. Traditionally, logicians were most interested in the most hidden conclusion; this is the last conclusion to appear in the deduction process (whether it is via separate rules of inference or via consensus generation). This is the conclusion that $E = 0$, which says that this example is not easy.

Example 9

This example is again adapted from Carroll (1955). Here, one wants to determine what can be concluded from the following premises:

1. All the dated letters in this room are written on blue paper;
2. None of them is in black ink, except those that are written in the third person;
3. I have not filed any of them that I can read;
4. None of them, that are written on one sheet, is undated;
5. All of them, that are not crossed, are in black ink;
6. All of them, written by Ali, begin with "Dear Sir";
7. All of them, written on blue paper, are filed;
8. None of them, written on more than one sheet, is crossed;
9. None of them, that begins with "Dear Sir" is written in the third person.

We use the following switching variables to symbolize the pertinent propositions:

D = The letter is dated,

B = The letter is written on blue paper,

I = The letter is in black ink,

T = The letter is written in the third person,

F = The letter is filed,

R = The letter can be read,

O = The letter is written on one sheet,

C = The letter is crossed,

A = The letter is written by Ali,

S = The letter begins with "Dear Sir".

The premises can be stated as:

Clausal form	Equational form
$D \rightarrow B$	$D\,\overline{B} = 0$
$I \rightarrow T$	$I\,\overline{T} = 0$
$R \rightarrow \overline{F}$	$R\,F = 0$
$O \rightarrow D$	$O\,\overline{D} = 0$
$\overline{C} \rightarrow I$	$\overline{C}\,\overline{I} = 0$
$A \rightarrow S$	$A\,\overline{S} = 0$
$B \rightarrow F$	$B\,\overline{F} = 0$
$\overline{O} \rightarrow \overline{C}$	$\overline{O}\,C = 0$
$S \rightarrow \overline{T}$	$S\,T = 0$

Then the given data are equivalent to the propositional equation $f = 0$, where f is given by

$$\begin{aligned} f = {} & D\,\overline{B} \vee I\,\overline{T} \vee R\,F \vee O\,\overline{D} \vee \overline{C}\,\overline{I} \vee A\,\overline{S} \\ & \vee B\,\overline{F} \vee \overline{O}\,C \vee S\,T = 0 \end{aligned} \quad . \qquad (38)$$

The complete sum of f is obtained via the improved Tison method of Rushdi and Al-Yahya (2001) as:

$$
\begin{aligned}
CS(f) = {} & D\,\overline{B} \vee I\,\overline{T} \vee R\,F \vee O\,\overline{D} \vee \overline{C}\,\overline{I} \vee A\,\overline{S} \vee B\,\overline{F} \vee \overline{O}\,C \vee S\,T \\
& \vee \overline{B}\,O \vee D\,\overline{F} \vee O\,\overline{F} \vee \overline{T}\,\overline{C} \vee I\,S \vee \overline{C}\,S \vee R\,B \vee R\,D \\
& \vee R\,O \vee \overline{D}\,C \vee \overline{B}\,C \vee \overline{F}\,C \vee R\,C \vee \overline{I}\,\overline{O} \vee \overline{T}\,\overline{O} \vee \overline{O}\,S \\
& \vee \overline{D}\,\overline{I} \vee \overline{B}\,\overline{I} \vee \overline{F}\,\overline{I} \vee R\,\overline{I} \vee \overline{D}\,\overline{T} \vee \overline{B}\,\overline{T} \vee \overline{F}\,\overline{T} \vee R\,\overline{T} \\
& \vee \overline{D}\,S \vee \overline{B}\,S \vee \overline{F}\,S \vee R\,S \vee A\,T \vee I\,A \vee A\,\overline{C} \vee A\,\overline{O} \\
& \vee A\,\overline{D} \vee A\,\overline{B} \vee A\,\overline{F} \vee A\,R = 0.
\end{aligned}
\tag{39}
$$

To the given 9 premises, we have added 36 new conclusions, the most hidden of which (the last to appear in our consensus generation) is:

$$A\,R = 0 \text{ or } \{A \rightarrow \overline{R}\},$$

which means that I cannot read any of Ali's letters. This conclusion is usually the only one sought for in traditional logic, with all other new conclusions being ignored or viewed as less important or uninteresting.

