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Impulsive Linear Extensions of Dynamical Systems on Torus

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نظام المجموعات الديناميكية النبضية الموسعة على حلقة

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قسم الرياضيات – الكلية الجامعية بالقنفذة – جامعة أم القرى- القنفذة – المملكة العربية السعودية

من خلال دراسة سلوك المجموعات الديناميكية النبضية على حلقة، تظهر لدينا دراسة أخرى هي الاستقرار الآسي والتفرع الثنائي (الانقسام الآسي) لهــذه المجموعات وذلك من خلال قوانيــن مجموعة المصفوفات الخطية.

ويجب علينا من خلال هذا البحث أن نراعي شروط لهذه المجموعات وهي شروط ليبشيتز للدوال الدورية الموجودة، وذلك بفرض أن هذه الدوال مستمرة على فضاء مفروض.

من خلال هذا البحث تظهر لدينا أيضا شروط على الاستقرار وذلك بوجود حلول لجملة المعادلات المدروسة ومنها الحل الصفري وأن هذه الحلول لا تملك نقاط حرجة، ومع العلم أن هذه الدراسة تتم بوجود تأثير نبضي أو ما يسمى- بتعبير آخر- دفع لحظي.

Abstract

Throughout studying the behavior of the dynamical impulsive systems on a torus, another study appears to us. It is the stability and the dichotomy of these systems, from the laws of the systems of the linear matrices.

Therefore, we must throughout this research, be careful in applying the Lipchitz conditions for systems of periodical sequences, which are existed. And this is by the supposition that these sequences are continuing on subspace.

Also from this research the conditions on stability appear to us, by existing the solution to a collection of equations that are studied. And this solution, is the zero solution.

These solution also have not troublesome points. And this research is studied completely by the existence of impulsive influence on which is called the momental impulsive.

Keywords:
Differential equations; linear matrix; linear differential equations; Impulsive equations; dynamical system

INTRODUCTION

In physics sciences , and technique sciences and other field of the sciences ,we found private problems seek for the dynamical systems[7], and these problems possess impulsive characteristics or called impulsion characteristics.
But in developed mathematics this process is defined called as a term of differential equations under influence of pulse [1]. This is in the empirically continuous methods of the watches design by Kerilov - Bhalobov[4].
In the nineteenth century and early of twentieth century through the theorem of differential equations either linear differential equation or nonlinear not give any importance to impulsive characteristics.
Almost the oscillation motions or circulation has impulsive characteristics to suffer from data curves[1]. For this differential groups with influence of impulsive characteristics distinguish would be process through the paragraphs of this research. Also we will discuss some problems of the linear theorems under influence of pulse.
The exponential dichotomy we connect the helpful treatments with linear matrices to give us in the conclusion true frequencies. These treatments have dichotomy influence, because this connection has two opposite linear matrices.
During the study of these dynamical systems, in condition of stability and exponential dichotomy appeared to us, and form these the usefulness on the matrix of exponential stability, and the usefulness of The function of Green- Samoilenko of exponential dichotomy[3].

Consider a system of differential equations with impulsive influence

$$\frac{d\varphi}{dt} = a(\varphi)$$
$$\frac{dx}{dt} = P(\varphi)x \quad ; \quad \varphi \in T_m \qquad (1)$$
$$\Delta\varphi\Big|_{\varphi\in T_t} = B(\varphi)x$$

$\Gamma = \{\varphi \in T_m : \psi(\varphi) = 0\}$ dimensional torus;- where $x \in R^n$, $\varphi \in T_m$; m , also $a(\varphi)$and$B(\varphi)$are periadof 2π $\Delta\varphi \equiv q(t+0) - q(t-o)$periodic functions from the space. and$C^1(T_m)$.
In present paper we study the conditions of the exponential stability and exponential $x = 0$,and $\varphi \in T_m$dichotomy of the trivial tours: of system (I) for the systems without impulsive. .Γ First let us introduce some restriction to the surface of influence$\varphi_0(\varphi) = \varphi$ the solution of the first equation (1), such that , $\varphi_t(\varphi)$We denote. Then it is obvious, that the moments of impulsive influence are determined by zero's $\psi(\varphi_t(\varphi)) = 0$.of the equation.

Arranging this in ascending order, we obtain, which elements fulfilled to such equality $\{\tau_t(\varphi)\}$ sequence

$$\tau_i(\varphi_\theta(\varphi))+\theta=\tau_i(\varphi) \qquad (2)$$

has not finite limit points T in the torus are $\{\tau_i\}$ Next we suppose that, the sequence transversal, i.e.