Example 10

Three balls are colored red, white and blue, but not necessarily respectively. Of the following three statements, one only is true.

A is red; B is not red; C is not blue.

What color is each ball?

We can use 9 switching variables to symbolize the 3 color possibilities for each of the 3 balls. Each switching variable is represented by the upper-case letter representing a ball subscribed by a small-case letter representing a color, where r stands for red, w stands for white, and b stands for blue. Figure 3 shows that only 4 dependent switching variables A_r, A_w, B_r and B_w suffice to describe the problem as the other 5 variables are completely given in terms of these 4 variables.

The given requirements can be stated as:

$$A_r\,B_r\,C_b \vee \overline{A_r}\,\overline{B_r}\,C_b \vee \overline{A_r}\,B_r\,\overline{C_b} = 1, \tag{40}$$

subject to the conditions in Figure 3, i.e.,

$$A_r\,A_w = A_r\,B_r = A_w\,B_w = B_r\,B_w = \overline{A_r}\,\overline{A_w}\,\overline{B_r}\,\overline{B_w} = 0, \tag{41}$$

$$C_b = A_r B_w \vee A_w B_r\,. \tag{42}$$

Equation (40) can be rewritten as:

$$
\begin{aligned}
& (\overline{A_r} \vee \overline{B_r} \vee \overline{C_b})\,(A_r \vee B_r \vee \overline{C_b})\,(A_r \vee \overline{B_r} \vee C_b) \\
& = (\overline{A_r}\,B_r \vee A_r\,\overline{B_r} \vee \overline{C_b})\,(A_r \vee \overline{B_r} \vee C_b) \\
& = A_r\,\overline{B_r} \vee (A_r \vee \overline{B_r})\,C_b \vee \overline{A_r}\,B_r\,C_b = 0,
\end{aligned}
$$

in which the value of C_b can be substituted from (42), and to which the conditions in (41) can be added to give:

$$\begin{aligned} f &= A_r\,\overline{B_r} \vee (A_r \vee \overline{B_r})\,(\overline{A_r} \vee \overline{B_w})\,(\overline{A_w} \vee \overline{B_r}) \vee \overline{A_r}\,B_r\,(A_r\,B_w \vee A_w\,B_r) \\ &\quad \vee A_r\,A_w \vee A_r\,B_r \vee A_w\,B_w \vee B_r\,B_w \vee \overline{A_r}\,\overline{A_w}\,\overline{B_r}\,\overline{B_w} \\ &= A_r\,\overline{B_r} \vee A_r\,\overline{A_w}\,\overline{B_w} \vee A_r\,\overline{B_r}\,\overline{B_w} \vee \overline{A_r}\,\overline{B_r} \vee \overline{B_r}\,\overline{B_w} \vee \overline{A_r}\,A_w\,B_r \vee A_r\,A_w \\ &\quad \vee A_r\,B_r \vee A_w\,B_w \vee B_r\,B_w \vee \overline{A_r}\,\overline{A_w}\,\overline{B_r}\,\overline{B_w} \\ &= A_r\,\overline{B_r} \vee A_r\,\overline{A_w}\,\overline{B_w} \vee \overline{A_r}\,\overline{B_r} \vee \overline{B_r}\,\overline{B_w} \vee \overline{A_r}\,A_w\,B_r \\ &\quad \vee A_r\,A_w \vee A_r\,B_r \vee A_w\,B_w \vee B_r\,B_w = 0. \end{aligned} \tag{43}$$

The complete sum of f is obtained in Fig. 3 via the improved Tison method (Rushdi and Al-Yahya 2001) as:

$$CS(f) = A_r \vee \overline{B_r} \vee A_w \vee B_w = 0. \tag{44}$$

Note that in Fig. 3, after consensi were generated with respect to two variables only, the complete sum emerged without any need to consider consensi with respect to the other two variables. Equation (44) yields the solution:

$$A_r = \overline{B_r} = A_w = B_w = 0,$$

from which one obtains:

$$\begin{aligned} A_b &= \overline{A_r}\,\overline{A_w} = 1, \\ B_b &= \overline{B_r}\,\overline{B_w} = 0, \\ C_r &= \overline{A_r}\,\overline{B_r} = 0, \\ C_w &= \overline{A_w}\,\overline{C_w} = 1, \\ C_b &= A_r\,B_w \vee A_w\,B_r = 0, \end{aligned}$$

which means that ball A is blue, ball B is red, and ball C is white. In passing, we note that the problem studied in this example is not as easy to formulate via the modern syllogistic method as the problems in the former examples. However, the details of this example are interesting since they show how the method can combine a variety of requirements and conditions into a single function.

Example 11

Flip flops are basic memory elements used in logic or digital design (Muroga 1979). A well known type of flip flops is the JK flip flop whose next state Y is defined in terms of its inputs J and K and its present state *y* by:

$$Y = J\,\bar{y} \vee \bar{k}\,y\,,$$

The foregoing equation is equivalent to the single equation $f = 0$, where f is:

$$f = Y\,\overline{(J\,\bar{y} \;\vee\; \bar{k}\,y)} \vee \bar{Y}\,(J\,\bar{y} \;\vee\; \bar{k}\,y) = 0$$

$$= Y\,(\bar{J}\,\bar{y} \;\vee\; k\,y) \vee \bar{Y}\,J\,\bar{y} \;\vee\; \bar{Y}\,\bar{k}\,y \;=\; 0$$

$$= Y\,\bar{J}\,\bar{y} \;\vee\; Y\,k\,y \;\vee\; \bar{Y}\,J\,\bar{y} \;\vee\; \bar{Y}\,\bar{k}\,y \;=\; 0 \qquad (45)$$

The complete sum of f is:

$$CS(f) = Y\,\bar{J}\,\bar{y} \vee Y\,k\,y \vee Y\,\bar{J}\,k \vee \bar{Y}\,J\,\bar{y} \vee \bar{Y}\,\bar{k}\,y \vee \bar{Y}\,J\,\bar{k} \;=\; 0 \qquad (46)$$

The associated prime clauses are:

1. $Y \rightarrow J \vee y$
2. $Y \rightarrow \bar{k} \vee \bar{y}$
3. $Y \rightarrow J \vee \bar{k}$
4. $\bar{Y} \rightarrow \bar{J} \vee y$
5. $\bar{Y} \rightarrow k \vee \bar{y}$
6. $\bar{Y} \rightarrow \bar{J} \vee k$

Which may be interpreted as follows:

1. If the next state is high, then the J input is high or the present state is high.
2. If the next state is high, then the K input is low or the present state is low.
3. If the next state is high, then the J input is high or the k input is low.
4. If the next state is low, then the J input is low or the present state is high.
5. If the next state is low, then the K input is high or the present state is low.
6. If the next state is low, then the J input is low or the K input is high.

These 6 conclusions can be easily verified from the excitation table of the JK flip flop shown in Fig. 4, which is rearranged in Fig. 5 for convenience.

Example 12

This example is adapted from Copi and Cohen (2002). Consider the following set of premises:

1. If the airplane had engine trouble (T), then it would have landed in Riyadh (R).
2. If the airplane did not have engine trouble ($\overline{T}$), then it would have landed in Jeddah (J).
3. The airplane did not land at either Riyadh or Jeddah.

These premises have the following formulation

Clausal form	Equational form
$T \to R$	$T\,\overline{R} = 0$
$\overline{T} \to J$	$\overline{T}\,\overline{J} = 0$
$(\overline{R \vee J})$	$R \vee J = 0$

These premises combine to give the function

$$f = T\overline{R} \vee \overline{T}\,\overline{J} \vee R \vee J = 0, \tag{47}$$

whose complete sum is

$$CS\,(f) \equiv 1, \tag{48}$$

which leads to the contradiction 1= 0. These mean that the set of premises is inconsistent. There is no way to make all the premises true at the same time. Moreover, the given set of premises validly yields any conclusion, no matter how irrelevant. For example, consider the statements:

$D =$ The airplane landed in Dammam,

$\overline{D} =$ The airplane did not land in Dammam;

Since *CS*(*f*) = 1, and the term 1 is subsumed by any term including each of the terms $\overline{\mathrm{D}}$ and D, then each of the results ($\overline{D} = 0$) and ($\mathrm{D} = 0$) follows, leading to a simultaneous paradoxical assertion of the irrelevant statement D and its denial $\overline{\mathrm{D}}$.