$\frac{\partial\psi}{\partial\varphi}(\varphi_t)a(\varphi_t)\neq 0$, $\forall\varphi\in T_m$. of the system of equations is called $x=0$, $\varphi\in T_m$

Definition. An invariant torus exponentially dichotomous and the system of equations itself exponentially, the space R^n can be represented as a direct $\varphi_0\in T_m$ dichotomous whenever for any of the dimensions *r,n-r* ,in such a way any solution $R^+(\varphi_0)$ *and* $R^-(\varphi_0)$ sum of satisfies the inequality $x_0\in E^+(\varphi_0)$ of system(1) with $\varphi_t(\varphi_0), x_t(\varphi_0,x_0)$

$$\|x_t(\varphi_0,x_0)\|\leq Ke^{-j(t-\tau)}\|x_0(\varphi_0,x_0)\| \quad ; \; t\geq\tau \qquad (3)$$

satisfies the inequality $\varphi_t(\varphi_0), x_t(\varphi_0,x_0)$ with $x_0\in E^-(\varphi_0)$ And any solution

$$\|x_t(\varphi_0,x_0)\|\leq K_1e^{j(t-\tau)}\|x_0(\varphi_0,x_0)\| \quad ; \; t\leq\tau \qquad (4)$$

And some positive constants k , j , k_1 independent of $\tau\in R^1, \varphi_0\in T_m$ For arbitrary. φ_0, x_0, τ is called exponentially stable . $\varphi_0\in T_m$ then the torus $x=0$, *if* $R^n=R^+$,
Similar problem at the terms of matrix are study early in [2] , for quase periodic winding in [3] .

At first we find out the condition of exponential stability of invariant torus h=0 of system (1) . When the impulse are absent , i.e. when system (1) is of the following form:

$$\frac{d\varphi}{dt}=a(\varphi) \quad , \quad \frac{dx}{dt}=P(\varphi)x \qquad (5)$$

Such conditions are obtained both in the terms of functions of Green- Samoilenko and functions of Lyapunov [1] . Next we suppose, that the system accordant to (1) without impulsive action (5) possesses exponential stable invariant w with the coefficient of shade in $K>0$ and exponent of shade in $j>0$. $x=0$, $\varphi\in T_m$ toeus

Theorem 1. Let the system (5) is exponentially stable (with the parameter K) $\|E+B(\varphi)\|\leq\frac{1}{K}$ for $\varphi_0\in T_m$; $E-$ unit matrix, then the system (1) *is* .and, if exponential stable.

Proof. The exponential of the system (5) means that for every solution, the inequality $\varphi_t(\varphi_0), x_t(\varphi_0, x_0)$ *satisfying the condition* $\varphi_0(\varphi_0) = \varphi_0,\ x_0(\varphi_0, x) = x$

$$\left\| x_t(\varphi_0, x) \right\| \le K e^{-j(t-\tau)} \left\| x_0(\varphi_0, x) \right\| \qquad (6)$$

$\tau \le t$, and $\varphi \in T, x \in R^n, \tau \in R^1$. is satisfied for all.

We shall analogous estimate for the solutions of system (1), in this aim .we assume the arbitrary now is defined there is define the fix $\varphi_t(\varphi_0), x_t(\varphi_0, x_0)$ of system (1).Since $\varphi_t(\varphi)$ solution at the every of $x_t(\varphi_0, x)$ such that $t_i = \tau_i(\varphi_0)$ moments of impulsive influence accordingly to the $x_t(\varphi_0, x)$ contain from "Parts" of solutions $[t_j, t_{j+1})$ intervals $\tau \in [t_j, t_{j+1})$, where t_j, t_{j+1} – moments of let $\tau \in R^1$ system (5) . For arbitrary impulsive influence.

Therefore, we have $t \in [\tau, t_{j+1})$. Then , it is obviously , estimate (6) holds for

$$\left\| x_t(\varphi_0, x) \right\| \le K e^{-j(t-\tau)} \left\| x_0(\varphi_0, x) \right\|$$

i.e

$$x_{t_{j+1}+0}(\varphi_0, x) = \left(E + B\left(\varphi_{t_{j+1}}(\varphi_0)\right)\right) x_{t_{j+1}}(\varphi_0, x)$$

Hence

$$\left\| x_{t_{j+1}+0}(\varphi_0, x) \right\| \le \left\| E + B(\varphi) \right\| \cdot \left\| x_{t_{j+1}}(\varphi_0, x) \right\| \le \frac{1}{k} \left\| x_{t_{j+1}}(\varphi_0, x) \right\| \qquad (7)$$

we have $[t_{j+1}, t_{j+2})$ From the estimate (6) at the intervals

$$\left\| x_t(\varphi_0, x) \right\| \le \left\| x_{t_{j+1}+0}(\varphi_0, x) \right\| K e^{-j(t-t_{j+1})} \le \frac{1}{k} \left\| x_{t_{j+1}}(\varphi_0, x) \right\| K e^{-j(t-t_{j+1})} \le K e^{-j(t-\tau)} \left\| x_\tau(\varphi_0, x) \right\|$$