CONCLUSIONS

This paper describes the modern syllogistic method, which ferrets out from a given set of premises all the consequents that can be concluded from it, and casts these consequents in the simplest compact form. The modern syllogistic method is similar to all other techniques of propositional logic in two respects: (a) it deals with arguments of many varieties on many topics including science, engineering, medicine, ethics, games, and simple affairs of everyday life, (b) it concerns itself only with the form or quality of the arguments it handles and has nothing to do with their subject matter. The modern syllogistic method distinguishes itself, however, among techniques of propositional logic, since it is the most powerful method among them and it encompasses each other's technique as a special case. We believe that the modern syllogistic method can serve as a useful tool for any researcher, as it can help him reason well about his discipline. A person mastering the method cannot be guaranteed to reason well or correctly; but he is more likely to reason correctly than one who is unaware of it. Partly this is because a person knowledgeable about the method can easily avoid the misuse of inconsistent premises to establish irrelevant conclusions, and can also detect many kinds of common formal fallacies or errors in reasoning.

A:

	AB	$A\overline{C}$	$ABCD$	$A\overline{B}C$	$\overline{B}C\overline{D}$
$\overline{A}B\overline{C}D$	$B\overline{C}D$	$B\overline{C}D$	---------	---------	

B:

	AB	$B\overline{C}D$	$A\overline{C}$
$A\overline{B}C$	AC	---------	
$\overline{B}C\overline{D}$	$AC\overline{D}$	---------	

C:

	AC	$\overline{B}C\overline{D}$	AB
$B\overline{C}D$	ABD	---------	
$A\overline{C}$	A	$A\overline{B}\,\overline{D}$	

D:

	$B\overline{C}D$	A
$\overline{B}C\overline{D}$	---------	

Fig. **1**. Derivation of the complete sum for f in (20.e) by the improved Tison method.

Ball \ Color	**Red= *r***	**White = *w***	**Blue = *b***
A	A_r	A_w $\{A_r\, A_w = 0\}$	$A_b = \overline{A_r}\, \overline{A_w}$
B	B_r $\{A_r\, B_r = 0\}$	B_w $\{A_w\, B_w = B_r\, B_w = 0\}$	$B_b = \overline{B_r}\, \overline{B_w}$ $\{A_b\, B_b = \overline{A_r}\, \overline{A_w}\, \overline{B_r}\, \overline{B_w} = 0\}$
C	$C_r = \overline{A_r}\, \overline{B_r}$	$C_w = \overline{A_w}\, \overline{B_w}$ $\{C_r\, C_w = \overline{A_r}\, \overline{B_r}\, \overline{A_w}\, \overline{B_w} = 0\}$	$C_b = A_r\, B_w \vee A_w\, B_r$

Fig. 2. The switching variables to symbolize the 3 color possibilities for each of the 3 balls.

A_r :

	$A_r \overline{B_r}$	$A_r \overline{A_w}\, \overline{B_w}$	$A_r A_w$	$A_r B_r$	$\overline{B_r}\, \overline{B_w}$ $A_w B_w$ $B_r B_w$
$\overline{A_r}\, \overline{B_r}$	B_r	$\overline{A_w}\, \overline{B_r}\, \overline{B_w}$	$A_w \overline{B_r}$	---	
$\overline{A_r} A_w B_r$	---	---	$A_w B_r$	$A_w B_r$	

B_r :

	$A_r B_r$	$A_w B_r$	$B_r B_w$	$A_r \overline{A_w}\, \overline{B_w}$ $A_r A_w$ $A_w B_w$
$\overline{B_r}$	A_r	A_w	B_w	

Fig. 3. Derivation of the complete sum of *f* in (43) by the improved Tison method.

y	Y	J	K
0	0	0	d
0	1	1	d
1	0	d	1
1	1	d	0

Fig. 4. Excitation table of the JK flip flop.

Y	y	J	K
0	0	0	d
0	1	d	1
1	0	1	d
1	1	d	0

Fig. 5. A rearrangement of the excitation table of the JK flip flop that facilitates the verification of the example conclusions.

Table 1. Summary of Replacement Rules (Logically Equivalent Expressions).

Rule name	Antecedent (premise)	Consequent (conclusion)
DOUBLE NEGATION	A	$\overline{\overline{A}}$
DUPLICATION (IDEMPOTENCY) (TAUTOLOGY)	A	$A \vee A$
	A	$A \wedge A$
COMMUTATION	$A \vee B$	$B \vee A$
	$A \wedge B$	$B \wedge A$
ASSOCIATION	$(A \vee B) \vee \mathrm{C}$	$A \vee (B \vee C)$
	$(A \wedge B) \wedge \mathrm{C}$	$A \wedge (B \wedge C)$
TRANSPOSITION (CONTRAPOSITION)	$A \rightarrow B$	$\overline{B} \rightarrow \overline{A}$
DE MORGAN'S	$\overline{(A \vee B)}$	$\overline{A} \wedge \overline{B}$
	$\overline{(A \wedge B)}$	$\overline{A} \vee \overline{B}$
MATERIAL EQUIVALENCE (BICONDITIONAL EXCHANGE)	$A \equiv B$	$(A \rightarrow B) \wedge (B \rightarrow A)$
	$A \equiv B$	$(A \wedge B) \vee (\overline{A} \wedge \overline{B})$
MATERIAL IMPLICATION (CONDITIONAL EXCHANGE)	$A \rightarrow B$	$\overline{A} \vee B$
DISTRIBUTION	$A \wedge (B \vee C)$	$(A \wedge B) \vee (A \wedge C)$
	$A \vee (B \wedge C)$	$(A \vee B) \wedge (\mathrm{A} \vee \mathrm{C})$
EXPORTATION	$(A \wedge B) \rightarrow \mathrm{C}$	$A \rightarrow (B \rightarrow C)$

Table 2. Summary of Rules of Inference (Elementary Valid Argument Forms).

Rule name	Antecedents (premises)	Consequent (conclusion)
MODUS PONENS	$A \rightarrow B$ A	B
MODUS TOLLENS	$A \rightarrow B$ $\overline{B}$	$\overline{A}$
HYPOTHETICAL SYLLOGISM	$A \rightarrow B$ $B \rightarrow C$	$A \rightarrow C$
SIMPLIFICATION (SPECIALIZATION))	$A \wedge B$	A
	$A \wedge B$	B
CONJUNCTION	A B	$A \wedge B$
CONSTRUCTIVE DILEMMA	$A \rightarrow B$ $C \rightarrow D$ $A \vee C$	$B \vee D$
DISJUNCTIVE SYLLOGISM	$A \vee B$ $\overline{A}$	B
	$A \vee B$ $\overline{B}$	A
ADDITION	A	$A \vee B$
	B	$A \vee B$
ABSORPTION	$A \rightarrow B$	$A \rightarrow AB$
CASES	A $A \rightarrow (C \vee D)$ $C \rightarrow B$ $D \rightarrow B$	B
CASE ELIMINATION	$A \vee B$ $A \rightarrow (C \wedge \overline{C})$	B
REDCTIO AD ABSURDUM (CONTRADICTION)	$\overline{A} \rightarrow (B \wedge \overline{B})$	A

Table 3. Proof that each of the Rules of Inference is derivable by the Syllogistic Method {The particular conclusion of a rule is highlighted in bold}.