Continuing the spending reasoning we obtain the fulfilling of estimate (6) for all the theorem is proofed . $\tau \le t$

Next we study the problem of exponential dichotomy of the system (1) . We shall Suppose again that the system without impulses (5) is exponentially dichotomous The present condition is equivalent to the existence of the function of Green- Samoilenko [1] .

$$G_0(\tau, \varphi) = \begin{cases} \Omega_\tau^0(\varphi) C(\varphi_\tau(\varphi)) & for \quad \tau \le 0 \\ -\Omega_\tau^0(\varphi)[E - C(\varphi_\tau(\varphi))] & for \quad \tau > 0 \end{cases} \qquad (8)$$

Satisfying the inequality

$$\left\| G_0(\tau, \varphi) \right\| \le K e^{-j|\tau|} \quad ; \quad \tau \in R^1 \qquad (9)$$

where we denote:

$$C(\varphi) = G_0(0, \varphi) \qquad (10)$$

is projector, therefore its Eigen value is equal to 1 or 0. $C(\varphi)$ It is known that the matrix.

We connect the problems of exponential dichotomy with the eigenvectors [8] of matrix defined as $C(\varphi)$

$$C(\varphi)x = x \qquad (M^{+}) \qquad (11)$$
$$C(\varphi)x = 0 \qquad (M^{-}) \qquad (12)$$

satisfying $T_m \times R^n$, of the phase space (φ, x) We denote by M^{+}, the set of points. The equality (11), and set (12), by M^{-}.
We note, that since it might be considering the evolution of system to left for exponential dichotomy, then, it is necessary, to suppose the under generation of the it is hold the following theorem. $(E + B(\varphi))$ *for all* $\varphi \in T_m$. matrix

Theorem 2 . Let system (5) is exponentially dichotomous, such that $M^{-}\big|_{\varphi_0=const}$, and $B(\varphi)$ is invariant with respect to the operator $M^{+}\big|_{\varphi_0=const}$ holds estimate $M^{+}\big|_{\varphi_0=const}$ $(E + B(\varphi))^{-1}$,such that for is invariant with respect to

$$\|(E + B(\varphi))x\| \le \frac{1}{K}\|x\| \quad ; \quad \varphi \in T_m \qquad (13)$$

the following estimate is true $M^{-}\big|_{\varphi_0=const}$ And for

$$\left\|(E + B(\varphi))^{-1} x\right\| \le \frac{1}{K}\|x\| \quad ; \quad \varphi \in T_m \qquad (14)$$

The system (1) is exponential dichotomous.[5] respect to $B(\varphi)$ Not. it is obviously, that the condition of invariant of operator and $C(\varphi)$ is hold, if the matrix $M^{-}\big|_{\varphi_0=const}$ and $M^{+}\big|_{\varphi_0=const}$ the subspace $(E + B(\varphi))$. is un degenerate for $B(\varphi)$ are commutative and $(E + B(\varphi))^{-1}$ and $C(\varphi)$ and also . $\varphi \in T_m$ and $\varphi_t(\varphi_0), x_t(\varphi_0, x_0)$ of system (1).
proof. We take, again, the arbitrary solution impulse for present solution. $\tau \in [t_j, t_{j+1})$, where t_j, t_{j+1} − moments of let $\tau \in R^1$ for Since the solutions of system (1) contain from parts of solution of system (5), then the estimate (3) is hold. Hence according to $x \in M^{+}\big|_{\varphi_0=const}$ and for $[\tau, t_{j+1})$. at the interval $x(t_{j+1} + 0) = x(t_{j+1}) + B(\varphi_{t_j}(\varphi_0)) \cdot x(t_j) \in M^{+}\big|_{\varphi_0=const}$ condition the estimate (3) is hold. Next the proof $[\tau, t_{j+1})$. and to condition (13) at the interval the estimate (4) is analogous proved. $x \in M^{-1}\big|_{\varphi_0=const}$ is analogous to proof of 1. for The theorem is proved From proof of theorem 2 it is seen, that the impulsive influence may corrupt the dichotomy of the system (1), to - knock out - the solutions from the separatrix manifolds.