Rule name	Premises as separate equations $f_i = 0, i = 1, 2, \ldots$	Premises as a single equation $f = 0$	Conclusions as a single equation $\mathrm{CS}(f) = 0$	Conclusions as separate equations $p_i = 0, i = 1, 2, \ldots$
MODUS PONENS	$A\bar{B}=0$ $\bar{A}=0$	$A\bar{B} \vee \bar{A}=0$	$\bar{B} \vee \bar{A}=0$	$\bar{\mathbf{B}}= 0$ $\bar{A}=0$
MODUS TOLLENS	$A\bar{B}=0$ $B=0$	$A\bar{B} \vee B=0$	$A \vee B=0$	$\mathbf{A}= 0$ $B=0$
HYPOTHETICAL SYLLOGISM	$A\bar{B}=0$ $B\bar{C}=0$	$A\bar{B} \vee B\bar{C}=0$	$A\bar{B} \vee B\bar{C} \vee A\bar{C}=0$	$A\bar{B}=0$ $B\bar{C}=0$ $\mathbf{A}\bar{\mathbf{C}}= 0$
SIMPLIFICATION	$\bar{A} \vee \bar{B}=0$	$\bar{A} \vee \bar{B}=0$	$\bar{A} \vee \bar{B}=0$	$\bar{\mathbf{A}}= 0$ $\bar{\mathbf{B}}= 0$
CONJUNCTION	$\bar{A}=0$ $\bar{B}=0$	$\bar{A} \vee \bar{B}=0$	$\bar{A} \vee \bar{B}=0$	$\bar{\mathbf{A}} \vee \bar{\mathbf{B}}= 0$
CONSTRUCTIVE DILEMMA	$A\bar{B}=0$ $C\bar{D}=0$ $\bar{A}\bar{C}=0$	$A\bar{B} \vee C\bar{D} \vee \bar{A}\bar{C}=0$	$A\bar{B} \vee C\bar{D} \vee \bar{A}\bar{C}$ $\vee \bar{B}\bar{C} \vee \bar{A}\bar{D}$ $\vee \bar{B}\bar{D}=0$	$A\bar{B}=0$ $C\bar{D}=0$ $\bar{A}\bar{C}=0$ $\bar{B}\bar{C}=0$ $\bar{A}\bar{D}=0$ $\bar{\mathbf{B}}\bar{\mathbf{D}}= 0$
DISJUNCTIVE SYLLOGISM	$\bar{A}\bar{B}=0$ $A=0$	$\bar{A}\bar{B} \vee A=0$	$\bar{B} \vee A=0$	$\bar{\mathbf{B}}=\mathbf{0}$ $A=0$
	$\bar{A}\bar{B}=0$ $B=0$	$\bar{A}\bar{B} \vee B=0$	$\bar{A} \vee B=0$	$\bar{\mathbf{A}}= 0$ $B=0$
ADDITION	$\bar{A}=0$	$\bar{A}=0$	$\bar{A}=0$	$\bar{A}=0$ Any subsuming term $=0$ $\bar{\mathbf{A}}\bar{\mathbf{B}} = 0$
	$\bar{B}=0$	$\bar{B}=0$	$\bar{B}=0$	$\bar{B}=0$ Any subsuming term $=0$ $\bar{\mathbf{A}}\bar{\mathbf{B}} = 0$
ABSORPTION	$A\bar{B}=0$	$A\bar{B}=0$	$A\bar{B}=0$	$A\bar{B}=0$ *replaced by* $A(\bar{A} \vee \bar{B})=0$
CASES	$\bar{A}=0$ $A\bar{C}\bar{D}=0$ $C\bar{B}=0$ $D\bar{B}=0$	$A \vee A\bar{C}\bar{D}$ $\vee C\bar{B}$ $\vee D\bar{B}=0$	$\bar{A} \vee \bar{B}$ $\vee \bar{C}\bar{D}=0$	$\bar{A}=0$ $\bar{\mathbf{B}}= 0$ $\bar{C}\bar{D}=0$
CASE ELIMINATION	$\bar{A}\bar{B}=0$ $A(\bar{C} \vee C)=0$	$\bar{A}\bar{B} \vee A=0$	$\bar{B} \vee A=0$	$\bar{\mathbf{B}}=0$ $A=0$
REDUCTIO AD ABSURDUM	$\bar{A}(\bar{B} \vee B)=0$	$\bar{A}=0$	$\bar{A}=0$	$\bar{A}=0$

REFERENCES

Al-Maidani, A. H. H., 1993. Knowledge Constraints and Fundamentals of Deduction and Debating: A Formulation of Logic and Research Fundamentals Compatible with Islamic Ideology, Darul Qalam, Damascus (In Arabic), 471 pp.

Anderson, J. A., 2003. *Discrete Mathematics with Combinatorics* (Second Edition). Prentice Hall, Upper Saddle River, NJ, 928 pp.

Boole, G., 1847. *The Mathematical Analysis of Logic*. G. Bell, London. (Reprinted by Thoemmes Press, 1998, 96 pp.).

Boole, G., 1854. *An Investigation of the Laws of Thought*. Walton, London. (Preprinted by Dover Publications, Mineola, NY, 1954, 424 pp).

Brown, F. M., 1974. "Equational Logic", IEEE Transactions on Computers, C-23(12): 1228-1237.

Brown, F. M. 1975. "Equational realizations of switching functions", IEEE Transactions on Computers. C-24(11): 1054-1066.

Brown, F. M. 1990. *Boolean Reasoning: The logic of Boolean Equations*, Kluwer Academic Publishers, Boston, MA, 273 pp. (Second Edition, 2003, by Dover Publications, Mineola, NY, 291 pp.).

Carroll, L., 1955. Symbolic Logic (Fourth edition), Berkeley Enterprises, 199 pp.

Chandru, V. & Hooker, J. N., 1999**.** Optimization Methods for Logical Inference, Wiley, New York, NY, 365 pp.

Chang, Chin-Liang & Lee, R. C. T., 1997. *Symbolic Logic and Mechanical Theorem Proving*, Academic Press, New York, NY, 331 pp.

Copi, I. M. 1979. *Symbolic Logic* (Fifth edition), Prentice hall, New York, NY, 411 pp.

Copi, I. M. & Cohen, C., 2002. *Introduction to Logic* (Eleventh Edition), Prentice-Hall, Upper Saddle River, NJ, 671 pp.

Gregg, J. R., 1998. *Ones and Zeroes: Digital Circuits and the Logic of Sets*, Wiley, New York, NY, 296 pp.

Haken, A., 1985. "The intractability of resolution. Theoretical Computer Science", 39: 297-308.

Kamel, O, A., 2004. *A Note on Logic Simplification*, Bisan Publications, Beirut (*In Arabic*), 117 pp.

Klenk, V., 2007**.** *Understanding Symbolic Logic* (Fifth Edition), Prentice-Hall, Englewood Cliffs, NJ, 480 pp.

Klir, G. J & Marin M. A., 1969. "New considerations in teaching switching theory", IEEE Transactions on Education. E-12(4): 257-261.

Kuo, B. C., 1995. *Automatic Control* (Seventh edition), Wiley, New York, NY, 928 pp.

Lynch, E. P., 1980. *Applied Symbolic Logic*, New York, NY, 272 pp.

Muroga, S., 1979. *Logic Design and Switching Theory*. Wiley, New York, NY, 618 pp.

Nelson, I. T., Ratliff, R. L., Steinhoff, G. & Mitchell, G. J., 2003. "Teaching logic to auditing students: can training in logic reduce audit judgment errors", *Journal of Accounting Education*. 21: 215-237.

Robinson, J. A., 1965. "A machine oriented logic based on the resolution principle", *Journal of the Association for Computing Machinery*. 12(1): 23-41.

Rushdi, A. M., 2001 (a). "Using variable-entered Karnaugh maps to solve Boolean equations", *International Journal of Computer Mathematics*, 78: 23-38.

Rushdi, A. M., 2001 (b). "Prime-implicant extraction with the aid of the variable-entered Karnaugh map", *Umm Al-Qura University Journal: Science, Medicine and Engineering*, 13(1): 53-74.

Rushdi, A. M. & Al-Yahya, H. A., 2001. "Derivation of the complete sum of a switching function with the aid of the variable-entered Karnaugh map", *King Saud University Journal: Engineering Sciences*, 13(2): 239-269.

Rushdi, A. M. & Al-Shehri, A. S., 2002. "Logical reasoning and its role in serving security and justice", *Security Research Journal* 11(22): 114-153 (In Arabic with an English abstract).

Rushdi, A. M. & BaRukab, O. M., 2007. "Some Engineering Applications of the Modern Syllogistic Method", SEC7 Paper 226. *Proceedings of the Seventh Saudi Engineering Conference* (SEC7), Riyadh, KSA, 4(2007): 389-401.

Rushdi A. M. & BaRukab, O. M., 2008. "The Modern Syllogistic Method as a Tool for Engineering Problem Solving", *Journal of Engineering and Computer Science*, Qassim University. 1(1): 57-70.

Russel, S. & Norvig, P., 2002. *Artificial Intelligence: A Modern Approach* (Second Edition), Prentice-Hall , Englewood Cliffs: NJ, 1132 pp.

Wheeler, R. F., 1981. *Rethinking Mathematical Concepts*. Prentice Hall Europe, 314 pp.

Winnie, J. A., 1970. "The completeness of Copi's system of natural deduction", Notre Dame Journal of Formal Logic. (11): 379-382.

Received 10/11/1428; 20/11/2007, accepted 8 /1/1430; 6/1/2009