But one may to choose impulsive influence in such away, that to make exponential dichotomous tours from unstable one. It 's may be, for example, in the case when the. We give accordant example $M^{+}\big|_{\varphi_0=const}$ will be projector at $(E + B(\varphi))$operator. Let the system (1) has the following view[6]

$$\begin{aligned} \frac{d\varphi}{dt} &= \omega \quad ; \qquad \omega = const \\ \frac{dx}{dt} &= A\,x \\ \Delta\varphi\Big|_{\omega t+\varphi_0=2\pi k\ ,\ k\in Z} &= Bx \end{aligned} \qquad (15)$$

where

$$A=\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \quad , \quad B=\begin{pmatrix} a & b \\ 0 & -1 \end{pmatrix} \ ; \ |1+a|+|b|\le 1$$

-[6] $|1+a|+|b|\le 1$ Such that of system without impulses is unstable $x=0\ ,\ \varphi\in T_m$ It obviously, that the torus of the system with $x=0\ ,\ \varphi\in T_m$ (exponentially dichotomous). But the torus impulses will be exponentially stable . It is really, that the solution of system (15) $\varphi_t(\varphi_0), x_t(\varphi_0,x)$ is moments of impulses for solution t_j, t_{j+1}, where $\lfloor t_j, t_{j+1})$ when has such view,[9].

$$\varphi_t(\varphi_0)=\omega_t+\varphi_0$$

$$x_t(x)=\begin{pmatrix} x_0^1 & e^{-t-t_j} \\ x_0^2 & e^{t-t_j} \end{pmatrix}$$

we hold [9] $t\in(t_{j+1}, t_{j+2}\rfloor$ Then for

$$x_{t_{j+t}+0}=\begin{pmatrix} 1+a & b \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x_0^1 & e^{-t} \\ x_0^2 & e^{t} \end{pmatrix}=\begin{pmatrix} (1+a)\cdot x_0^1\cdot e^{-t_{j+1}-t_j}+b\cdot x_0^2\cdot e^{t_{j+1}-t_j} \\ o \end{pmatrix}$$

where $\|x_{t_{j+t}+0}\|\le\|x_{t_{j+t}}\|$

we have $t\in(t_{j+1}, t_{j+2}\rfloor$ Next, when

$$x(t)=\begin{pmatrix} y_0\cdot e^{-t-t_{j+1}} \\ o \end{pmatrix}$$

where $y_0=(1+a)\cdot x_0^1\cdot e^{-t_{j+1}-t_j}+b\cdot x_0^2\cdot e^{t_{j+1}-t_j}$

$x=0\ ,\ \varphi\in T_m$ Whence it appears the be exponentially stability of the torus

DISCUSIONS

The results of this study may be used in vibrational systems exist in applied problems which depend on impulsive influence. However, then has been no considerable

importance for rotational or vibrational motions in linear or nonlinear differential equations since 19th century and early beginnings of the 20th centaury. For, actually, the graphical curves of these
motions have been suffering from impulsive influence as dynamic impulses given Δx .
$\Delta x = q(t+0) - q(t-0)$
Throughout the analysis of results, these results may be used in general context of these large and open issues .

RESULTS

Based on various questions and mathematical problems that arise, which has not been answered yet, the exponential stability for impulsive dynamic system on torus has been studied, Using the system of linear matrices laws . Based on such study, the following results are concluded:

1- The behavior of exponential stability and branching dichotomy for dynamical systems on torus of linear matrices has been studied under impulsive influence.

2- Restrictions on impulsive has been reached during the reservation of exponential stability without impulsive influence.

Implications for further research

I hope from those who are interested in impulsive differential equations to contact on my e-mail to help solving some of the nelted mathematical problems, I have so many issues that are in need for solution in this field.

REFERENCES

Bermant A., 1975, Mathematical Analysis, Moscow, Mir.

Elsgolts L., 1973, Differential Equations and the Calculus of Variations Moscow, Mir.

Eruge N. Y, 1972, An Elementary Course of Differential Equations, Mensk .SSR.

Gohn A. T, 1989, Differential Equations.

Metropolske U. F, 1989, Virtual rotational surfaces and the conditions of the system existence under impulsive influence, Ukrane –Kiev, *Magazine of scientific mathematics*, **10**:1302-1313.

Richard B., 1997, Schaum,s 2500 Solved Problems In Differential Equations, Academia Arabic Copyright.

Samoilenko A. M, 1987, Vibrational Frequencies and Virtual Torus Faculty of Science - Ukrane -Kiev

Samoilenko A. M, Perestyuk N. A., 1995, Impulsive Differential Equations, New Jersey,

London, Hong Kong – *World - Scientific Publ.*, **14**:462-.
William R.D, 1984, Elementary Differential Equations with Applications, London.

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